

THE *Arithmetician's Guide.*

In Two PARTS.

PART I.

Containing Practical ~~ARITHMETIC~~ thro' all its several Denominations; in EXAMPLES wrought at large, and in a plain and easy Manner exhibited, and the Work of FRACTIONS, both *Vulgar* and *Decimal*, compleated.

PART II.

Containing the Principles of ALGEBRA, in a more plain and intelligible a Manner, than any heretofore extant.

Illustrated in the Numerical and Literal SOLUTION of many EXAMPLES, both in *Simple* and *Quadratic Equations*. Wherein every Process, both by Figures and Letters, are from Rules, so plain, easy, and clear, that Persons of a mean Understanding may comprehend them.

By THO. CROSBY,

Teacher of the MATHEMATICS, upon *Horsely-Down*,
in SOUTHWARK.

L O N D O N :

Printed for the AUTHOR; and sold by JOHN CLARK,
Bookseller, under the *Royal Exchange*; and JOHN
WILCOX, Bookseller, near the New Church in the Strand.

M. DCC. XLVI.

512

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Miss B. H. L.

Lincoln Green and
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induce the *World* to be,
at least, civil to it.

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but I chose rather to address them to
you, as it was your Encouragement
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are but nominally so; and generally
make but small Reckoning of such
Presents made to them.

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GENTLEMEN,

Your most Obliged,

Most Obedient, and

Very Humble Servant,

Horsely-down, in
SOUTHWARK,
Sept. 20, 1746.

THO. CROSBY.



T H E
P R E F A C E.

I*T must be granted, that Arithmetic and Geometry, are the Foundations of all useful Sciences. Plato therefore very justly called them the two Wings whereby the Minds of Men might mount up to Heaven, that is, in searching after the Motions and Properties of the Celestial Bodies.*

It were to be wished, that these Sciences only were taught in all our Writing-Schools, and the learned Gentlemen in the Languages left wholly to their own Province: For the learning of Grammar, blended with Writing and Accompts, too much practised in our common Schools, apparently spoils both; and the Generality of Youth, after six or seven Years Exercise in this Manner, turn out with Qualifications hardly fit for the meanest Employments.

To

The P R E F A C E.

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This Treatise sets forth the Method only I have used, in the Principles of Arithmetic and Algebra. In which is first exhibited, the general Rules of Numeration, Addition, Subtraction, Multiplication, and Division, in whole Numbers; and the same applied to Money, Weights, and Measure.

Then follows an easy, plain, and practical Method, to determine the Amount of any Quantity at any Price, by one general Rule; and some easy and very plain Rules to account for Commissions, Brokage, purchasing of public Stocks, the Interest of Money, and Exchanges.

In the next Place you have another Method, by one general Rule, to determine the Amount of any Quantity at any Price, which, in the Language of Arithmeticians, is generally stiled Practice.

The P R E F A C E.

Practice. Then I proceed to shew the Use of Fractions, both Vulgar and Decimal; and because the Work of Decimals is the most useful, and differs not from whole Numbers, but in separating the Decimals from the Integers, I begin with Decimal Arithmetic, and apply the same to the Computation of the Duties payable at the Custom-House, on Importation and Exportation of Goods, and in extracting the Square and Cube Roots. Then follows Vulgar Fractions, in which all the Examples are likewise wrought decimally, to lead the Learner into a right Understanding of this excellent Rule of Decimal Arithmetic.

I conclude with some Examples in the various Rules of Arithmetic, generally stiled the Rule of Three, Direct, Inverse, Double Rule, Fellowship, Interest, &c. all which I comprehend under the Title of the Golden Rule. And here, to perfect the Knowledge and Use of Decimals, every Example is wrought also decimally, and compleats the first Part of this Treatise.

The second Part begins with Addition, Subtraction, Multiplication, and Division of Algebraic Quantities. Then follows the Rule of Proportion, the Reduction of Equations, and the Solution of many Examples, in Simple Equations;

THE PREFACE.

Equations; the Extraction of Roots by an Algebraic Theorem, with Arithmetical and Geometrical Progression. The Interest of Money, the Computation of Annuities, or Pensions in Arrear, and the present Worth of them. And concludes with the Solution of many Examples in Quadratic Equations.

All which I have endeavoured to deliver in Rules so clear, plain, and easy, that he that runs may read, without confounding his Judgment, but really informing it: And therefore I commit it to the candid Reader, and in particular to those who have been under my Instructions.

Harrison



1807

For the printer.

On 6. 11. 92

THE



THE
Arithmetician's Guide.

ARITHMETIC



Is an Art of Accompting by *Numbers*, and doth consist of 5 general Parts, *viz.* *Numeration*, *Addition*, *Subtraction*, *Multiplication*, and *Division*.



NUMERATION

TEACHETH to read, express, or write down any *Sum* or *Number*, propounded, known, or named, and doth consist of 2 Parts, *viz.*

1. The due Order of placing down *Figures*.
2. The true valuing of each *Figure* in its Place.

Hundreds of Thousands.	Tens of Thousands.	Thousands.	Hundreds.	Tens.	Units.
1	2	3	4	5	6

By this *Table* it is plain, That the *Figure* standing in the Place of *Units*, doth signify so many *Units* as
 B the

2 NUMERATION.

the *Figure* represents: And the *Figure* in the Place of *Tens*, so many *Tens*: And the *Figure* in the Place of *Hundreds*, so many *Hundreds*, &c. That is, 6 *Units*, 5 *Tens*, (or fifty) 4 *Hundreds*, 3 *Thousands*, 2 *Tens* of *Thousands*, (or twenty Thousand) 1 *Hundred Thousand*. The whole to be read or expressed together, thus, viz.

One Hundred Twenty-three Thousand, Four Hundred Fifty-six.

Note. In reading *Numbers*, you must always begin with the *first Figure* towards the Left Hand, and so many *Figures* as are placed together in a Line, without any Point or Note of Distinction between them, are all but *one Sum*; and must be read as such: As in the *Examples* following.

Example 1	- - - - -	759423
2	- - - - -	86427
3	- - - - -	9503
4	- - - - -	270
5	- - - - -	94
6	- - - - -	8

The first *Example* is read thus, *Seven Hundred Fifty-nine Thousand, Four Hundred Twenty-three.*

The second *Example* thus, *Eighty-six Thousand, Four Hundred Twenty-seven.*

The third thus, *Nine Thousand, Five Hundred and Three.*

The fourth thus, *Two Hundred and Seventy.*

The fifth thus, *Ninety-four.*

The sixth thus, *Eight.*

But if the *Sum* consist of more than 6 *Figures*, place a Period, or Comma, over the *sixth Figure*, (counting from the Right Hand towards the Left) and the other *Figures*, to the Number of *six*, will be so many *Millions*; as in the following *Examples*.

Example

Example 1 - - - - 297671485321

2 - - - - - 27192537129

3 - - - - - 5072860724

4 - - - - - 570200683

5 - - - - - 97124654

6 - - - - - 8307291

In the first *Example*, the *Sum* to the *Period*, over the *sixth Figure*, is read (according to the *Table*) thus, *Two Hundred Ninety-seven Thousand, Six Hundred Seventy-one*; which is so many *Millions*: So that the whole *Example* must be thus read, *Two Hundred Ninety-seven Thousand, Six Hundred Seventy-one Million, Four Hundred Eighty-five Thousand, Three Hundred Twenty-one*.

The second *Example* is read thus, *Twenty-seven Thousand, One Hundred Ninety-two Million, Five Hundred Thirty-seven Thousand, One Hundred Twenty-nine*.

The *Third* thus, *Five Thousand Seventy-two Million, Eight Hundred Sixty Thousand, Seven Hundred Twenty-four*.

The *Fourth* thus, *Five Hundred Seventy Million, Two Hundred Thousand, Six Hundred Eighty-three*.

The *Fifth* thus, *Ninety-seven Million, One Hundred Twenty-four Thousand, Six Hundred Fifty-four*.

The *Sixth* thus, *Eight Million, Three Hundred Seven Thousand, Two Hundred Ninety-one*.

But if the *Sum* should be still farther increased, then by placing a *Comma*, or *Period*, over every *sixth Figure*, it may easily read, only observing; That from the *first Period* to the *second*, the *Sum* is so many *Millions*, from the *second* to the *third*, so many *Billions*, from the *third* to the *fourth*, so

4 NUMERATION.

many *Trillions*, &c. to the End of the annexed TABLE.

Millions		Let the following <i>Example</i> be considered, which I think will be sufficient to the understanding of the whole, viz.
Billions		
Trillions		
Quadrillions		
Quintillions		
Sextillions		1712346719402719467192312
Septillions		
Octillions		
Nonillions		To be read thus, viz.

One *Quadrillion*, Seven Hundred Twelve Thousand, Three Hundred Forty-six Trillion, Seven Hundred, Nineteen Thousand, Four Hundred Two Billion, Seven Hundred Nineteen Thousand, Four Hundred Sixty-seven Million, One Hundred Ninety-two Thousand, Three Hundred Twelve.

From whence it may be observed, That if the young Learner be first taught to read a *Sum*, to the Number of *six Figures*, how easily afterwards he may read any greater *Sum*.



ADDITION

IS the gathering, or collecting of many Numbers into one *Sum*, called the *Total*; and doth consist of three Varieties, viz. *Money*, *Weight*, and *Measure*: To which may be added *Time*: Which have *Tables* of the several denominative Parts belonging thereto, from the least to the greatest Denomination.

To add *Integers* together, you must carefully observe, to place *Units* under *Units*, *Tens* under *Tens*,

Tens, Hundreds under Hundreds, &c. and then proceed according to the following Rule.

R U L E.

First, add together all the *Figures*, that stand in the Place of *Units*; and if their Sum be under *Ten*, set it down below the Line under its own Place: But if their Sum be *Ten*, or above *Ten*, it consists of 2 *Figures*, one whereof, *viz.* the *Units*, must be set down, and the other, *viz.* the *Tens*, must be carried to the next Row of *Figures*, &c. as in the following Examples.

Exa. 1.	Exa. 2.	Exa. 3.
246719	3126719	42671982
<hr/>	<hr/>	<hr/>
327194	4213716	21467194
671283	9212744	32671942
421537	5267392	22467297
443126	7146837	53716729
571928	4159326	31246719
<hr/>	<hr/>	<hr/>
2681787	33126734	204241863
<hr/>	<hr/>	<hr/>
2435068	30000015	161569881
<hr/>	<hr/>	<hr/>
2681787	33126734	204241863

In the *first* Example, beginning at the Place of *Units*, I say, 8 and 6 is 14, and 7 is 21, and 3 is 24, and 4 is 28, and 9 is 37, I set down the 7 *Units*, and carry the 3 to the next Row of *Figures*, saying, 3 and 2 is 5, and 2 is 7, and 3 is 10, and 8 is 18, and 9 is 27, and 1 is 28, I set down 8, and carry 2 to the next Place, saying, 2 and 9 is 11, and 1 is 12, and 5 is 17, and 2 is 19, and 1 is 20, and 7 is 27, I set down 7, and carry 2 to the next Place, saying, 2 and 1 is 3, and 3 is 6, and 1 is 7, and 1 is 8, and 7 is 15, and 6 is 21, I set down 1, and carry 2 to the next Place, saying,

saying, 2 and 7 is 9, and 4 is 13, and 2 is 15, and 7 is 22, and 2 is 24, and 4 is 28, I set down 8, and carry 2 to the next Place, saying, 2 and 5 is 7, and 4 is 11, and 4 is 15, and 6 is 21, and 3 is 24, and 2 is 26. This being the last Row of *Figures*, I set down the whole Number 26, as in the *Example*.

The second and third *Examples*, are done after the same Manner.

The common Method of proving the Work of *Addition*, is, by separating the *first* Line from the Work, and adding together the *Residue*. Then to the *separated* Line add the *Total* of the *Residue*, it gives the *whole Sum*: The Reason of which is plain, from that natural Truth, of the *whole* being equal to all the Parts taken together.

The *Proof* of the first *Example*.

The separated Line is	246719
The Total of the Residue	2435068
	<hr/>
	2681787

Note. It may be sufficient in *Practice*, to prove *Addition*, by beginning at the Top, and adding together, the same Numbers downwards.



S U B T R A C T I O N

IS the taking of a *less* Number, from a *greater*, that so the *Remainder*, *Difference*, or *Excess*, may be known.

To *subtract* *Integers*, observe to place *Units* under *Units*, *Tens* under *Tens*, *Hundreds* under *Hundreds*, &c. and the *less* Number under the *greater*, and then follow this general Rule.

R U L E.

SUBTRACTION.

7

RULE.

First *subtract* the *lower* Figure in the *Units* Place, from the *upper* Figure in the *Units* Place, and set down the *Remainder*; but if the *lower* Figure be greatest, you must call the *upper* Figure *ten* more than it is, and *subtract*; always remembering, that when you have so done, you must *carry one* to the next Figure *below*, and proceed to *subtract* in that Place, as in the former; and so gradually on, from one Row of *Figures* to another, till all be done.

	Exa. 1.	Exa. 2.	Exa. 3.
From	7194672	9467294	8127953
Take	5171231	5371697	6257428
	<hr/>	<hr/>	<hr/>
Rem.	2023441	4095597	1870525

In Example the *first*, I begin with the *lower* Figure, in the *Units* Place, saying, 1 from 2, and there *remains* 1, I set down 1, and proceed to the next Place, saying, 3 from 7, and there *remains* 4, I set down 4, and proceed to the next Place, saying, 2 from 6, and there *remains* 4, I set down 4, and proceed to the next, saying, 1 from 4, and there *remains* 3, I set down 3, and proceed to the next, saying, 7 from 9, and there *remains* 2, and so on; 1 from 1, and there *remains* 0, 5 from 7, and there *remains* 2, and set down the Figures, 2, 0, 2, 3, 4, 4, 1, in their Places, as in the Example.

In Example the *second*, I begin as afore, saying, 7 from 4 I cannot, (calling it 10 more) but 7 from 14, and there *remains* 7, I set down 7, and proceed to the next Place, (carrying 1) saying, 1 and 9 is 10, from 9 I cannot, but 10 from 19, and there *remains* 9, I set down 9, and proceed to the next Place, (carrying 1) saying, 1 and 6 is 7, from 2 I cannot, but 7 from 12, and there *remains* 5, I set down 5, and proceed to the next Place, (carry-
ing

8 SUBTRACTION.

ing 1) saying, 1 and 1 is 2, from 7, and there *remains* 5, (here I carry nothing, because I did not call 7, 17) having set down 5, I proceed, saying, 7 from 6 I cannot, but 7 from 16, and there *remains* 9, I set down 9, and proceed to the next Place, (carrying 1) saying, 1 and 3 is 4, from 4, and there *remains* 0, I set it down, and proceed, saying, 5 from 9, and there *remains* 4, which being set down, I have done.

The third *Example*, is wrought after the same Manner.

If you *add* the *Remainder* to the *under* Line (which is called the *Subtrahend*) it will produce the *upper* Line, and is the *Proof* of your Work.



MULTIPLICATION

IS a compendious Way of *Addition*, consisting of 3 Parts, *viz.* *Multiplicand*, *Multiplier*, and *Product*.

The *Multiplicand*, is the Sum or Number given to be multiplied.

The *Multiplier*, is the Sum or Number given, by which the *Multiplicand* is to be multiplied, and denotes the Number of Times, that the *Multiplicand* is to be added to itself.

The *Product*, is the Sum or Number produced, by the *Multiplicands*, being multiplied, into or with the *Multiplier*.

Before any Operation can be readily performed in Multiplication, the following *Table* must be perfectly learned.

T A B L E

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TABLE.

2 Times	5 Times	11 Times	12 Times
2 - is - 4	5 - is - 25	2 - is - 22	2 - is - 24
3 - - - 6	6 - - - 30	3 - - - 33	3 - - - 36
4 - - - 8	7 - - - 35	4 - - - 44	4 - - - 48
5 - - - 10	8 - - - 40	5 - - - 55	5 - - - 60
6 - - - 12	9 - - - 45	6 - - - 66	6 - - - 72
7 - - - 14		7 - - - 77	7 - - - 84
8 - - - 16	6 Times	8 - - - 88	8 - - - 96
9 - - - 18	6 - is - 36	9 - - - 99	9 - - - 108
	7 - - - 42	10 - - - 110	10 - - - 120
3 Times	8 - - - 48	11 - - - 121	11 - - - 132
3 - is - 9	9 - - - 54	12 - - - 132	12 - - - 144
4 - - - 12			
5 - - - 15	7 Times		
6 - - - 18	7 - is - 49		
7 - - - 21	8 - - - 56		
8 - - - 24	9 - - - 63		
9 - - - 27			
	8 Times		
4 Times	8 - is - 64		
4 - is - 16	9 - - - 72		
5 - - - 20			
6 - - - 24	9 Times		
7 - - - 28	9 - is - 81		
8 - - - 32			
9 - - - 36			

Note. 3 Times 2, is the same with 2 Times 3; and 4 Times 2, the same with 2 Times 4; and 5 Times 2, the same with 2 Times 5, &c.

The like is to be understood, of all the rest in the TABLE, I shall not repeat.

Having perfectly learnt the foregoing Table, the Work of *Multiplication* will be very easy, if the following *Examples* be carefully observed.

	Exa. 1.	Exa. 2.
Multiplicand	3719467	5714936
Multiplier	2	30
Product	7438934	171448080
Exa. 3.	Exa. 4.	Exa. 5.
5712943	527194	137194671
4	700	12
22851772	369035800	1646336052

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In

10 **MULTIPLICATION.**

In the first *Example*, I begin with the *Units* Place, in the *Multiplicand*, (and so must always) saying, 2 Times 7 is 14, *I set down* 4, and *carry* 1 to the next Place, saying, 2 Times 6 is 12, and 1 is 13, *I set down* 3, and *carry* 1 to the next Place, saying, 2 Times 4 is 8, and 1 is 9, *I set down* 9, and have nothing to carry. Therefore I go on, saying, 2 Times 9 is 18, *I set down* 8, and *carry* 1, saying, 2 Times 1 is 2, and 1 is 3, *I set down* 3, and proceed, saying, 2 Times 7 is 14, *I set down* 4, and *carry* 1, saying, 2 Times 3 is 6, and 1 is 7, which being set down, compleats the Work, as in the *Example*.

In the second *Example*, where the *Multiplier* is 30, I proceed as afore, setting the *Cypher* one Place forward: And in the fourth *Example*, where the *Multiplier* is 700, setting the *Cyphers* 2 Places forward, as appears by the *Examples*.

In the fifth *Example*, beginning at the *Units* Place, I say, 12 Times 1 is 12, *I set down* 2, and *carry* 1 to the next Place, saying, 12 Times 7 is 84, and 1 that *I carry* is 85, *I set down* 5, and *carry* 8, saying, 12 Times 6 is 72, and 8 that *I carry* is 80, *I set down* 0, and *carry* 8, saying, 12 Times 4 is 48, and 8 is 56, *I set down* 6, and *carry* 5, saying, 12 Times 9 is 108, and 5 is 113, *I set down* 3, and *carry* 11, saying, 12 Times 1 is 12, and 11 is 23, *I set down* 3, and *carry* 2, saying, 12 Times 7 is 84, and 2 is 86, *I set down* 6, and *carry* 8, saying, 12 Times 3 is 36, and 8 is 44, *I set down* 4, and *carry* 4, saying, 12 Times 1 is 12, and 4 is 16, which being set down, compleats the Operation, as in the *Example*.

The Work of these two *Examples*, I presume, is sufficient to shew the young Learner, all the rest: Observing, that when a *Cypher* or *Cyphers* is annexed to the End of the *Multiplier*, that they be
set

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set forward, and taken down in the Product, as in the 2d and 4th *Example*, as afore hinted.

But when the *Multiplier* doth consist of more than one Figure; then so many Figures as there are in the *Multiplier*, so many particular Products there must be, and the *Sum* of all those Products, will be the true Product required.

Note. The *first* Figure or *Cypher* of every *Product*, must be placed directly underneath the *multiplier* Figure, as in the following Examples.

57192343 13	49271946 69	42719467 908
171577029	443447514	341755736
57192343	295631676	3844752030
743500459	3399764274	38789276036

(6)	(7)
57194672 422	567213719 57008
114389344	4537709752
114389344	397049603300
228778688	2836068595
24136151584	32335719692752

(8)	(9)
57294671 4592	7129467 39804
114589342	28517868
515652039	570357360
286473355	64165203
229178684	21388401
263097129232	283781304468

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(10)	(11)
3827194	5719400
5268	54007020
30617552	114388000
22963164	400358000
7654388	2287760000
19135970	28597000
20161657992	308887750188000

These *Examples* are sufficient to instruct the Learner, in all the Varieties that can happen in *multiplying* of whole Numbers: And here I might shew the Method of performing *Multiplication* by *Addition* only; but because that Method is not preferable to this, I shall therefore omit inserting of it.

The common Method of *proving* Multiplication, by casting away the *Nines* in the Multiplicand, Multiplier, and Product, being not infallible, I shall omit it, and here only acquaint you, that the best Way of *proving* Multiplication, is by dividing the *Product*, by the *Multiplicand*, or *Multiplier*; the *Quotient* without any *Remainder*, shews the *Multiplier*, or *Multiplicand*, as will plainly appear at the Close of *Division*, to which I proceed.

DIVISION

IS a compendious Way of *Subtraction*, consisting of 3 Parts, viz. *Dividend*, *Divisor*, and *Quotient*.

The *Dividend*, is the Sum or Number given, to be divided.

The *Divisor*, is the Sum or Number given, by which the *Dividend* is to be divided, and denotes the



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the Number of Parts, that the *Dividend* is to be divided into.

The *Quotient*, is the Number produced by the *Dividend*, and *Divisor*, and shews how often the *Divisor* is subtracted from, or contained in the *Dividend*, or into what Number of equal Parts the *Dividend* is divided.

All the Difficulty in *Division*, lieth in making Choice of the *Quotient* Figure, so that it be neither too much, nor too little, which will soon be overcome, if you carefully observe the following *Examples*.

Dividend,	(2)
Divisor 2)47926712	3)59467291
Quotient 23963356	19822430:1 Rem.

(3)	(4)
4)372946729	5)3872946719
93236682:1	774589343:4

(5)	(6)
9)42836714291	12)2489367201
4759634921:2	207447266:9

In the *first Example*, I first see how many Times 2 the *Divisor*, is contained in 4, the *first* Figure of the *Dividend*, which is 2 Times, I place 2 in the *Quotient*, and proceed, enquiring how many Times 2 is contained in 7, the next Figure, which is 3 Times, for 3 Times 2 is 6, from 7, and there remains 1, place 3 in the *Quotient*, and carry the Remainder 1, to the next Figure 9, and see how often 2 is contained in 19, which is 9 Times, and 1 remains, for 9 Times 2 is 18, from 19, and there remains 1, place 9 in the *Quotient*, and carry the remainder

remainder 1, to the next Figure 2, then see how often 2 is contained in 12, which is just 6 Times, without any Remainder, *place 6* in the *Quotient*, and see how often 2 is contained in the next Figure 6, which is just 3 Times, without any Remainder, *place 3* in the *Quotient*, and see how often 2 is contained in the next Figure 7, which is 3 Times, and 1 remains, *place 3* in the *Quotient*, and carry the *remainder 1*, to the next Figure 1, and see how often 2 is contained in 11, which is 5 Times, and 1 remains, *place 5* in the *Quotient*, and carry the remainder 1 to the next Figure 2, and see how often 2 is contained in 12, which is just 6 Times, without any remainder, *place 6* in the *Quotient*, and the Work is done.

In *Example 2*, I begin and proceed as aforegoing; but in *Examples 3d*, 4th, and 5th, I begin as is shewed in *Example 5th*.

Example 5th. The *first* Figure of the *Dividend*, being less than the *Divisor*, begin with the two first, *viz.* 42, and see how often 9 the *Divisor*, is contained in them, which is 4 Times, for 4 Times 9 is 36, from 42, and there remains 6, *place 4* in the *Quotient*, *carrying 6* to the next Figure 8, it makes 68, then see how often 9 is contained in 68, which is 7 Times, and 5 remains, *place 7* in the *Quotient*, and *carry 5* to the next Figure 3, see how often 9 is contained in 53, which is 5 Times, and 8 remaining, *place 5* in the *Quotient*, and *carry 8* to the next Figure 6, and see how often 9 is contained in 86, which is 9 Times, and 5 remaining, *place 9* in the *Quotient*, and *carry 5* to the next Figure 7, see how often 9 is contained in 57, which is 6 Times, and 3 remaining, *place 6* in the *Quotient*, and *carry 3* to the next Figure 1, see how often 9 is contained in 31, which is 3 Times, and 4 remaining, *place 3* in the *Quotient*, and *carry 4* to the next Figure 4, see how often 9 is contained in

44, which is 4 Times, and 8 remaining, *place 4* in the *Quotient*, and *carry 8* to the next Figure 2, see how often 9 is contained in 82, which is 9 Times, and 1 remaining, *place 9* in the *Quotient*, and *carry 1* to the next Figure 9, see how often 9 is contained in 19, which is 2 Times, and 1 remaining, *place 2* in the *Quotient*, and *carry 1* to the next Figure 1, and see how often 9 is contained in 11, which is 1 Time, and 2 remaining, *place 1* in the *Quotient*, and set down the remainder 2, separated by a *Semicolon*, or other Note of Distinction, from the *Quotient*, as in the *Example*, and your Work is done.

Understand the like of all the rest, and all other *Examples*, that may occur with one single Figure for a *Divisor*.

There are several Methods taught, and treated of, by Masters of *Arithmetic*, for the Operation of *Division*, when the *Divisor* consists of two or more Figures, but the most plain and easy to be understood, is the following Method.

(7)	(8)	(9)
<u>61128</u>	<u>17307</u>	<u>8432</u>
13)794672	27)467295	34)286719
<u>78</u>	<u>27</u>	<u>272</u>
14	197	147
<u>13</u>	<u>189</u>	<u>136</u>
16	82	111
<u>13</u>	<u>81</u>	<u>102</u>
37	195	99
<u>26</u>	<u>189</u>	<u>68</u>
112	6	31
<u>104</u>		
8		

(10)

14807

572)8469728

572

2749

2288

4617

4576

4128

4004124

(11)

37345

876)32714972

2628

6434

6132

3029

2628

4017

3504

5132

4380752

(12)

74808

4972)371946719

34804

23906

19888

40187

39776

41119

397761343

(13)

4597

92837)426837194

371348

554891

464185

907069

835533

715364

64985965507

(14)

(15)

(14)

$$\begin{array}{r}
 8327196 \\
 \hline
 647082 \overline{) 5388378642072} \\
 \underline{5176656} \\
 2117226 \\
 \underline{1941246} \\
 1759804 \\
 \underline{1294164} \\
 4656402 \\
 \underline{4529574} \\
 1268280 \\
 \underline{647082} \\
 6211987 \\
 \underline{5823738} \\
 3882492 \\
 \underline{3882492} \\
 0
 \end{array}$$

Example 7, I observe how often 1 (the *first* Figure of the *Divisor*) is contained in 7, (the *first* Figure of the *Dividend*) which is 7 Times; but 7 Times 13 is 91, which exceeds 79 (the two first Figures of the *Dividend*) therefore I take 1 less than 7 (*viz.* 6) and place it in the *Quotient*; then multiply the *Divisor* 13, by the *Quotient* Figure 6, it makes 78, drawing a Line underneath; I *subtract*, and the *Remainder* is 1, to which I bring down the next Figure of the *Dividend*, (*viz.* 4) and so have 14 for a new *Dividend*, as you see in the *Example*. Then I see again, how often 1, the *first* Figure of the *Divisor*, is contained in 1, the *first* Figure of the new *Dividend*, which is 1 Time, and place it in the *Quotient*, by which I again multiply
D the

the *Divisor*, it makes 13, and *subtract* as afore, the Remainder is again 1, to which I bring down the next Figure 6, which makes 16, for a new *Dividend*; now again, I see how often 1 is contained in 1, and is 1 Time, which placed in the *Quotient*, I therewith multiply the *Divisor*, it makes 13, which being *subtracted* from the last new *Dividend*, leaves 3, to which I bring down the next Figure 7, and see how often 1 is contained in 3, which for my purpose is but 2 Times, by which I multiply the *Divisor*, it makes 26, and *subtract*, the Remainder is 11, to which I bring down the next Figure 2, and it makes 112; I see how often 1 is contained in 11, which for my Purpose is 8 Times, wherewith I multiply the *Divisor*, it makes 104, and *subtract*, the Remainder is 8, and having no more Figures to bring down, this *Example* is finished.

Example 11th. The *first* Figure of the *Divisor*, being greater than the *first* Figure of the *Dividend*, I see how often 8 is contained in 32, which is just 4 Times, and is too much for my Purpose; for the Product of the *Divisor* multiplied by 4, will be greater than 3271, therefore I set down but 3 in the *Quotient*, and multiply the *Divisor* therewith, it produces 2628, which *subtracted* from 3271, leaves 643, to which I bring down the next Figure 4, and have 6434, for a new *Dividend*; then I see how often 8 is contained in 64, which for my Purpose is 7 Times, wherewith I multiply the *Divisor*, it produces 3504, which *subtracted* from 6434, leaves 302, to which I bring down the next Figure 9, and see how often 8 is contained in 30, which is 3 Times, wherewith I multiply the *Divisor*, and it produces 2628, which being *subtracted* from 3029, leaves 401; to which I bring down the next Figure 7, and see how often 8 is contained in 40, which for my Purpose is 4 Times, wherewith I multiply the *Divisor*, it produces 3504, which being

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ing *subtracted* from 4017, leaves 513, to which I bring down the next Figure 2, and see how often 8 is contained in 51, which is 5 Times, wherewith I multiply the *Divisor*, it produces 4380, which being *subtracted* from 5132, leaves 752, and so the Operation is finished.

Example 12. Having in the preceeding Method, carried on the Operation, I find when 1 is brought down to the Remainder 411, that the new *Dividend*, viz. 4111, is less than the *Divisor*, therefore I place 0 in the *Quotient*, and must always do so, in the like Case, and bring down the next Figure 9, and then proceed as in the *Example*.

Note. If the *Remainder* after *Subtraction*, before the next Figure is brought down to it, be greater than the *Divisor*, your last *Quotient* Figure, is too little, and must be increased.

Also if the *Product* of the *Divisor*, by the *Quotient* Figure, be greater than the *Dividend*, from which it is to be subtracted, then is your last *Quotient* Figure too great, and must be decreased.

For the *Proof* of *Division*, If the *Quotient*, and *Divisor*, be multiplied together, and the *Remainder*, if any, be added to it, it will produce the *Dividend*.

Exa. 14.

Quotient 8327196
Divisor 647082

16654392
66617568
582903720
33308784
49963176

5388378642072

Exa. 13.

Divisor 92837
Quotient 4597

649859
835533
464185
371348

426771689
Rem. 65505

426837194

P 2

Thus

Thus having plainly and briefly gone through the 5 general Parts of *Aritbmetic*, viz. *Numeration*, *Addition*, *Subtraction*, *Multiplication*, and *Division*; being the *Foundation*, upon which all *Operations* whatsoever, that are possible to be wrought by Numbers are built, I shall therefore here advise the Learner, to be well acquainted with what hath been delivered, before he proceed.

I shall pass by all the other Ways of *Division*, by cancelling the Figures, as we proceed in the Work, because in Practice, those Methods are best, that are plainest, and least liable to Mistakes; I would therefore advise the *Practitioner*, to commit as little to his Head as possible; and follow the Practice of *Division*, as I have here exhibited it; and proceed to the Application of these general Rules, in *Money*, *Weights*, and *Measures*.



Of M O N E Y.

THE least Piece used in *England*, is a *Farthing*, and from thence ariseth the other Denominations, as in the Table following, viz.

4 Farthings	make	1 Penny.
12 Pence	- - - - -	1 Shilling.
20 Shillings	- - - - -	1 Pound.

To add Sums of Money together, due Care must be taken, to place all the Numbers of the same Denomination, exactly underneath each other, viz. place *Pounds* under *Pounds*, *Shillings* under *Shillings*, *Pence* under *Pence*, and *Farthings* under *Farthings*.

The like is to be understood in *Weights*, and *Measures*, according to their several Denominations. Then observe this general Rule.

R U L E.

R U L E.

Begin with the Figures of the *least* Denomination, and add them altogether into one Sum; then consider how many of the next *superior* Denomination, are contained in that Sum, so many *Units* must be carried to the said next *superior* Denomination, and the *Overplus*, if there be any, must be set down underneath its own *Denomination*, and so proceed from one *Denomination* to the other, till all be finished.

(1)		(2)			(3)		
l.	s.	l.	s.	d.	l.	s.	d.
2794	13	371	14	10	5672	11	9 $\frac{1}{4}$
1724	14	219	15	11	3826	12	8
3716	15	439	16	10	4271	13	7 $\frac{1}{2}$
2143	16	526	17	9	5372	14	6 $\frac{1}{2}$
9167	17	729	18	8	9137	15	5
4283	18	832	19	7	4672	16	4 $\frac{1}{2}$
£.23831	13	£.3119.	3	7	£.32954	4	4 $\frac{1}{2}$

In the *first Example*, according to the Rule, I begin with the *Shillings*, being there the least *Denomination*; and adding them together, I find the Sum to be 93 *Shillings*, that is, 4 *Pounds*, 13 *Shillings*, I set down 13 *Shillings*, and carry 4 *Pounds* to the Place of *Pounds*, saying, 4 and 3 is 7, and 7 is 14, and so on, as in *Addition* of Integers, till the Work is finished, as in the Example.

In the 2d *Example*, I begin with the *Pence*, and find their Sum to be 55, that is, 4 *Shillings* and 7 *Pence*, because 12 Times 4 is 48, and 7 is 55, I set down the 7 *Pence*, and carry the 4 *Shillings* to the Place of *Shillings*, adding them, and all the *Shillings* together, I find the Sum of 103 *Shillings*, that is, 5 *Pounds* 3 *Shillings*, I set down 3 *Shillings*, and carry 5 *Pounds* to the Place of *Pounds*, and proceed as afore. In

In the 3d *Example*, I begin with the *Farthings*, and find their Sum to be 7, that is, 1 *Penny* 3 *Farthings*, I set down 3 *Farthings*, and carry 1 *Penny* to the Place of *Pence*, and proceed as aforegoing.

Each *Example* is proved by separating the *first* Line, as in *Addition* of Integers, which I leave to the Practitioner.

To *subtract* Money from Money, observe this general Rule.

R U L E.

Begin with the *least* Denomination, and *subtract* the *under* Figure or Figures from the *upper*, of the same Denomination, and set down the *Remainder*; but if the *under* Figure or Figures be the greatest, increase the upper, with one of the next *superior* Denomination, and subtract, and so proceed to the next Place, where you must pay the 1 borrowed, by *adding* 1 to the under Figure, or Figures in that Place, &c. as in *Subtraction* of Integers.

(1)				(2)				(3)			
	<i>l.</i>	<i>s.</i>	<i>d.</i>		<i>l.</i>	<i>s.</i>	<i>d.</i>		<i>l.</i>	<i>s.</i>	<i>d.</i>
Lent	75	97	16 8		24	68	14 5		37	19	18 10½
Paid	31	54	19 7		19	75	18 3		26	64	19 11½
Rem.	£. 44	42	17 1		£. 49	2	16 2		£. 10	54	18 10½

In the 3d *Example*, I begin with the *Farthings*, saying, $\frac{3}{4}$ from $\frac{1}{4}$, I cannot, but 3 from 5, the *upper* increased by 1 *Penny*, and there remains 2, I set down 2, and proceed to the next Place, *viz.* *Pence*, saying, the 1 that I borrowed, and 11 is 12 *Pence*, from 10 *Pence* I cannot, but 12 from 22, the *upper* increased by 1 *Shilling*, and there remains 10, I set down 10, and proceed to the *Shillings*, saying, 1 that I borrowed and 19 *Shillings* is 20 *Shillings*.

Shillings from 18 I cannot, but 20 from 38, the upper increased by 1 *Pound*, and there remains 18, I set down 18, and proceed to the *Pounds*, saying, 1 that I borrowed, and 4 is 5 from 9, and there remains 4, and so on, 6 from 1 I cannot, but 6 from 11, and there remains 5, 1 that I borrowed and 6 is 7, from 7, and there remains 0, 2 from 3, and there remains 1, all which being set down, the *Remainder* is 1054*l.* 18*s.* 10*d.* $\frac{1}{2}$ as in the *Example*.

The 2 other *Examples* are wrought in the same Manner.

Note. The *Remainder*, is by Merchants called the *Ballance* of an Account, and is that Sum of Money, which makes the *Dr.* and *Cr.* Side both equal, as in the *Examples* following.

(1)

Mr. A. B. Dr.				per Contra, Cr.			
	<i>l.</i>	<i>s.</i>	<i>d.</i>		<i>l.</i>	<i>s.</i>	<i>d.</i>
To -----	7597	14	7	By -----	322	15	8
				By -----	456	16	9
				By -----	594	17	6
				By -----	322	18	5
				By Ball.	5900	06	3
					7597	14	7

(2)

Mr. A. B. Dr.				per Contra, Cr.			
	<i>l.</i>	<i>s.</i>	<i>d.</i>		<i>l.</i>	<i>s.</i>	<i>d.</i>
To -----	326	14	9	By -----	1794	15	8
To -----	572	19	8	By Ball.	87	07	10
To -----	654	17	7				
To -----	327	11	6				
	1882	03	6		1882	03	6

(3)

(3)

Mr. A. B. Dr.			per Contra Cr.		
To	- - -	444 15 9	By	- - -	356 11 5
To	- - -	356 14 6	By	- - -	271 10 3
To	- - -	272 11 8	By	- - -	369 09 2
To	- - -	554 10 2	By Ball.		631 01 3
<hr/>			<hr/>		
1628 12 1			1628 12 1		

Note. In the preceeding Examples, the several Sums, Dr. and Cr. are first added together, and then Subtraction is made to find the Ballance, as in Example 1. The several Sums Cr. added together, is 1697*l.* 8*s.* 4*d.* which subtracted from the Dr. 7597*l.* 14*s.* 7*d.* leaves 5900*l.* 6*s.* 3*d.* which is the Ballance of that Account, and makes the Dr. and Cr. fide equal.

In Example 2. The several Sums Dr. added together, is 1882*l.* 3*s.* 6*d.* from which the Cr. 1794*l.* 15*s.* 8*d.* is subtracted, and leaves 87*l.* 7*s.* 10*d.* the Ballance of that Account.

In Example 3. The several Sums Dr. added together, is 1628*l.* 12*s.* 1*d.* and the several Sums Cr. added together, is 997*l.* 10*s.* 10*d.* which subtracted from the Dr. 1628*l.* 12*s.* 1*d.* leaves 631*l.* 1*s.* 3*d.* the Ballance of that Account.

The Proof of Subtraction of Money is, by adding the Remainder to the Sum paid, and it will produce the Sum lent.

		<i>l.</i>	<i>s.</i>	<i>d.</i>
Example 1.	The Sum paid is	3154	19	7
	The Remainder	4442	17	1
		<hr/>		
		7597	16	8

		<i>l.</i>	<i>s.</i>	<i>d.</i>
Example 3.	The Sum paid	2664	19	11½
	The Remainder	1054	18	10½
		<hr/>		
		3719	18	10½
		To		

To multiply Money in the Manner here delivered, is one of the most useful Parts in *Arithmetic*; and has been but lightly touched upon, or not at all, by most *Authors*, and *Teachers* of *Arithmetic* that I have seen. It is many Years ago, that I published a Specimen of it, and have indeed since, seen some little Use made thereof, both by *Authors*, and *Teachers* of *Arithmetic*; but its intire Usefulness will be evident in the Sequel of this *Treatise*. For *Multiplication* is no other, but the *Addition*, or repeating the same Thing, a certain Number of Times, and consequently produces the *Position*, or *Affirmation* of the Thing, so many Times more than it was before. The following *Examples* I doubt not, will be very plain and easy to them, who have well learned the preceeding Parts of this *Treatise*.

(1)

	l.	s.	d.
Multiply	7248	15	9
By	-----	7	
Product	£.50741	10	3

(2)

	l.	s.	d.
Mult.	6275	14	8
By	-----	9	
	£.56481	12	0

(3)

	l.	s.	d.
Multiply	6754	18	9½
By 19		9	10
	£.67549	7	8½
	60794	8	11½
	£.128343	16	7½

(4)

	l.	s.	d.
Mult.	3825	18	10
By 17		7	10
	£.38259	8	6½
	26781	11	11½
	£.65041	00	6½

(5)

	l.	s.	d.
Multiply	8327	18	10½
By 14		4	10
	£.83279	8	9
	33311	15	6
	£.116591	4	3

(6)

	l.	s.	d.
Multiply	4529	19	10½
By 16		6	10
	£.45299	18	9
	27179	19	3
	£.72479	18	0

E

(7)

(7)

	<i>l.</i>	<i>s.</i>	<i>d.</i>
Multiply	3768	18	9½
By 36		6	10

£. 37689	7	8½
		3

£. 113068	3	1½
22613	12	7½

£. 135681	15	9
-----------	----	---

(8)

	<i>l.</i>	<i>s.</i>	<i>d.</i>
Multiply	8275	19	11½
By 27		7	10

£. 82759	19	9½
		2

£. 165519	19	7
57931	19	10½

£. 223451	19	5½
-----------	----	----

(9)

	<i>l.</i>	<i>s.</i>	<i>d.</i>
Mult.	2856	17	9½
By 47		7	10

£. 28568	17	8½
		4

£. 114275	10	10
19998	4	4½

£. 134273	15	2½
-----------	----	----

(10)

	<i>l.</i>	<i>s.</i>	<i>d.</i>
Mult.	8256	17	9½
By 72		2	10

£. 82568	17	8½
		7

£. 577982	3	11½
16513	15	6½

£. 594495	19	6
-----------	----	---

(11)

	<i>l.</i>	<i>s.</i>	<i>d.</i>
Mult.	7528	19	11½
By 273		3	10

£. 75289	19	3½
		7 10

£. 752899	12	8½
		2

£. 1505799	5	5
527029	14	10½
22586	19	9½

£. 2055416	00	1½
------------	----	----

(12)

	<i>l.</i>	<i>s.</i>	<i>d.</i>
Mult.	6397	17	9½
By 592		2	10

£. 63978	17	9½
		9 10

£. 639788	18	1½
		5

£. 3198944	10	7½
575810	00	3½
12795	15	6½

£. 3787550	6	6
------------	---	---

Note. In these two last and the two following Examples, the Penny is divided into 8 Parts, and so

so consequently $\frac{1}{4}$ is *one Farthing*, and $\frac{1}{2}$ is *one Half-penny*, and $\frac{3}{4}$ is *three Farthings*; the which will be found of great Use in the Computation of *Foreign Exchanges*, and such Commodities where they come so near in the Price, as to $\frac{1}{4}$ Part of a Penny.

(13)

	<i>l.</i>	<i>s.</i>	<i>d.</i>
Mult.	3768	18	$9\frac{1}{8}$
By 374		4	10
<hr/>			
£.	37689	8	$0\frac{1}{8}$
		7	10
<hr/>			
£.	376894	00	$2\frac{1}{8}$
			3
<hr/>			
£.	1130682	00	$7\frac{1}{8}$
	263825	16	$1\frac{1}{8}$
	15075	15	$2\frac{1}{8}$
<hr/>			
£.	1409583	11	$11\frac{1}{8}$

(14)

	<i>l.</i>	<i>s.</i>	<i>d.</i>
Mult.	8249	19	$11\frac{7}{8}$
By 675		5	10
<hr/>			
£.	82499	19	$10\frac{6}{8}$
		7	10
<hr/>			
£.	824999	18	$11\frac{1}{8}$
			6
<hr/>			
£.	4949999	13	9
	577499	19	$3\frac{1}{8}$
	41249	19	$11\frac{3}{8}$
<hr/>			
£.	5568749	12	$11\frac{5}{8}$

(15)

	<i>l.</i>	<i>s.</i>	<i>d.</i>
Mult.	2167	14	$3\frac{1}{2}$
By 2375		5	10
<hr/>			
£.	21677	2	11
		7	10
<hr/>			
£.	216771	9	2
		3	10
<hr/>			
£.	2167714	11	8
			2
<hr/>			
£.	4335429	3	4
	650314	7	6
	151740	0	5
	10838	11	$5\frac{1}{2}$
<hr/>			
£.	5148322	2	$8\frac{1}{2}$

(16)

	<i>l.</i>	<i>s.</i>	<i>d.</i>
Mult.	3251	6	$1\frac{1}{2}$
By 4759		9	10
<hr/>			
£.	32513	1	$0\frac{1}{2}$
		5	10
<hr/>			
£.	325130	10	5
		7	10
<hr/>			
£.	3251305	4	2
			4
<hr/>			
£.	13005220	16	8
	2275913	12	11
	162565	5	$2\frac{1}{2}$
	29261	14	$11\frac{1}{2}$
<hr/>			
£.	15472961	9	$8\frac{1}{2}$

E 2

(17)

(17)

	l.	s.	d.
Mult. 5243	2	3	$\frac{1}{2}$
By 5356	6	10	

£. 52431	3	$1\frac{1}{2}$	
	5	10	

£. 524311	11	3	
	3	10	

£. 5243115	12	6	
	5		

£. 26215578	2	6	
1572934	13	9	
262155	15	$7\frac{1}{2}$	
31458	13	$10\frac{1}{2}$	

£. 28082127	5	9	
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(19)

	l.	s.	d.
Mult. 6573	6	$8\frac{1}{4}$	
By 25378	8	10	

£. 65733	6	$10\frac{1}{4}$	
	7	10	

£. 657333	8	9	
	3	10	

£. 6573334	7	6	
	5	10	

£. 65733343	15	0	
		2	

£. 131466687	10	0	
32866671	17	6	
1972000	6	3	
460133	8	$1\frac{1}{2}$	
52586	13	6	

£. 166818079	15	$4\frac{1}{2}$	
--------------	----	----------------	--

(18)

	l.	s.	d.
Mult. 7124	1	$2\frac{1}{2}$	
By 7284	4	10	

£. 71240	12	1	
	8	10	

£. 712406	00	10	
	2	10	

£. 7124060	8	4	
		7	

£. 49868422	18	4	
1424812	1	8	
569924	16	8	
28496	4	10	

£. 51891656	1	6	
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(20)

	l.	s.	d.
Mult. 4187	8	$5\frac{1}{4}$	
By 38563	3	10	

£. 41874	4	$9\frac{1}{2}$	
	6	10	

£. 418742	7	11	
	5	10	

£. 4187423	19	2	
	8	10	

£. 41874239	11	8	
		3	

£. 125622718	15	0	
33499391	13	4	
2093711	19	7	
251245	8	9	
12562	5	$5\frac{1}{4}$	

£. 161479630	2	$1\frac{1}{4}$	
--------------	---	----------------	--

In

In the *first Example*, I begin with the least Denomination, *viz. Pence*, saying, 7 Times 9 is 63 *Pence*, that is, 5 *Shillings* and 3 *Pence*, I set down 3 *Pence*, and carry the 5 *Shillings* to the Place of *Shillings*, saying, 7 Times 5 is 35, and 5 is 40, I set down 0 in the first Place of *Shillings*, and carry 4, saying, 7 Times 1 is 7, and 4 that I carry is 11, the *half* of 11 is 5 for *Pounds*, and 1 remains, I set down the *remainder* 1 in the second Place of *Shillings*, and carry 5 to the *Pounds*, saying, 7 Times 8 is 56, and 5 that I carry is 61, I set down 1 in the Place of *Pounds*, and carry 6, saying, 7 Times 4 is 28, and 6 is 34, I set down 4, and carry 3, saying, 7 Times 2 is 14, and 3 is 17, I set down 7, and carry 1, saying, 7 Times 7 is 49, and 1 is 50, which being set down, the whole *Product* is 50741*l.* 10*s.* 3*d.* as in the *Example*.

Example 2d, I begin with the *Pence*, as afore, saying, 9 Times 8 is 72 *Pence*, that is, 6 *Shillings* just, I set down 0 in the Place of *Pence*, and carry 6 *Shillings* to the Place of *Shillings*, saying, 9 Times 4 is 36, and 6 is 42, I set down 2, and carry 4, saying, 9 Times 1 is 9, and 4 is 13, the *half* of 13 is 6 for *Pounds*, and 1 remains, I set down 1, and carry 6, saying, 9 Times 5 is 45, and 6 is 51, I set down 1 in the Place of *Pounds*, and carry 5, saying, 9 Times 7 is 63, and 5 is 68, I set down 8, and carry 6, saying, 9 Times 2 is 18, and 6 is 24, I set down 4, and carry 2, saying, 9 Times 6 is 54, and 2 is 56, which being set down, gives 56481*l.* 12*s.* 0*d.* for the *Product*, as in the *Example*.

Example 3, 4, 5, and 6, I first multiply the given Sums by 10, which gives me the *Product* of 10, then in *Example 3d*, the given Sum by 9, it gives the *Product* of 9. Then the *Product* of 10, and the *Product* of 9 added together, gives the *Product* of 19, as in the *Example*. So likewise the

Product

Product of 10, and the *Product* of 7 added together, gives the *Product* of 17, as in the 4th Example. And the *Product* of 10, and the *Product* of 4 added together, gives the *Product* of 14, as in the 5th Example. And the *Product* of 10, and the *Product* of 6, added together, gives the *Product* of 16, as in the 6th Example.

Example 7, 8, 9 and 10. The *Products* of 10 is obtain'd as afore. Then in Example 7, the *Product* of 10 is multiplied by 3, and it gives the *Product* of 30, because 3 Times 10 is 30. To which I add the *Product* of 6, it gives the *Product* of 36, as in the Example. Again in Example 8, the *Product* of 10, is multiplied by 2, and it gives the *Product* of 20, because 2 Times 10 is 20, to which I add the *Product* of 7, it gives the *Product* of 27, as in the Example.

In Example 9. The *Product* of 10, is multiplied by 4, and it gives the *Product* of 40, because 4 Times 10 is 40; to which I add the *Product* of 7, it gives the *Product* of 47, as in the Example.

In Example 10. The *Product* of 10, is multiplied by 7, and it gives the *Product* of 70, because 7 Times 10, is 70, to which I add the *Product* of 2, it gives the *Product* of 72, as in the Example.

Example 11, 12, 13 and 14. The *Product* of 10, multiplied by 10, gives the *Product* of 100, because 10 Times 10, is 100; then in Example 11, the *Product* of 100, is multiplied by 2, and it gives the *Product* of 200; the *Product* of 10, is multiplied by 7, for the *Product* of 70, and the given Sum multiplied by 3, for the *Product* of 3; then the *Product* of 200, the *Product* of 70, and the *Product* of 3, added together, gives the *Product* of 273, as in the Example.

In Example 12. The *Product* of 100 is multiplied by 5, for the *Product* of 500, and the *Product* of 10 by 9, for the *Product* of 90, and the
given

given Sum by 2, for the *Product* of 2, these 3 *Products* added together, gives the *Product* of 592, as in the Example.

In *Example 13*. The *Product* of 100 is multiplied by 3, for the *Product* of 300, the *Product* of 10 by 7, for the *Product* of 70, and the given Sum by 4, for the *Product* of 4; these 3 *Products* added together, gives the *Product* of 374, as in the Example.

Example 14. The *Product* of 100 is multiplied by 6, for the *Product* of 600, and the *Product* of 10 by 7, for the *Product* of 70, and the given Sum by 5, for the *Product* of 5; these 3 *Products* added together, gives the *Product* of 675, as in the Example.

Example 15, 16, 17, and 18. The *Products* of 100 are multiplied by 10, and it gives the *Products* of 1000, because 10 Times 100 is 1000. Then in *Example 15*. The *Product* of 1000 is multiplied by 2, for the *Product* of 2000, and the *Product* of 100 by 3, for the *Product* of 300, and the *Product* of 10 by 7, for the *Product* of 70, and the given Sum by 5, for the *Product* of 5. These 4 *Products* added together, gives the *Product* of 2375, as in the Example. The like is to be understood with respect to the 16, 17, and 18 Examples.

Example 19 and 20. The *Products* of 1000 are multiplied by 10, for the *Products* of 10000; and in *Example 19*, the *Product* of 10000 is multiplied by 2, for the *Product* of 20000, and the *Product* of 1000 by 5, for the *Product* of 5000, and the *Product* of 100 by 3, for the *Product* of 300, and the *Product* of 10 by 7, for the *Product* of 70, and the given Sum by 8, for the *Product* of 8; these 5 *Products* added together, gives the *Product* of 25378, as in the Example.

The like is to be understood, with respect to the 20th Example.

To

To divide Money by a certain Number proposed, comes next under Consideration, and herein I shall be brief, because its Use, in accompting for the Parts of Quantity, falls generally under some one or other of these Numbers, viz. 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, or 12, as will be evident hereafter.

$$\begin{array}{r} \text{l.} \quad \text{s.} \quad \text{d.} \\ 2)5785 \quad 16 \quad 8\frac{1}{2} \\ \hline \text{£.}2892 \quad 18 \quad 4\frac{1}{4} \end{array}$$

$$\begin{array}{r} \text{l.} \quad \text{s.} \quad \text{d.} \\ 3)6524 \quad 11 \quad 10\frac{1}{2} \\ \hline \text{£.}2174 \quad 17 \quad 3\frac{1}{4} - 2 \end{array}$$

$$\begin{array}{r} \text{l.} \quad \text{s.} \quad \text{d.} \\ 4)3248 \quad 19 \quad 4\frac{1}{2} \\ \hline \text{£.}812 \quad 4 \quad 10 - 3 \end{array}$$

$$\begin{array}{r} \text{l.} \quad \text{s.} \quad \text{d.} \\ 5)9728 \quad 14 \quad 11\frac{1}{4} \\ \hline \text{£.}1945 \quad 14 \quad 11\frac{1}{4} - 2 \end{array}$$

$$\begin{array}{r} \text{l.} \quad \text{s.} \quad \text{d.} \\ 6)6275 \quad 14 \quad 11\frac{1}{2} \\ \hline \text{£.}1045 \quad 19 \quad 1\frac{1}{4} - 4 \end{array}$$

$$\begin{array}{r} \text{l.} \quad \text{s.} \quad \text{d.} \\ 7)5834 \quad 16 \quad 7\frac{1}{2} \\ \hline \text{£.}833 \quad 10 \quad 11\frac{1}{4} - 3 \end{array}$$

$$\begin{array}{r} \text{l.} \quad \text{s.} \quad \text{d.} \\ 8)6785 \quad 16 \quad 8\frac{3}{4} \\ \hline \text{£.}848 \quad 4 \quad 7 - 3 \end{array}$$

$$\begin{array}{r} \text{l.} \quad \text{s.} \quad \text{d.} \\ 9)5326 \quad 19 \quad 11\frac{1}{4} \\ \hline \text{£.}591 \quad 17 \quad 9\frac{1}{4} - 2 \end{array}$$

$$\begin{array}{r} \text{l.} \quad \text{s.} \quad \text{d.} \\ 10)8257 \quad 19 \quad 7\frac{1}{4} \\ \hline \text{£.}825 \quad 15 \quad 11\frac{1}{2} - 3 \end{array}$$

$$\begin{array}{r} \text{l.} \quad \text{s.} \quad \text{d.} \\ 12)2794 \quad 18 \quad 11\frac{1}{2} \\ \hline \text{£.}232 \quad 18 \quad 2\frac{1}{4} - 10 \end{array}$$

In the 1st *Example*, I begin with the *Pounds*, and see how many Times 2 the *Divisor*, is contained in 5, which is 2 Times, and 1 remains, I set down 2, and carry 1 to the next Figure, and see how often 2 is contained in 17, which is 8 Times, and 1 remains, I set down 8, and carry 1 to the next Figure, and see how often 2 is contained in 18, which is 9 Times, and 0 remains, I set

set down 9, and because 0 remained, I see how often 2 is contained in 5, which is 2 Times, and 1 remains, I set down 2, and the 1 *Pound* which remained I carry to the *Shillings*, saying, 1 *Pound* is 20 *Shillings*, and 16 *Shillings* is 36 *Shillings*, then I see how often 2 is contained in 36, which is 18, and 0 remains, I set down 18 in the Place of *Shillings*, and proceed to the *Pence*, and see how often 2 is contained in 8, which is just 4 Times, I set down 4 in the Place of *Pence*, and proceed to the *Farthings*, and see how often 2 is contained in 2 *Farthings*, which is just one Time, I set down 1 *Farthing*, and the *Example* is finished.

Example 10. I begin with the *Pounds* as afore, and see how often 12 is contained in 27, which is 2 Times, and 3 remains, I set down 2, and carry 3 to the next Figure, and see how often 12 is contained in 39, which is 3 Times, and 3 remains, I set down 3, and carry 3 to the next Figure, and see how often 12 is contained in 34, which is 2 Times, and 10 remains, I set down 2, and carry the 10 *Pound*, which is 200 *Shillings* that remained, to the *Shillings*, and it makes 218 *Shillings*, then I see how often 12 is contained in 218, which is 18 Times, and 2 remains, I set down 18 in the Place of *Shillings*, and carry 2 *Shillings*, which is 24 *Pence*, to the *Pence*, and it makes 35 *Pence*, then I see often 12 is contained in 35, which is 2 Times, and 11 remains, I set down 2 in the Place of *Pence*, and carry 11 *Pence*, which is 44 *Farthings*, to the *Farthings*, and it makes 46 *Farthings*, then I see how often 12 is contained in 46, which is 3 Times, and 10 remains, I set down the 3 *Farthings* in the Place of *Farthings*, and the 10 that remained, separated by a Mark of Distinction, and the *Example* is finished, so that 2794*l.* 18*s.* 11*d.* $\frac{1}{2}$, divided by 12, or into 12 equal Parts, is 232*l.* 18*s.* 2*d.* $\frac{3}{4}$. and $\frac{10}{12}$, or $\frac{5}{6}$ of a *Farthing*.

In like Manner are the rest of the *Examples* wrought, which I leave to the Learner.



Of WEIGHT.

THERE are three Sorts of *Weight* used in *Great Britain*, viz. *Troy*, *Apothecary's*, and *Averdupoize*.

The least Denomination of *Troy Weight* is a *Grain*, from whence ariseth the following Table, viz.

24 Grains	make	1 Pennyweight.
20 Pennyweight	-----	1 Ounce.
12 Ounces	-----	1 Pound.

By *Troy Weight*, is weighed, *Jewels*, *Gold*, *Silver*, *Corn*, *Bread*, and all *Liquors*,

The least Denomination of *Apothecary's Weight*, is also a *Grain*, from whence ariseth this Table, viz.

20 Grains	-----	a Scruple.
3 Scruples	-----	a Dram.
8 Drams	-----	an Ounce.
12 Ounces	-----	a Pound.

By *Apothecary's Weight*, is weighed such *Commodities* only, wherewith the *Apothecary* compounds his *Medicines*.

The least Denomination of *Averdupoize Weight*, is a *Dram*, from whence ariseth the following Table, viz.

16 Drams	make	1 Ounce.
16 Ounces	-----	1 Pound.
28 Pounds	-----	1 Quarter.
4 Quarters	-----	1 Hundred w ^r .
20 Hundred w ^r .	---	1 Ton.

By

Of WEIGHT.

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By *Averdupoize Weight*, is weighed all such Commodities, as are either very coarse and droffy, or very subject to waste, as all Kinds of *Grocery Wares*, and *Pitch, Tar, Rosin, Wax, Tallow, Soap, Flax, &c.* *Copper, Tin, Steel, Iron, Lead, &c.* also *Flesh, Butter, Cheese, Salt, &c.*

The Pound *Troy* and *Apothecary's* are both equal, and less than the Pound *Averdupoize*, by about a *sixth Part*; for the Pound *Averdupoize* contains 14 Ounces, 11 Pennyweight, 15 $\frac{1}{2}$ Grains *Troy*.

lb	oz.	dwt	gr.	lb	3	dr.	3	gr.
2719	10	11	13	5167	11	3	2	10
3126	11	12	19	2842	10	2	2	11
4719	10	13	21	3719	9	1	2	12
2832	9	14	23	5672	8	3	2	13
7193	8	15	11	1428	7	4	1	14
4268	11	16	14	3719	6	5	1	15
lb. 24862	3	5	5	lb. 22551	5	6	1	15

Ton.	q.	gr.	lb	3	dr.
6719	13	2	10	11	10
3126	14	3	11	12	11
7149	15	2	19	13	12
2873	16	2	21	14	13
2197	17	1	25	15	14
4137	18	3	27	14	15
Tons 26205	17	1	6	3	11

	lb	oz.	dwt	gr.	lb	dwt	dr.	sc.	gr.
From	2279	10	7	11	3712	10	1	1	10
Take	1976	7	13	5	2719	11	2	2	19
Rem. lb.	303	2	14	6	lb. 992	10	6	1	11

F 2

Ton.

	Ton.	Ⓔ	q ^r .	lb	3	dr.
Received	157	10	3	11	10	10
Delivered	79	13	1	19	11	7
Remains Ton.	77	17	1	19	15	3

I think it is needless to explain the *aforegoing Operations*; what has been already done in *Money*, is sufficient.

For the like is to be understood, both in *Weight* and *Measure*, to which I proceed.



Of MEASURE.

THERE are 5 Sorts of *Measure* used in Great Britain, viz. *Liquid Measure*, *Dry Measure*, *Long Measure*, *Land Measure*, *Cloth Measure*. To which may be added, the *Measure of Time*.

The least Denomination of *Liquid Measure*, is a *Pint* (and 2 Sorts of *Liquid Measure* are used, viz. *Wine*, and *Beer*) from whence ariseth the following Tables.

Wine Measure.

2 Pints	make	1 Quart.
4 Quarts	----	1 Gallon.
63 Gallons	----	1 Hoghead.
4 Hogheads	--	1 Tun.

Beer Measure.

2 Pints	make	1 Quart.
4 Quarts	----	1 Gallon.
8½ Gallons	----	1 Firkin.
2 Firkins	----	1 Kilderkin.
2 Kilderkins	--	1 Barrel.
1½ Barrel	----	1 Hoghead.
2 Hogheads	--	1 Butt.
2 Butts	----	1 Tun.

Note. In *London*, there is a Distinction between *Ale* and *Beer Measure*, viz. that 8 Gallons make a *Firkin*

Firkin of Ale, and 9 Gallons a *Firkin of Beer*. But in all other Parts, it is measured, as in the Table.

The *Ale* and *Beer Pint* are both equal, and do exceed the *Wine Pint*, by almost *one fifth Part*; accounting 28 $\frac{1}{2}$ Cubic Inches, to contain a *Pint of Wine*.

The least Denomination of *Dry Measure* is also a *Pint*, which is less than the *Ale* and *Beer Pint*, by almost a *two and twentieth Part*, and greater than a *Wine Pint*, by a little more than *one sixth Part*; from whence ariseth the following Table, viz.

2 Pints	make	1 Quart.
4 Quarts	-----	1 Gallon.
2 Gallons	-----	1 Peck.
4 Pecks	-----	1 Bushel.
8 Bushels	-----	1 Quarter.
10 Quarter	-----	1 Last.

The least Denomination of *Long Measure* is a *Barly Corn*; from whence ariseth this Table, viz.

3 Barly Corns	make	1 Inch.
12 Inches	-----	1 Foot.
3 Feet	-----	1 Yard.
20 Yards	-----	1 Score.
11 Score	-----	1 Furlong.
8 Furlongs	-----	1 Mile.
3 Miles	-----	1 League.
20 Leagues	-----	1 Degree.

Note. By an Experiment made by Mr. Norwood, betwixt *London* and *York*, Anno 1635, it was found that 23 Leagues, 4 Furlongs, 80 Yards do make a Degree on the Surface of the Earth and Sea; though it is still the Practice of Mariners to account but 20 Leagues to a Degree.

The

38 Of MEASURE.

The least Denomination of *Land Measure*, that is generally used, is a Foot; from whence ariseth this Table, *viz.*

3 Feet	make	1 Yard.
5½ Yards	- - - - -	1 Pole or Perch.
40 Square Pole	- -	1 Rood
4 Rood	- - - - -	1 Acre.

The least Denomination of *Cloth Measure*, is a *Nail*; from whence ariseth this Table, *viz.*

4 Nails	make	1 Quarter.
4 Quarters	- - - - -	1 Yard.
5 Quarters	- - - - -	1 Ell.

A *Flemish Ell* is sometimes used, which contains 3 *Quarters*.

The least Part of *Time*, that can be measured by a mechanick Instrument, as a Clock, Watch, &c. is a *Second*; from whence ariseth the following Table, *viz.*

60 Seconds	make	1 Minute.
60 Minutes	- - - - -	1 Hour.
24 Hours	- - - - -	1 Day.
7 Days	- - - - -	1 Week.
4 Weeks	- - - - -	1 Month.
13 m. 1 d. 6 h.	- - - - -	1 Year.

The Sun apparently compleats its Revolution, according to Modern Observations, in 365 *Days*, 5 *Hours*, 48 *Minutes*, 57 *Seconds*, &c. which Space of Time is called a Solar Year; and is 11 *Minutes*, 3 *Seconds* less than the common Year used with us: And from hence comes that Difference, there is between the *Old Stile* and the *New*.

Tuns

Tuns	hh ^{ds}	ga.	qu.	pi.
37124	2	21	3	1
26719	2	32	3	1
31274	2	44	3	0
65372	3	56	2	1
84268	3	61	2	1
27197	2	28	1	1

Last	qu.	bu.	pe	ga.	qu.	pi.
27597	5	5	2	1	2	1
31267	6	6	2	1	3	1
42719	7	7	3	1	2	1
56718	8	6	3	1	3	1
37192	9	5	2	1	2	1
82428	8	4	1	1	1	1

T. 271958 1 57 0 1 L. 277925 7 5 2 0 0 0

Deg.	le.	m.	fu.	sc.	ya.	fe.	in.	ba.
157	13	2	3	7	13	2	10	2
113	14	2	4	8	14	1	11	2
27	15	2	5	9	15	1	10	1
19	16	2	6	8	16	1	9	1
8	17	1	7	7	17	1	8	1
7	18	1	0	8	18	2	7	2

Acres	ro.	po.	ya.
27194	3	27	5
12346	2	39	4½
51297	2	18	3
18671	3	26	2½
42719	3	34	1
53718	1	15	2½

D. 335 17 1 5 7 17 0 10 0 A. 205949 2 2 2

Ells	qa.	na.
267	2	2
164	3	2
159	4	2
718	3	1
271	2	1
926	1	3

Yds.	qa.	na.
471	3	2
375	2	1
264	2	1
197	3	3
548	2	3
352	1	1

Mo.	w.	d.	h.	m.
971	2	5	13	27
719	3	6	14	30
719	1	5	15	22
137	1	4	19	19
719	2	3	22	50
837	2	2	11	40

E. 2508 2 3 Y. 2210 3 3 M. 4105 3 1 1 8

	Tuns	hh ^{ds}	ga.	qu.	pi.
Bought	7129	2	21	2	1
Sold	4827	3	13	3	1

Last	qu.	bu.	pe	ga.	qu.	pi.
359	3	7	3	1	2	0
260	6	5	3	1	3	1

Unfol T. 2301 3 7 3 0 L. 98 7 1 3 1 2 1

	Deg.	le.	m.	fu.	sc.	ya.	fe.	in.	ba.
From	259	13	2	7	10	13	1	10	2
Take	168	12	1	5	7	10	2	11	1

Acr.	ro.	po.	ya.
719	3	13	3
532	2	20	4

Re. Deg. 91 01 1 2 3 2 1 11 1 A. 187 0 32 4½

	Ells	qa.	na.
From	5794	2	3
Take	5729	4	1

	Yds.	qa.	na.
	3679	1	2
	2572	2	3

Mon.	w.	da.	ho.	m.	se.
946	2	3	17	11	10
372	3	4	10	13	23

Rem. E. 64 3 2 Y. 1106 2 3 M. 73 2 6 6 57 47

I have

I have omitted giving any Examples of *Multiplication* and *Division*, in *Weight* and *Measure*, as but of little use, and differing from that of *Money*, only in the Denomination of its Parts; it being my Design to render this Treatise as compendious and useful as I am able, and therefore shall now proceed to shew, how easily the *Amount* of any *Quantity* at any *Price*, may be obtained by one easy general Rule, viz.

R U L E.

Multiply the *Price* by the *Quantity*.

$$\begin{array}{r}
 \text{9 Pound at } 10\frac{1}{2} \text{ per lb} \\
 \underline{9} \\
 \text{Sh. 7 } 10\frac{1}{2}
 \end{array}$$

$$\begin{array}{r}
 \text{10 Pounds at } 9\frac{1}{4} \text{ per lb} \\
 \underline{10} \\
 \text{Sh. 8 } 1\frac{1}{2}
 \end{array}$$

$$\begin{array}{r}
 \text{17 Yards at } 3 \text{ } 6 \text{ per Y.} \\
 \underline{7 \text{ } 10} \\
 \text{£. 1 } 15 \text{ } 0 \\
 \text{1 } 4 \text{ } 6 \\
 \hline
 \text{£. 2 } 19 \text{ } 6
 \end{array}$$

$$\begin{array}{r}
 \text{18 Yds. at } 13 \text{ } 4 \text{ per Y.} \\
 \underline{8 \text{ } 10} \\
 \text{£. 6 } 13 \text{ } 4 \\
 \text{5 } 6 \text{ } 8 \\
 \hline
 \text{£. 12 } 0 \text{ } 0
 \end{array}$$

$$\begin{array}{r}
 \text{25 Ells at } 5 \text{ } 10\frac{1}{4} \text{ per E.} \\
 \underline{5 \text{ } 10} \\
 \text{£. 2 } 18 \text{ } 6\frac{1}{2} \\
 \underline{2} \\
 \text{£. 5 } 17 \text{ } 1 \\
 \text{1 } 9 \text{ } 3\frac{1}{4} \\
 \hline
 \text{£. 7 } 6 \text{ } 4\frac{1}{4}
 \end{array}$$

$$\begin{array}{r}
 \text{36 Ells at } 11 \text{ } 5\frac{1}{4} \text{ per E.} \\
 \underline{6 \text{ } 10} \\
 \text{£. 5 } 14 \text{ } 7 \\
 \underline{3} \\
 \text{£. 17 } 3 \text{ } 9 \\
 \text{3 } 8 \text{ } 9 \\
 \hline
 \text{£. 20 } 12 \text{ } 6
 \end{array}$$

Because

Practical Examples.

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Because 5 Times 5 is 25, and 6 Times 6 is 36:
The two preceeding Examples may be more com-
pendiously wrought as follows.

$$\begin{array}{r}
 \text{25 Ells at } 5 \text{ } 10\frac{1}{4} \text{ per E.} \\
 \hline
 \text{£. } 1 \text{ } 9 \text{ } 3\frac{1}{4} \\
 \hline
 \text{£. } 7 \text{ } 6 \text{ } 4\frac{1}{4}
 \end{array}$$

$$\begin{array}{r}
 \text{36 Ells at } 11 \text{ } 5\frac{1}{2} \text{ per E.} \\
 \hline
 \text{£. } 3 \text{ } 8 \text{ } 9 \\
 \hline
 \text{£. } 20 \text{ } 12 \text{ } 6
 \end{array}$$

$$\begin{array}{r}
 \text{244 Doz. at } 6 \text{ } 8 \text{ per D.} \\
 \hline
 \text{£. } 3 \text{ } 6 \text{ } 8 \\
 \hline
 \text{£. } 33 \text{ } 6 \text{ } 8 \\
 \hline
 \text{£. } 66 \text{ } 13 \text{ } 4 \\
 \hline
 \text{£. } 81 \text{ } 6 \text{ } 8
 \end{array}$$

$$\begin{array}{r}
 \text{353 Doz. at } 16 \text{ } 9 \text{ per D.} \\
 \hline
 \text{£. } 8 \text{ } 7 \text{ } 6 \\
 \hline
 \text{£. } 83 \text{ } 15 \text{ } 0 \\
 \hline
 \text{£. } 251 \text{ } 5 \text{ } 0 \\
 \hline
 \text{£. } 295 \text{ } 12 \text{ } 9
 \end{array}$$

$$\begin{array}{r}
 \text{2475 } \textcircled{\text{P}} \text{ at } 1 \text{ } 3 \text{ } 6 \text{ per } \textcircled{\text{P}} \\
 \hline
 \text{£. } 11 \text{ } 15 \text{ } 0 \\
 \hline
 \text{£. } 117 \text{ } 10 \text{ } 0 \\
 \hline
 \text{£. } 1175 \text{ } 0 \text{ } 0 \\
 \hline
 \text{£. } 2350 \text{ } 0 \text{ } 0 \\
 \hline
 \text{£. } 2908 \text{ } 2 \text{ } 6
 \end{array}$$

$$\begin{array}{r}
 \text{3895 } \textcircled{\text{P}} \text{ at } 2 \text{ } 14 \text{ } 5 \text{ per } \textcircled{\text{P}} \\
 \hline
 \text{£. } 27 \text{ } 4 \text{ } 2 \\
 \hline
 \text{£. } 272 \text{ } 1 \text{ } 8 \\
 \hline
 \text{£. } 2720 \text{ } 16 \text{ } 8 \\
 \hline
 \text{£. } 8162 \text{ } 10 \text{ } 0 \\
 \hline
 \text{£. } 10597 \text{ } 12 \text{ } 11
 \end{array}$$

G

Yds.

Practical Examples.

Yds.	l.	s.	d.	
7½	at	1	3	6 per Y.
			7	
	£.	8	4	6
½ Y.		5	10½	
	£.	8	10	4½

Yds.	l.	s.	d.	
8½	at	1	19	6 per Y.
			8	
	£.	15	16	0
¼ Y.			9	10½
	£.	16	5	10½

Yds.	l.	s.	d.	
16½	at	2	14	8 per Y.
			6	10
	£.	27	6	8
		16	8	0
½ Y.		1	7	4
	£.	45	2	0

Yds.	l.	s.	d.	
15½	at	3	12	10 per Y.
			5	10
	£.	36	8	4
		18	4	2
½ Y.		1	16	5
	£.	56	8	11

⊕	l.	s.	d.	
35½	at	1	17	9 per ⊕
			5	10
	£.	18	17	6
			3	
	£.	56	12	6
		9	8	9
⅓ ⊕			18	10½
¼ ⊕			9	5½
	£.	67	9	6½

⊕	l.	s.	d.	
54½	at	1	12	11 per ⊕
			4	10
	£.	26	9	2
			5	
	£.	132	5	10
		10	11	8
⅓ ⊕		1	6	5½
¼ ⊕			13	2½
	£.	144	17	2½

The preceeding *Examples* are so plain, that I think it superfluous to add any thing thereupon.

Wares sold by the Stone.

If the Stone consist of 8 Pounds. Then

4 lb	will be	½	} of a Stone.
3	-----	⅓	
2	-----	¼	
1	-----	⅕	
		⅛	

Wares

fl. lb. s. d.
6 7 at 2 3 per Stone.
6

Sb. 13 6
4 lb. 1 1 1/2
2 6 1/2
1 3 1/2
Sb. 15 5 1/2

fl. lb. s. d.
8 7 at 3 4 1/2 per Stone.
8

L. 1 6 10
4 lb. 1 8
2 10
1 5
L. 1 9 9

fl. lb. s. d.
12 5 at 14 8 1/2 per Stone.
2 10

L. 7 7 3 1/2
1 9 5 1/2
4 lb. 7 4 1/2
1 1 10
L. 9 5 11 1/2

fl. lb. s. d.
14 5 at 15 9 1/2 per Stone.
4 10

L. 7 17 8 1/2
3 3 1
4 lb. 7 10 1/2
1 1 11 1/2
L. 11 10 7 1/2

fl. lb. s. d.
25 3 at 3 6 1/2 per Stone.
5 10

L. 1 15 5
2
L. 3 10 10
17 8 1/2
2 lb. 10 1/2
1 5 1/2
L. 4 9 10 1/2

fl. lb. s. d.
36 3 at 4 11 1/2 per Stone.
6 10

L. 2 9 9 1/2
3
L. 7 9 4 1/2
1 9 10 1/2
2 lb. 1 2 1/2
1 7 1/2
L. 9 1 1

fl. lb. l. s. d.
496 1 at 1 7 6 per S.
6 10

L. 13 15 0
9 10
L. 137 10 0
4
L. 550 0 0
123 15 0
8 5 0
1 lb. 3 5 1/2
L. 682 3 5 1/2

fl. lb. l. s. d.
578 1 at 2 14 8 per S.
8 10

L. 27 6 8
7 10
L. 273 6 8
5
L. 1366 13 4
191 6 8
21 17 4
1 lb. 6 10
L. 1580 4 2

In the preceeding *Examples*, to obtain the Price of the *Pounds*, the Price of one *Stone* is divided by 2, for the Price of 4 *lb.* because 4 *lb.* is the half of a *Stone*, the half of the Price of 4 *lb.* is the Price of 2 *lb.* and half Price of 2 *lb.* is the Price of 1 *lb.*

Also the Price of 4 *lb.* divided by 4, gives the Price of 1 *lb.* again, the Price of a *Stone*, divided by 4, gives the Price of 2 *lb.* and the Price of a *Stone* divided by 8, gives the Price of 1 *lb.*

The *Stone* of 8 *lb.* is used by *Butchers*, and *Sellers* of *Flesh*. But the *Stone* used in *Merchandise*, contains 14 *lb.* the even Parts of which are, viz.

7 *lb.* the $\frac{1}{2}$ of a *Stone*,
2 $\frac{1}{4}$

sto. lb.	s.	d.	
7 13	at	5 0	per Stone.
		7	
£. 1	15	0	
7 lb.	2	6	
2	0	8 $\frac{1}{2}$	
2	0	8 $\frac{1}{2}$	
2	0	8 $\frac{1}{2}$	
£. 1	19	7 $\frac{1}{2}$	

sto. lb.	s.	d.	
9 13	at	7 6	per Stone.
		9	
£. 3	7	6	
7 lb.	3	9	
2	1	0 $\frac{1}{4}$	
2	1	0 $\frac{1}{4}$	
2	1	0 $\frac{1}{4}$	
£. 3	14	5 $\frac{1}{4}$	

sto. lb.	s.	d.	
16 12	at	11 4 $\frac{1}{2}$	per Stone.
		6 10	
£. 5	13	9	
3	8	3	
7 lb.	5	8 $\frac{1}{4}$	
2	1	7 $\frac{1}{2}$	
2	1	7 $\frac{1}{2}$	
1		9 $\frac{1}{4}$	
£. 9	11	9	

sto. lb.	s.	d.	
18 12	at	16 10 $\frac{1}{4}$	per Stone.
		8 10	
£. 8	8	6 $\frac{1}{2}$	
6	14	10	
7 lb.	8	5	
2	2	4 $\frac{1}{4}$	
2	2	4 $\frac{1}{4}$	
1	1	2 $\frac{1}{4}$	
£. 15	17	9 $\frac{1}{4}$	

sto.

Practical Examples.

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$$\begin{array}{r}
 \text{sto. lb.} \quad l. \quad s. \quad d. \\
 35 \quad 8 \quad \text{at} \quad 1 \quad 4 \quad 9 \quad \text{per Sto.} \\
 \hline
 \quad \quad \quad 5 \quad 10 \\
 \hline
 \text{£.} \quad 12 \quad 7 \quad 6 \\
 \hline
 \quad \quad \quad 3 \\
 \hline
 \text{£.} \quad 37 \quad 2 \quad 6 \\
 \quad \quad 6 \quad 3 \quad 9 \\
 7 \text{ lb.} \quad 12 \quad 4 \frac{1}{2} \\
 1 \quad \quad \quad 1 \quad 9 \\
 \hline
 \text{£.} \quad 44 \quad 0 \quad 4 \frac{1}{2}
 \end{array}$$

$$\begin{array}{r}
 \text{sto. lb.} \quad l. \quad s. \quad d. \\
 46 \quad 8 \quad \text{at} \quad 1 \quad 14 \quad 7 \quad \text{per Stone;} \\
 \hline
 \quad \quad \quad 6 \quad 10 \\
 \hline
 \text{£.} \quad 17 \quad 5 \quad 10 \\
 \hline
 \quad \quad \quad 4 \\
 \hline
 \text{£.} \quad 69 \quad 3 \quad 4 \\
 \quad \quad 10 \quad 7 \quad 6 \\
 7 \text{ lb.} \quad 17 \quad 3 \frac{1}{2} \\
 1 \quad \quad \quad 2 \quad 5 \frac{1}{2} \\
 \hline
 \text{£.} \quad 80 \quad 10 \quad 7
 \end{array}$$

$$\begin{array}{r}
 \text{sto. lb.} \quad l. \quad s. \quad d. \\
 245 \quad 6 \quad \text{at} \quad 2 \quad 6 \quad 7 \frac{1}{4} \quad \text{per Sto.} \\
 \hline
 \quad \quad \quad 5 \quad 10 \\
 \hline
 \text{£.} \quad 23 \quad 6 \quad 0 \frac{1}{2} \\
 \quad \quad 4 \quad 10 \\
 \hline
 \text{£.} \quad 233 \quad 0 \quad 5 \\
 \quad \quad \quad 2 \\
 \hline
 \text{£.} \quad 466 \quad 0 \quad 10 \\
 \quad \quad 93 \quad 4 \quad 2 \\
 \quad \quad 11 \quad 13 \quad 0 \frac{1}{4} \\
 2 \text{ lb.} \quad 6 \quad 7 \frac{3}{4} \\
 2 \text{ lb.} \quad 6 \quad 7 \frac{3}{4} \\
 2 \text{ lb.} \quad 6 \quad 7 \frac{3}{4} \\
 \hline
 \text{£.} \quad 571 \quad 17 \quad 11 \frac{1}{2}
 \end{array}$$

$$\begin{array}{r}
 \text{sto. lb.} \quad l. \quad s. \quad d. \\
 386 \quad 6 \quad \text{at} \quad 2 \quad 18 \quad 5 \frac{1}{2} \quad \text{per Sto.} \\
 \hline
 \quad \quad \quad 6 \quad 10 \\
 \hline
 \text{£.} \quad 29 \quad 4 \quad 7 \\
 \quad \quad 8 \quad 10 \\
 \hline
 \text{£.} \quad 292 \quad 6 \quad 5 \\
 \quad \quad \quad 3 \\
 \hline
 \text{£.} \quad 876 \quad 19 \quad 3 \\
 \quad \quad 233 \quad 16 \quad 8 \\
 \quad \quad 17 \quad 10 \quad 9 \\
 2 \text{ lb.} \quad 8 \quad 4 \\
 2 \text{ lb.} \quad 8 \quad 4 \\
 2 \text{ lb.} \quad 8 \quad 4 \\
 \hline
 \text{£.} \quad 1129 \quad 11 \quad 8
 \end{array}$$

$$\begin{array}{r}
 \text{sto. lb.} \quad l. \quad s. \\
 3729 \quad 1 \quad \text{at} \quad 3 \quad 10 \quad \text{per Sto.} \\
 \hline
 \quad \quad \quad 9 \quad 10 \\
 \hline
 \text{£.} \quad 35 \quad 0 \\
 \quad \quad 2 \quad 10 \\
 \hline
 \text{£.} \quad 350 \quad 0 \\
 \quad \quad 7 \quad 10 \\
 \hline
 \text{£.} \quad 3500 \quad 0 \\
 \quad \quad \quad 3 \\
 \hline
 \text{£.} \quad 10500 \quad 0 \\
 7 \frac{1}{3} \quad 10 \quad 2450 \quad 0 \\
 2 \frac{1}{2} \quad 10 \quad 70 \quad 0 \\
 \hline
 \quad \quad 31 \quad 10 \\
 5 \quad 1 \text{ lb.} \quad 5 \\
 \hline
 \text{£.} \quad 13051 \quad 15
 \end{array}$$

$$\begin{array}{r}
 \text{sto. lb.} \quad l. \quad s. \quad d. \\
 4987 \quad 1 \quad \text{at} \quad 4 \quad 11 \quad 5 \quad \text{per} \\
 \text{Stone.} \quad \quad \quad 7 \quad 10 \\
 \hline
 \text{£.} \quad 45 \quad 14 \quad 2 \\
 \quad \quad 8 \quad 10 \\
 \hline
 \text{£.} \quad 457 \quad 1 \quad 8 \\
 \quad \quad 9 \quad 10 \\
 \hline
 \text{£.} \quad 4570 \quad 16 \quad 8 \\
 \quad \quad \quad 4 \\
 \hline
 \text{£.} \quad 18283 \quad 6 \quad 8 \\
 7 \frac{1}{4} \quad 11 \quad 5 \quad 4113 \quad 15 \quad 0 \\
 2 \frac{1}{2} \quad 13 \quad 0 \frac{1}{2} \quad 364 \quad 13 \quad 4 \\
 \hline
 \quad \quad 6 \quad 6 \frac{1}{4} \quad 31 \quad 19 \quad 11 \\
 1 \text{ lb.} \quad \quad \quad 6 \quad 6 \frac{1}{4} \\
 \hline
 \text{£.} \quad 22794 \quad 1 \quad 5 \frac{1}{4}
 \end{array}$$

In

In the preceeding Examples, the Price of a Stone is divided by 2, for the Price of 7 th , and the Price of a Stone by 7, for the Price of 2 th , and the Price of 2 th by 2, for the Price of 1 th .

Wares sold by the Dozen.

All the even Parts of a Dozen will be, *viz.*

6	th	the	$\frac{1}{2}$	} of a Dozen.
4	-	-	$\frac{1}{3}$	
3	-	-	$\frac{1}{4}$	
2	-	-	$\frac{1}{6}$	
1	-	-	$\frac{1}{12}$	

<i>D. lb.</i>	<i>s.</i>	<i>d.</i>	
7 11	at	7 6	per Dozen.
		<u>7</u>	
<i>£.</i>	<i>s.</i>	<i>d.</i>	
6 lb.	2 12	6	
3		3 9	
2		1 10 $\frac{1}{2}$	
		1 3	

<i>D. lb.</i>	<i>s.</i>	<i>d.</i>	
9 11	at 13	5	per Dozen.
		9	
	<hr/>		
<i>£.</i> 6	0	9	
6 <i>lb.</i>	6	8 $\frac{1}{2}$	
3	3	4 $\frac{1}{4}$	
2	2	2 $\frac{1}{4}$	

D. lb.	l.	s.	d.	
14 9 at	3	6	per D.	
	4	10		
	£. 11	15	0	
	4	14	0	
6 lb.		11	9	
3		5	10 $\frac{1}{2}$	

D. lb.	l. s. d.	
16 9 at	1 14 9	per D.
	6 10	
	<hr/>	
£.	17 7 6	
	10 8 6	
6 lb.	17 4 $\frac{1}{2}$	
3	8 8 $\frac{1}{4}$	

D. lb. *s. d.*
 56 7 at 8 $4\frac{1}{2}$ per Doz.
 6 10

 £. 4 3 9
 5

 £. 20 18 9
 2 10 3
 6 lb. 4 $2\frac{1}{4}$
 8 $\frac{1}{4}$

 £. 23 13 $10\frac{1}{2}$

D. lb. *s.* *d.*
 72 7 at 12 $5\frac{3}{4}$ per Doz.
 2 10

 £. 6 4 $9\frac{1}{2}$
 7

 £. 43 13 $6\frac{1}{2}$
 1 4 $11\frac{1}{2}$
 6 lb. 6 $2\frac{3}{4}$
 1 0 $\frac{1}{4}$

 £. 45 5 9

D.

Practical Examples.

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doz. lb. l. s. d.
275 5 at 1 11 6 per Doz.

5 10
£ 15 15 0
7 10
£. 157 10 0
2
£. 315 0 0
110 5 0
7 17 6
4 lb 10 6
1 2 7½
£. 433 15 7½

doz. lb. l. s. d.
482 5 at 1 14 10 per Doz.

2 10
£. 17 8 4
8 10
£. 174 3 4
4
£. 696 13 4
139 6 8
3 9 8
4 lb. 11 7½
1 2 10½
£. 840 4 2

doz. lb. s. d.
2756 3 at 8 11½ per Doz.

6 10
£. 4 9 7
5 10
£. 44 15 10
7 10
£. 447 18 4
2
£. 895 16 8
313 10 10
22 7 11
2 13 9
2 lb. 1 5¾
1 8¾
£. 1234 11 4½

doz. lb. s. d.
4795 3 at 19 5¼

5 10
£. 9 14 4½
9 10
£. 97 3 9
7 10
£. 971 17 6
4
£. 3887 10 0
680 6 3
87 9 4½
4 17 2¾
2 lb. 3 2¾
1 1 7¾
£. 4660 7 7¾

In the preceeding *Examples*, of Goods sold by the *Dozen*, the Price of the Pounds are obtained by dividing the Price of 1 *Dozen* by 2, it gives the Price of 6 *Pounds*, the Price of 6 *Pounds* by 2, it gives the Price of 3 *Pounds*, also the Price of 6 *Pounds* by 3, gives the Price of 2 *Pounds*. Again, the Price of 6 *Pounds* divided by 6, gives the Price of

of 1 Pound, the Price of 1 Dozen divided by 3, gives the Price 4 Pounds, and the Price of 4 Pounds by 4, gives the Price of 1 Pound, the Price of 1 Dozen divided by 6, gives the Price of 2 Pounds, and the Price of 2 Pounds by 2, gives the Price of 1 Pound.

Wares sold by the Pound.

The even Parts of a Pound *Averdupoize*, will be, viz.

$$\left. \begin{array}{r} 8 \frac{3}{4} \text{ the} \\ 4 \text{ -----} \\ 2 \text{ -----} \end{array} \right\} \frac{1}{2} \frac{1}{4} \frac{1}{8} \text{ of a Pound.}$$

lb	3	s.	d.	
3	15	at	4	6 per lb
			3	
		<i>Sh.</i>	13	6
8	3		2	3
4			1	1½
2				6¼
1				3¼
		<i>Sh.</i>	17	8½

lb	3	s.	d.	
5	15	at	13	9 per lb
			5	
		£.	3	8 9
8	3		6	10½
4			3	5¼
2			1	8½
1			0	10¼
		£.	4	1 7½

lb	3	l.	s.	d.	
14	13	at	1	3	4 per lb
			4	10	
		£.	11	13	4
			4	13	4
8	3		11	8	
4			5	10	
1			1	5½	
		£.	17	5	7½

lb	3	l.	s.	d.	
15	13	at	1	13	9 per lb
			5	10	
		£.	16	17	6
			8	8	9
8	3		16	10½	
4			8	5¼	
1			2	1¼	
		£.	26	13	8

lb

Practical Examples.

49

lb 3 s. d.
39 11 at 9 7½ per lb

9 10
£. 4 16 3
3
£. 14 8 9
4 6 7½
8 3 4 9¼
2 1 2¼
1 7
£. 19 1 11½

lb 3 s. d.
48 11 at 14 6½ per lb

8 10
£. 7 5 2½
4
£. 29 0 10
5 16 2
8 3 7 3
2 1 9¼
1 10¼
£. 35 6 11½

lb 3 l. s. d.
376 9 at 2 7 9½ per lb

6 10
£. 23 17 11
7 10
£. 238 19 2
3
£. 716 17 6
167 5 5
14 6 9
8 3 1 3 10¼
1 2 11¼
899 16 6½

lb 3 l. s. d.
485 9 at 3 16 4¼ per lb

5 10
£. 38 3 11½
8 10
£. 381 19 7
4
£. 1527 18 4
305 11 8
19 1 11¼
8 3 1 18 2¼
1 4 9¼
1854 14 11¼

lb 3 s. d.
2749 7 at 7 8¼ per lb

9 10
£. 3 16 9½
4 10
£. 38 7 8¼
7 10
£. 383 17 1
2
£. 767 14 2
268 13 11¼
15 7 1
3 9 1½
4 3 1 11
2 11¼
1 5½
£. 1055 7 7¼

lb 3 s. d.
3127 7 at 16 11¼ per lb

7 10
£. 8 9 5½
2 10
£. 84 14 9¼
1 10
£. 847 7 11
3
£. 2542 3 9
84 14 9¼
16 18 11¼
5 18 7½
4 3 4 2½
2 2 1½
1 1 C¼
£. 2650 3 6¼

H

lb

Practical Examples.

lb 3 l. s. d.
4 5 at 1 7 9½ per lb

4
£. 5 11 2½
4 3 6 11½
1 1 8½
£. 5 19 10½

lb 3 l. s. d.
6 5 at 2 8 6 per lb

6
£. 14 11 5½
4 3 12 1½
1 3 0½
£. 15 6 7½

lb 3 s. d.
12 3 at 4 3¼ per lb

2 10
£. 2 2 8½
8 6½
2 3 6¼
1 3
£. 2 12 0½

lb 3 s. d.
13 3 at 13 9½ per lb

3 10
£. 6 17 11
2 1 4½
2 3 1 8½
1 10½
£. 9 1 10½

lb 3 l. s. d.
56 1 at 2 7 11
per lb 6 10

£. 23 19 2
4 3 7 11
4 11 11½
Sb. 2 11½
£. 119 15 10
14 7 6
1 3 2 11½
£. 134 6 3½

lb 3 l. s. d.
65 1 at 3 18 9
per lb 5 10

£. 39 7 6
4 3 18 9
4 19 8½
Sb. 4 11
£. 236 5 0
19 13 9
1 3 4 11
£. 256 3 8

In the preceeding *Examples*, the Price of the *Ounces* are obtained as followeth, *viz.*

In *Example 1.* I divide the Price of 1 *Pound* by 2, for the Price of 8 *Ounces*, and the Price of 8 3 by 2, for the Price of 4 3, and the Price of 4 3 by 2, for the Price of 2 3, and the Price of 2 3 by 2, for the Price of 1 3.

In *Example 2.* The Price of 4 3 is divided by 4, for the Price of 1 3.

In

In *Example 3.* The Price of 8 oz. is divided by 4, for the Price of 2 oz. £3c.

In *Example 4.* The Price of 8 oz. is divided by 8, for the Price of 1 oz.

In *Example 5.* The Price of 1 lb. is divided by 4, for the Price of 4 oz. £3c.

In *Example 7.* The Price of 1 lb. is divided by 8, for the Price of 2 oz. £3c.

In *Example 8.* The Price of 1¹/₂ lb. is divided by 4, for the Price of 4 oz. and the Price of 4 oz. by 4, for the Price of 1 oz.

Wares sold by the Yard.

The even Parts of a Yard are,

2 Quarters, or 8 Nails, the $\frac{1}{2}$ } of a Yard.
 1 4 $\frac{1}{4}$
 2 $\frac{1}{8}$

ya.	qu.	na.	s.	
9	3	3	at	6 per Yard.
<hr/>				
				9
				£. 2 14
2	qrs.			3
1	qr.			6
2	nails.			9
1	nail.			4 $\frac{1}{2}$
<hr/>				
				£. 2 19 7 $\frac{1}{2}$

ya.	qu.	na.	s.	d.
8	3	3	at	17 9 per Yd.
<hr/>				
				8
				£. 7 2 0
2	qrs.			8 10 $\frac{1}{2}$
1	qr.			5 $\frac{1}{2}$
2	nails.			2 $\frac{1}{2}$
1	nail.			1 $\frac{1}{4}$
<hr/>				
				£. 7 18 7 $\frac{1}{2}$

ya.	qu.	na.	l.	s.	d.
13	3	1	at	1 7 6 per	
Yard.					
<hr/>					
					3 10
					£. 13 15 0
					4 2 6
2	qrs.				13 9
1	qr.				6 10 $\frac{1}{2}$
1	nail.				1 8 $\frac{1}{2}$
<hr/>					
					£. 18 19 10

ya.	qrs.	na.	s.	d.
17	3	1	at	4 10 per Yd.
<hr/>				
				7 10
				£. 2 8 4
				1 13 10
2	qrs.			2 5
1	qr.			1 2 $\frac{1}{2}$
1	nail.			3 $\frac{1}{2}$
<hr/>				
				£. 4 6 1

ya.	qu.	na.	s.	d.	
75	1	1	at	14	9 $\frac{1}{4}$ per
Yard.				5	10
				£. 7	7 8 $\frac{1}{2}$
					7
				£. 51	13 11 $\frac{1}{2}$
					3 13 10 $\frac{1}{4}$
1 qr.				3	8 $\frac{1}{4}$
1 nail					11
				£. 55	12 5

ya.	qu.	na.			
67	1	1	at	12	Shil. per Yd.
				7	10
				£. 6	0
					6
				£. 36	0
					4 4
1 qu.					3
1 nail.					0 9
				£. 40	7 9

yar.	qu.	na.	s.	d.	
352	2	1	at	5	9 per Yd.
				2	10
				£. 2	17 6
					5 10
				£. 28	15 0
					3
				£. 86	5 0
					14 7 6
					11 6
2 qrs.				2	10 $\frac{1}{2}$
1 nail					4 $\frac{1}{4}$
				£. 101	7 2 $\frac{3}{4}$

yar.	qu.	na.	l.	s.	d.
564	2	1	at	3	12 8 $\frac{1}{2}$
per Yard.				4	10
				£. 36	7 1
					6 10
				£. 363	10 10
					5
				£. 1817	14 2
					218 2 6
					14 10 10
2 qrs.				1	16 4 $\frac{1}{4}$
1 nail					4 6 $\frac{1}{2}$
				£. 2052	8 4 $\frac{3}{4}$

yards	qu.	na.	l.	s.	
3728	0	3	at	1	13
per Yard.				8	10
				£. 16	10
					2 10
				£. 165	0
					7 10
				£. 1650	0
					3
				£. 4950	0
					1155 0
					33 0
					13 4
2 nails				4	1 $\frac{1}{2}$
1 nail					0 $\frac{3}{4}$
				£. 6151	10 2 $\frac{1}{4}$

yards	qu.	na.	l.	s.	d.
4875	0	3	at	2	12 11
per Yard.				5	10
				£. 26	9 2
					7 10
				£. 264	11 8
					8 10
				£. 2645	16 8
					4
				£. 10583	6 8
					2116 13 4
					185 4 2
					13 4 7
2 nails				6	7 $\frac{1}{4}$
1 nail					3 3 $\frac{1}{2}$
				£. 12898	18 7 $\frac{3}{4}$

Practical Examples.

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ya. qu. na.	l. s. d.		ya. qu. na.	l. s. d.
7 0 1 at	2 7 8		9 0 1 at	3 5 6
per Yard.	7		per Yard.	9
4 2 7 8	£. 16 13 8		4 3 5 6	£. 29 9 6
4 11 11	1 nail . . 2 11 $\frac{3}{4}$		4 16 4 $\frac{1}{2}$	1 nail . . . 4 1
Sb. 2 11 $\frac{3}{4}$	£. 16 16 7 $\frac{3}{4}$		Sb. . 4 1	£. 29 13 7

The *Quarters*, and *Nails* in the preceeding *Ex-amples*, are obtained as followeth, *viz.*

In *Example 1.* I divide the Price of a Yard by 2, for the Price of 2 qrs. and the Price of 2 qrs. by 2, for the Price of 1 qr. then the Price of 1 qr. by 2, for the Price of 2 *Nails*, and the Price of 2 *Nails* by 2, for the Price of 1 *Nail*.

In *Example 2.* I divide the Price of 1 qr. by 4, for the Price of 1 *Nail*.

In *Example 3.* I divide the Price of 1 Yard by 4, for the Price of 1 qr. &c.

In *Example 4.* I divide the Price of 2 qrs. by 8, for the Price of 1 *Nail*.

In *Example 5.* I divide the Price of 1 Yard, by 8, for the Price of 2 *Nails*, &c.

In *Example 6.* I divide the Price of 1 Yard by 4, it gives the Price of 1 qr. which divided by 4, gives the Price of 1 *Nail*.

Wares sold by the Hundred Weight.

The even Parts of an Hundred Weight.

2 qrs. or 56 lb.	the $\frac{1}{2}$	} of an ⌘
1 28	$\frac{1}{4}$	
16	$\frac{1}{7}$	
14	$\frac{1}{8}$	

Note, 16 Pounds being $\frac{1}{7}$ Part of 112 Pounds, or One hundred Weight, I shall wholly make Use thereof, in obtaining the Price of the Pounds, in the following *Examples*.

⊕ qu. lb. s.
9 3 27 at 14 per ⊕
9

£. 6 6

2 qu. ----- 7
1 qu. ----- 3 6
16 lb. ----- 2 0
8 lb. ----- 1 0
2 lb. ----- 3
1 lb. ----- 1 $\frac{1}{2}$

£. 6 19 10 $\frac{1}{2}$

⊕ qu. lb. l. s. d.
17 2 25 at 1 13 10
per ⊕ 7 10

£. 16 18 4
11 16 10

2 qu. ----- 16 11
16 lb. ----- 4 10
8 lb. ----- 2 5
1 lb. ----- 3 $\frac{1}{2}$

£. 29 19 7 $\frac{1}{2}$

⊕ qu. lb. s. d.
37 2 23 at 18 9 $\frac{1}{2}$ per
⊕ 7 10

£. 9 7 11
3

£. 28 3 9
6 11 6 $\frac{1}{2}$

2 qu. ----- 9 4 $\frac{1}{2}$
16 lb. ----- 2 8
4 ----- 8
2 ----- 4
1 ----- 2

£. 35 8 6 $\frac{1}{4}$

⊕ qu. lb. s. d.
8 3 27 at 16 11 per
⊕ 8

£. 6 15 4

2 qu. ----- 8 5 $\frac{1}{2}$
1 qu. ----- 4 2 $\frac{1}{4}$
16 lb. ----- 2 5
8 lb. ----- 1 2 $\frac{1}{2}$
2 lb. ----- 3 $\frac{1}{2}$
1 lb. ----- 1 $\frac{1}{4}$

£. 7 12 1

⊕ qu. lb. l. s. d.
15 2 25 at 2 16 8
per ⊕ 5 10

£. 28 6 8
14 3 4

2 qu. ----- 1 8 4
16 lb. ----- 8 1
8 lb. ----- 4 0 $\frac{1}{4}$
1 lb. ----- 6

£. 44 10 11 $\frac{1}{2}$

⊕ qu. lb. s. d.
49 2 23 at 19 11 $\frac{1}{4}$ per
⊕ 9 10

£. 9 19 9 $\frac{1}{2}$
4

£. 39 19 2
8 19 9 $\frac{1}{4}$

2 qu. ----- 9 11 $\frac{1}{4}$
16 lb. ----- 2 10 $\frac{1}{4}$
4 ----- 8 $\frac{1}{2}$
2 ----- 4 $\frac{1}{4}$
1 ----- 2

£. 49 13 0 $\frac{1}{4}$

⊕

Practical Examples.

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qu. lb	$l.$	$s.$	$d.$
375 3 21 at 2 17 8			
per qu	5	10	
	£. 28	16	8
		7	10
	£. 288	6	8
			3
	£. 865	0	0
	201	16	8
	14	8	4
2 qu. -----	1	8	10
1 qu. -----		14	5
16 lb -----		8	2 $\frac{1}{4}$
4 lb -----		2	0 $\frac{1}{2}$
1 lb -----			6
	£. 1083	19	0 $\frac{1}{4}$

qu. lb	$s.$	$d.$
3756 1 19 at 9 7		
per qu	6	10
	£. 4	15 10
		5 10
	£. 47	18 4
		7 10
	£. 479	3 4
		3
	£. 1437	10 0
	325	8 4
	23	19 2
	2	17 6
1 qu. ::::::::::	2	4 $\frac{3}{4}$
16 lb ::::::::::	1	4 $\frac{1}{4}$
2 lb ::::::::::		2
1 lb ::::::::::		1
	£. 1799	19 0

qu. lb	$l.$	$s.$	$d.$
486 3 21 at 2 16 10			
per qu	6	10	
	£. 28	8	4
		8	10
	£. 284	3	4
			4
	£. 1136	13	4
	227	6	8
	17	1	0
2 qu. -----	1	8	5
1 qu. -----		14	2 $\frac{1}{2}$
16 lb -----		8	1 $\frac{1}{4}$
4 lb -----		2	0 $\frac{1}{2}$
1 lb -----			6
	£. 1383	14	3

qu. lb	$s.$	$d.$
4287 1 19 at 10 11		
per qu	7	10
	£. 5	9 2
		8 10
	£. 54	11 8
		2 10
	£. 545	16 8
		4
	£. 2183	6 8
	109	3 4
	43	13 4
	3	16 5
1 qu. ::::::::::	2	8 $\frac{1}{4}$
16 lb ::::::::::	1	6 $\frac{1}{2}$
2 lb ::::::::::		2 $\frac{1}{4}$
1 lb ::::::::::		1
	£. 2340	4 3 $\frac{1}{2}$

The *Quarters* and *Pounds* in the preceeding Examples, are obtained as followeth, viz.

In *Example 1.* The Price of 1 ⌘ is divided by 2, for the Price of 2 *Quarters*, and the Price of 2 *Quarters* by 2, for the Price of 1 *Quarter*. Again, the Price of 1 ⌘ is divided by 7, for the Price of 16 *lb.* the Price of 16 *lb.* by 2, for the Price of 8 *lb.* the Price of 8 *lb.* by 4, for the Price of 2 *lb.* and the Price of 2 *lb.* by 2, for the Price of 1 *lb.*

In *Example 2.* The Price of 8 *lb.* is divided by 8, for the Price of 1 *lb.*

In *Example 3.* The Price of 16 *lb.* is divided by 4, for the Price of 4 *lb.* and the Price of 4 *lb.* by 2, for the Price of 2 *lb.* and the Price of 2 *lb.* by 2, for the Price of 1 *lb.*

In *Example 4.* The Price of 16 *lb.* divided by 4, gives the Price of 4 *lb.* and the Price of 4 *lb.* divided by 4, gives the Price of 1 *lb.*

In *Example 5.* The Price of 16 *lb.* divided by 8, gives the Price of 2 *lb.* &c.

It is to be observed, that in *Wares* bought and sold by the *Hundred Weight*; there is sometimes an Allowance to be made for *Tare* and *Trett*; and then, *Gross* is the Weight of a Commodity, with the *Hogshead*, *Chest*, or *Box*, or whatsoever else contains it.

Tare is the Weight of the *Hogshead*, *Chest*, or *Box*, and is sometimes rated at a certain Number of *Pounds per Hundred Weight*.

Trett is an Allowance of 4 *Pounds*, in every 104 *Pounds*, for Waste and Dust, on some Sorts of Goods; such as *Tobacco*, *Indico*, and all *Spices*.

Neat is the Weight of a Commodity, after the *Tare* and *Trett* is deducted.

Note. Where Allowance is given for *Trett*, when the *Tare* is taken from the *Gross*, the Remainder is called *Subtle*; which being divided by 26, gives the *Trett*, because 4 *Pounds* is the 26th Part of 104.

⌘

⊕ qu. ℥					
10	0	25	Gross		
1	8		Tare		
<hr/>					
9	3	17	Net at 13 per		
<hr/>					
		9			
<hr/>					
s. d.		£. 5	17		
8 1 10 $\frac{1}{4}$	2 qu.		6	6	
2 2 $\frac{1}{2}$	1 qu.		3	3	
d. 1 $\frac{1}{4}$	16 lb.		1	10 $\frac{1}{4}$	
	1 lb.			1 $\frac{1}{4}$	
<hr/>					
		£. 6	8	8 $\frac{1}{2}$	

⊕ qu. ℥					
8	0	25	Gross		
1	8		Tare		
<hr/>					
7	3	17	Net. at 14		
<hr/>					
per ⊕		7			
<hr/>					
s. d.		£. 5	0	11	
8 2 0 $\frac{3}{4}$	2 qu.		7	2 $\frac{1}{2}$	
2 3	1 qu.		3	7 $\frac{1}{4}$	
d. 1 $\frac{1}{2}$	16 lb.		2	0 $\frac{1}{2}$	
	1 lb.			1 $\frac{1}{2}$	
<hr/>					
		£. 5	13	10 $\frac{3}{4}$	

⊕ qu. ℥					
13	3	27	Gross		
2	12		Tare		
<hr/>					
13	1	15	Net. at 1		
<hr/>					
per ⊕		3	10		
<hr/>					
l. s. d.		£. 16	2	6	
7 1 12 3			4	16	9
2 4 7 $\frac{1}{2}$	1 qu.		8	0 $\frac{1}{4}$	
Sb. 2 3 $\frac{1}{2}$	8 lb.		2	3 $\frac{1}{2}$	
	4 lb.		1	1 $\frac{3}{4}$	
	2 lb.			6 $\frac{3}{4}$	
	1 lb.			3 $\frac{1}{4}$	
<hr/>					
		£. 21	11	7	

⊕ qu. ℥					
14	3	27	Gross		
2	12		Tare		
<hr/>					
14	1	14	Net. at 1		
<hr/>					
per ⊕		4	10		
<hr/>					
l. s. d.		£. 14	15	0	
7 1 9 6			5	18	0
2 4 2 $\frac{1}{2}$	1 qu.		7	4 $\frac{1}{2}$	
Sb. 2 1 $\frac{1}{2}$	8 lb.		2	1 $\frac{1}{4}$	
	4 lb.		1	0 $\frac{1}{2}$	
	2 lb.			6 $\frac{3}{4}$	
	1 lb.			3	
<hr/>					
		£. 21	4	3 $\frac{1}{2}$	

Practical Examples.

⊕ qu. lb

28 3 25 Gross
1 0 12 Tare

27 3 13 Neat at 18 per
⊕ 7 10

7 | 18
2 | 2 6 3/4
Sb. 1 3 1/4

9 0
2
£. 18 0
6 6

2 qu. 9
1 qu. 4 6
8 lb. 1 3 1/4
4 lb. 7 1/2
1 lb. 1 1/4

£. 25 1 6 1/2

⊕ qu. lb

37 3 25 Gross
1 0 12 Tare

36 3 13 Nt. at 19 4 per
⊕ 6 10

7 | 19 9
2 | 2 9
Sb. 1 4 1/2

£. 9 13 4
3
£. 29 0 0
5 16 0

2 qu. 9 8
1 qu. 4 10
8 lb. 1 4 1/4
4 lb. 8 1/4
1 qu. 2

£. 35 12 8 1/4

⊕ qu. lb

368 2 26 Gross
12 1 15 Tare

356 1 11 N. at 12 4
per ⊕ 6 10

7 | 1 12 4
2 | 4 7 1/4
Sb. 2 3 1/2

£. 16 3 4
5 10

£. 161 13 4 3

£. 485 0 0
80 16 8
9 14 0

1 qu. 8 1
8 lb. 2 3 1/4
2 lb. 6 1/4
1 lb. 3 1/4

£. 576 1 10 1/2

⊕ qu. lb

507 2 26 Gross
12 1 15 Tare

495 1 11 N. at 19 6
per ⊕ 5 10

7 | 1 19 6
2 | 4 7 1/2
Sb. 2 3 1/2

£. 19 15 0
9 10

£. 197 10 0 4

£. 790 0 0
177 15 0
9 17 6

1 qu. 9 10 1/4
8 lb. 2 9 1/4
2 lb. 8 1/4
1 lb. 4

£. 978 6 2 1/2

⊕

Practical Examples.

59

⊕ qu. lb.
7385 1 8 Gros
56 1 27 Tare

7328 3 9 N. at 7 per ⊕
8 10

7 | 7
2 | 1
—
56 0 6

£. 3 10
2 10

£. 35 0
3 10

£. 350 0
7

£. 2450 0
105 0
7 0
2 16

2 qu. 3 6
1 qu. 1 9
8 lb. 6
1 lb. 3/4

£. 2565 1 9 3/4

⊕ qu. lb.
6630 1 8 Gros
56 1 27 Tare

6573 3 9 Neat at 19 per
3 10

7 | 19
2 | 8 1/2
—
56 1 4 1/4

£. 9 10
7 10

£. 95 0
5 10

£. 950 0
6

£. 5700 0
475 0
66 10
2 17

2 qu. 9 6
1 qu. 4 9
8 lb. 1 4 1/4
1 lb. 2

£. 6245 2 9 1/4

⊕ qu. lb.
4297 2 14 Gros
729 0 6 Tare at 19 lb per ⊕

⊕ 3568 2 8 Subtle
137 1 0 Trett

⊕ 3431 1 8 Neat at 1 2 6 per ⊕
1 10

⊕ qu. lb.
7 | 4297 2 14
16 lb. 613 3 22
2 lb. 76 2 27
1 lb. 38 1 13
—
19 ⊕ 729 0 6

£. 11 5 0
3 10

£. 112 10 0
4 10

£. 1125 0 0
3

£. 3375 0 0
450 0 0
33 15 0
1 2 6

1 qu. 5 7 1/2
7 lb. 1 4 1/4
1 lb. 2 1/4

£. 3860 4 8 1/2
1 2

⊕ qu. lb.
137 1 0
26 | 3568 2 8
26

96
78

188
182

6
4

126
26

98

⊕

3829 3' 30 Gros
 410 1 11 $\frac{1}{4}$ Tare at 12 16 per 16

Φ 3419 2 $8\frac{3}{4}$ Subtle
 131 2 $2\frac{1}{4}$ Trett

$\oplus 3288 \circ 6\frac{1}{2}$ Neat at 2 $\begin{matrix} l. & s. & d. \\ 4. & 8\frac{1}{4} & \text{per } \oplus \end{matrix}$
8 10

\oplus gn. lb
 $\begin{array}{r} 7 \overline{) 3829 \ 3 \ 20} \\ 16 \text{ lb. } 547 \ 0 \ 15 \end{array}$

$$\begin{array}{r} \text{£. 22} \quad 6 \text{ } 10 \frac{1}{2} \\ \quad \quad 8 \text{ } 10 \end{array}$$

8 lb. 273 2 7 $\frac{1}{2}$
4 lb. 136 3 3 $\frac{3}{4}$

£. 223 8 9
 2 10

$$\oplus_{410} \quad 1 \quad 11\frac{1}{4}$$

L. 2234 7 6
3

	<i>l.</i>	<i>s.</i>	<i>d.</i>
	7	2	4 8 $\frac{1}{2}$
	<hr/>		
16 lb.	6		4 $\frac{1}{2}$

$$\begin{array}{r} \text{£. } 6703 \quad 2 \quad 6 \\ 446 \quad 17 \quad 6 \\ 178 \quad 15 \quad 0 \\ 17 \quad 17 \quad 6 \end{array}$$

4 lb. 1 7

4 lb.	7
2 lb.	9 $\frac{1}{2}$
$\frac{1}{2}$ lb.	2 $\frac{1}{2}$

£. 7346 15 0 $\frac{3}{4}$

$$\begin{array}{r}
 \text{H} \quad \text{qu.} \quad \text{H} \\
 131 \quad 2 \quad 2\frac{1}{2} \\
 26 \overline{) 341928\frac{1}{2}} \\
 \underline{26} \\
 81 \\
 \underline{78} \\
 39 \\
 \underline{26} \\
 13 \\
 \underline{4} \\
 154 \\
 \underline{52} \\
 2 \\
 \underline{28} \\
 164 \\
 \underline{52} \\
 12 \\
 \underline{4} \\
 148 \\
 \underline{26} \\
 22
 \end{array}$$

In *Example 1.* The Price of the *Quarters*, and the Price of 16 *lb.* is obtained as afore; then the Price of 16 *lb.* divided by 8, gives the Price of 2 *lb.* which divided by 2, gives the Price of 1 *lb.*

In *Example* 2, 3, 4, and 5. The Price of 1 q^{r} is divided by 7, for the Price of 16 *lb.* which divided by 2, gives the Price of 8 *lb.* 8*c.*

In *Example 6*, and *7*. The *Gross Weight* is divided by *7*, for the *Tare* of *16 lb. &c.* and the *Subtle Weight* by *26*, for the whole *Trett*, as in the *Examples*.

To render this Treatise compleat, I have added some further *Examples in Weight and Measure.*

For

Practical Examples.

61

Ton \oplus qr. lb.	l.	s.	d.
57 19 3 27 at	4	16	9
per Ton.	7	10	
	£. 48	7	6
4 19 4			5
Sb. 4 10	£. 241	17	6
	33	17	3
10 \oplus	2	8	4 $\frac{1}{2}$
5 \oplus	1	4	2 $\frac{1}{4}$
4 \oplus		19	4
2 grs.	2	5	
1 gr.	1	2	1 $\frac{1}{2}$
16 lb.		8	4
8 lb.			4
2 lb.			1
1 lb.			1 $\frac{1}{2}$
	£. 280	11	5

Ton \oplus qr. lb.	l.	s.	d.
65 19 3 27 at	5	18	4
per Ton.	5	10	
	£. 59	3	4
4 1 3 8			6
Sb. 5 11	£. 355	0	0
	29	11	8
10 \oplus	2	19	2
5 \oplus	1	9	7
4 \oplus	1	3	8
2 grs.	2	11	1 $\frac{1}{2}$
1 gr.	1		5 $\frac{1}{2}$
16 lb.			10
8 lb.			5
2 lb.			1 $\frac{1}{2}$
1 lb.			1 $\frac{1}{2}$
	£. 390	9	11

Ton \oplus qr. lb.	l.	s.	d.
154 17 1 25 at	5	18	7 $\frac{1}{4}$
per Ton.	4	10	
	£. 59	6	0 $\frac{1}{2}$
2 11 10 $\frac{1}{2}$		5	10
Sb. 5 11	£. 593	0	5
	296	10	2 $\frac{1}{2}$
	23	14	5
10 \oplus	2	19	3 $\frac{1}{2}$
5 \oplus	1	9	7
2 \oplus	11	10	1 $\frac{1}{2}$
1 qu.	1		5 $\frac{1}{4}$
16 lb.			10
8 lb.			5
1 lb.			1 $\frac{1}{2}$
	£. 918	8	7 $\frac{1}{4}$

Ton \oplus qr. lb.	l.	s.	d.
256 17 1 25 at	6	11	9
per Ton.	6	10	
	£. 65	17	6
2 13 2		5	10
Sb. 6 7	£. 658	15	0
			2
	£. 1317	10	0
	329	7	6
	39	10	6
10 \oplus	3	5	10 $\frac{1}{2}$
5 \oplus	1	12	11 $\frac{1}{2}$
2 \oplus		13	2
1 gr.	1		7 $\frac{1}{2}$
16 lb.			11 $\frac{1}{2}$
8 lb.			5 $\frac{1}{2}$
1 lb.			1 $\frac{1}{2}$
	£. 1692	3	0 $\frac{1}{2}$

In *Example 1.* 20 \oplus being a *Ton*, I divide the Price of 1 *Ton* by 2, it gives the Price of 10 \oplus . The Price of 10 \oplus divided by 2, gives the Price of 5 \oplus . The Price of 1 *Ton* divided by 5, gives the Price of 4 \oplus . The Price of 4 \oplus divided by 4, gives 4s. 10d. for the Price of 1 \oplus , from whence the *Quarters* and *Pounds* are obtained as afore.

In

Practical Examples.

In *Example 2*. The Price of 10 Φ is divided by 5, for the Price of 2 Φ , and the Price of 2 Φ divided by 2, gives 5s. 11d. for the Price of 1 Φ , from whence the *Quarters* and *Pounds* are obtained.

oz. dwt. gr.	s. d.	oz. dwt. gr.	s. d.
37 19 23 at	5 9½	55 19 23 at	6 8
per Ounce.	7 10	per Ounce.	5 10
<hr/>		<hr/>	
£. 2 17 11		£. 3 6 8	
3		5	
<hr/>		<hr/>	
£. 8 13 9		£. 16 13 4	
2 0 6½		1 13 4	
10 dwt. - - - -	2 10½	10 dwt. - - - -	3 4
5 dwt. - - - -	1 5½	5 dwt. - - - -	1 8
4 dwt. - - - -	1 1¼	4 dwt. - - - -	1 4
12 gr. - - - -	1½	12 gr. - - - -	2
6 gr. - - - -	¾	6 gr. - - - -	1
4 gr. - - - -	½	4 gr. - - - -	½
<hr/>		<hr/>	
£. 11 0 0		£. 18 13 3½	
oz. dwt. gr.	s. d.	oz. dwt. gr.	s. d.
259 11 3 at	6 8½	354 11 3 at	8 9
per Ounce.	9 10	per Ounce.	4 10
<hr/>		<hr/>	
£. 3 7 3½		£. 4 7 6	
5 10		5 10	
<hr/>		<hr/>	
£. 33 12 11		£. 43 15 0	
2		3	
<hr/>		<hr/>	
£. 67 5 10		£. 131 5 0	
16 16 5½		21 17 6	
3 0 6¼		1 15 0	
10 dwt. - - - -	3 4½	10 dwt. - - - -	4 4½
1 dwt. - - - -	4	1 dwt. - - - -	5½
3 gr. - - - -	½	3 gr. - - - -	½
<hr/>		<hr/>	
£. 87 6 7		£. 155 2 4½	

In

In *Example 1.* 20 Pennyweight being an Ounce, I divide the Price of 1 Ounce by 2, it gives the Price of 10 Pennyweight ; the Price of 10 Pennyweight by 2, gives the Price of 5 Pennyweight ; the Price of 1 Ounce divided by 5, gives the Price of 4 Pennyweight ; the Price of 4 Pennyweight divided by 8, gives the Price of 12 Grains ; the Price of 12 Grains by 2, gives the Price of 6 Grains, and the Price of 12 Grains by 3, gives the Price of 4 Grains ; the Price of 1 Grain being of no Value, is left out.

In *Example 2.* The Price of 10 Pennyweight is divided by 10, for the Price of 1 Pennyweight, and the Price of 1 Pennyweight divided by 8, for the Price of 3 Grains.

lb. oz. dwt. gr.	l. s. d.
459 11 5 7 per Pound.	at 5 16 8 9 10
	£. 58 6 8 5 10
	£. 583 6 8 4
	£. 2333 6 8 291 13 4 52 10 0
6 oz.	2 18 4
3 oz.	1 9 2
2 oz.	19 5 $\frac{1}{4}$
4 dwt.	1 11 $\frac{1}{2}$
1 dwt.	5 $\frac{3}{4}$
6 gr.	1 $\frac{1}{2}$
	£. 2682 19 5 $\frac{1}{2}$

lb. oz. dwt. gr.	l. s. d.
572 11 5 7 per Pound.	at 7 18 9 2 10
	£. 79 7 6 7 10
	£. 793 15 0 5
	£. 3968 15 0 555 12 6 15 17 6
6 oz.	3 19 4 $\frac{1}{2}$
3 oz.	1 19 8 $\frac{1}{2}$
2 oz.	1 6 5 $\frac{1}{2}$
4 dwt.	2 7 $\frac{3}{4}$
1 dwt.	7 $\frac{1}{4}$
6 gr.	1 $\frac{1}{2}$
1 gr.	$\frac{1}{4}$
	£. 4547 13 11 $\frac{1}{2}$

lb.	oz.	dwt.	gr.	l.	s.	d.
653	9	3	5	at	15	16 8 $\frac{1}{2}$
per Pound.					3	10
				£.	158	7 3 $\frac{1}{2}$
					5	10
				£.	1583	12 11
						6
				£.	9501	17 6
					791	16 5 $\frac{1}{2}$
					47	10 2 $\frac{1}{2}$
6 oz.					7	18 4 $\frac{1}{2}$
2 oz.					2	12 9 $\frac{1}{2}$
1 oz.					1	6 4 $\frac{1}{2}$
2 dwt.					2	7 $\frac{1}{2}$
1 dwt.					1	3 $\frac{1}{2}$
4 gr.						2 $\frac{1}{2}$
1 gr.						$\frac{1}{2}$
				£.	10353	5 10

lb.	oz.	dwt.	gr.	l.	s.	d.
278	9	3	5	at	18	19 7
per Pound.					8	10
				£.	189	15 10
						7 10
				£.	1897	18 4
						2
				£.	3795	16 8
					1328	10 10
					151	16 8
6 oz.				9	9	9½
2 oz.				3	3	3
1 oz.				1	11	7½
2 dwt.					3	1½
1 dwt.					1	6½
4 gr.						3
1 gr.						½
				£.	5290	13 10½

In Example 1 and 2. 12 Ounces being a Pound, I divide the Price of 1 Pound by 2, it gives the Price of 6 Ounces. The Price of 6 Ounces divided by 2, gives the Price of 3 Ounces, and the Price of 6 Ounces divided by 3, gives the Price of 2 Ounces, the Price of 2 Ounces divided by 10, gives the Price of 4 Pennyweight, &c.

Tun bbd. ga. qt.	l.	s.	d.
652 3 62 3 at	5	17	8
per Tun.		2	10
	£. 58	16	8
		5	10
	£. 588	6	8
			6
	£. 3530	0	0
	294	3	4
	11	15	4
2 bbd. . .	2	18	10
1 bbd. . . .	1	9	5
21 gal.	9	9	$9\frac{1}{2}$
21 gal.	9	9	$9\frac{1}{2}$
7 gal.	3	3	
7 gal.	3	3	
3 gal.	1	4	$4\frac{3}{4}$
3 gal.	1	4	$4\frac{3}{4}$
3 qts.			4
	£. 3841	16	$1\frac{1}{2}$

Tun	bbd.	ga.	qt.		l.	s.	d.
798	3	62	3	at	7	18	9
per Tun.						8	10
					£.	79	7 6
							9 10
					£.	793	15 0
							7
					£.	5556	5 0
						714	7 6
						63	10 0
2	bbds			3	19	4	$\frac{1}{2}$
1	bbd			1	19	8	$\frac{1}{4}$
21	gal.				13	2	$\frac{1}{2}$
21	gal.				13	2	$\frac{1}{2}$
7	gal.				4	4	$\frac{1}{2}$
7	gal.				4	4	$\frac{1}{2}$
3	gal.				1	10	$\frac{1}{4}$
3	gal.				1	10	$\frac{1}{4}$
3	qts.					5	$\frac{1}{2}$
					£.	6342	1 0 $\frac{1}{4}$

Practical Examples.

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Tn. hhd. ga. qu. pe. $\frac{1}{2}$
 796 1 11 2 1 at 7 per Tun.
 6 10

$\frac{1}{2}$ 1 15
 Sb. 11 8

£. 70
 9 10

£. 700
 7

£. 4900
 630
 42

1 Hoghead 1 15
 7 Gallons 3 10 $\frac{1}{2}$
 3 Gallons 1 8
 1 Gallon 6 $\frac{1}{4}$
 2 Quarts 3 $\frac{1}{2}$
 1 Pint $\frac{1}{4}$

£. 5574 1 5

Tn. hhd. ga. qu. pe. $\frac{1}{2}$ s.
 875 1 11 2 1 at 7 16 per
 Tun. 5 10

$\frac{1}{2}$ 1 19
 Sb. 13

£. 78 0
 7 10

£. 780 0
 8

£. 6240 0
 546 0
 39 0

1 Hoghead 1 19
 7 Gallons 4 4
 3 Gallons 1 10 $\frac{1}{2}$
 1 Gallon 7 $\frac{1}{4}$
 2 Quarts 3 $\frac{1}{2}$
 1 Pint 0 $\frac{1}{4}$

£. 6827 6 1 $\frac{1}{4}$

In Example 1. 4 Hogsheads being a Tun, I divide the Price of a Tun by 4, it gives the Price of 1 Hogshead, and the Price of 1 Hogshead divided by 2, gives the Price of 1 Hogshead; then because 63 Gallons makes 1 Hogshead, I divide the Price of 1 Hogshead by 3, it gives the Price of 21 Gallons, and the Price of 21 Gallons by 3, it gives the Price of 7 Gallons, and also the Price of 21 Gallons by 7, it gives the Price of 3 Gallons, then the Price of 3 Gallons by 4, it gives the Price of 3 Quarts.

In the 2d Example, I divide the Price of a Tun by 4, it gives the Price of 1 Hogshead, then the Price of 1 Hogshead by 3, it gives 11 s. 8 d. for the Price of 21 Gallons, then the Price of 21 Gallons divided by 3, gives the Price of 7 Gallons, &c.

K

qu

qu. bu. pe. l. s. d.
227 5 1 at 13 6 per
Quarter. 7 10

£. 6 15 0

5 10

£. 67 10 0

2

£. 135 0 0

33 15 0

4 14 6

4 Bushels - - - 6 9

1 Bushel - - - - 1 8½

1 Peck - - - - - 5

£. 173 18 4½

qu. bu. pe. l. s. d.
354 5 1 at 1 8 9 per
Quarter. 4 10

£. 14 7 6

5 10

£. 143 15 0

3

£. 431 5 0

71 17 6

5 15 0

4 Bushels - - - - 14 4½

1 Bushel - - - - - 3 7

1 Peck - - - - - 10½

£. 509 16 4½

Last qu. bu. pe. l. s.
2756 9 7 3 at 3 19 per
Last. 6 10

£. 39 10

5 10

£. 395 0

7 10

£. 3950 0

2

£. 7900 0

2765 0

197 10

23 14

5 Quarters - - - - 1 19 6

2 Quarters - - - - - 15 9½

2 Quarters - - - - - 15 9½

4 Bushels - - - - - 3 11¼

2 Bushels - - - - - 1 11½

1 Bushel - - - - - 11¾

4 Pecks - - - - - 5½

1 Peck - - - - - 2½

£. 10890 2 8

Last qu. bu. pe. l. s. d.
4279 9 7 3 at 4 12 6
per Last. 9 10

£. 46 5 0

7 10

£. 462 10 0

2 10

£. 4625 0 0

4

£. 18500 0 0

925 0 0

323 15 0

41 12 6

5 Quarters - - - - 2 6 3

2 Quarters - - - - - 18 6

2 Quarters - - - - - 18 6

4 Bushels - - - - - 4 7½

2 Bushels - - - - - 2 3½

1 Bushel - - - - - 1 1¼

2 Pecks - - - - - 6¼

1 Peck - - - - - 3½

£. 19794 19 8

In

In *Example 1.* A *Quarter* being 8 *Busbels*, I divide the Price of one *Quarter* by 2, it gives the Price of 4 *Busbels*, then the Price of 4 *Busbels* by 4, it gives the Price of 1 *Busbel*, then the Price of 1 *Busbel* by 4, it gives the Price of 1 *Peck*.

In *Example 2.* Because 10 *Quarters* is a *Last*, I divide the Price of 1 *Last* by 2, it gives the Price of 5 *Quarters*, and the Price of a *Last* by 5, it gives the Price of 2 *Quarters*, then I divide the Price of 2 *Quarters* by 4, it gives the Price of 4 *Busbels*, &c.

Thus have I gone through what I think is sufficient to render a Person a good Accomptant; and have established it upon such plain and easy Principles, that Persons of the meanest Capacities may attain it, and be enabled to give the *Amount* of any *Quantity* at any Price. I shall now proceed to account for *Commissions*, *Brokage*, *Purchasing of publick Stocks*, *Foreign Exchanges*, and *Interest of Money*, and establish the same upon the same Principles, with as little Variation as possible.

Commission is an Allowance given to a *Factor* or *Correspondent*, for selling of Goods put into his Hands by his *Employer*, and is generally at a certain Rate *per Cent.* according to the Custom of Places, or rather Agreement: To Account for which, observe this Rule.

R U L E.

Multiply the Sum of Money by the Rate *per Cent.*; and divide by 100.

What is the *Commission* of
256*l.* at 3 *per Cent.*

$$\begin{array}{r} \text{£. } 7 \overline{68} \\ \text{20} \\ \text{Sb. } 13 \overline{60} \\ \text{12} \\ \text{d. } 7 \overline{20} \end{array}$$

What is the *Commission* of
487*l.* 18*s.* at 2½ *per C.*

$$\begin{array}{r} \text{£. } 975 \overline{16} \\ \frac{1}{2} - 121 \overline{19} \ 6 \\ \text{£. } 10 \overline{97} \ 15 \ 6 \\ \text{20} \\ \text{Sb. } 19 \overline{15} \\ \text{12} \\ \text{d. } 1 \overline{86} \\ \text{4} \\ \text{f. } 3 \overline{44} \end{array}$$

At 2½ *per Cent.* what is the
Com. of 3456*l.* 19*s.* 10*d.*

$$\begin{array}{r} \text{£. } 6913 \overline{19} \ 8 \\ \frac{1}{2} - 1728 \overline{9} \ 11 \\ \text{£. } 86 \overline{42} \ 9 \ 7 \\ \text{20} \\ \text{Sb. } 8 \overline{49} \\ \text{12} \\ \text{d. } 5 \overline{95} \\ \text{4} \\ \text{f. } 3 \overline{80} \end{array}$$

At 2½ *per Cent.* what is the
Com. of 7528*l.* 14*s.* 6*d.* ½

$$\begin{array}{r} \text{£. } 15057 \overline{9} \ 1\frac{1}{2} \\ \frac{1}{2} - 3764 \overline{7} \ 3\frac{1}{2} \\ \frac{1}{4} - 1882 \overline{3} \ 7\frac{1}{2} \\ \text{£. } 207 \overline{04} \ 0 \ 0\frac{1}{4} \\ \text{20} \\ \text{Sb. } 0 \overline{80} \\ \text{12} \\ \text{d. } 9 \overline{60} \\ \text{4} \\ \text{f. } 2 \overline{41} \end{array}$$

In *Example 1.* I multiply the Sum of Money by 3, then cut off 2 Figures in the *Pounds*, which is dividing them by 100, you have 7 *Pounds* and 68 Parts of 100 for the *Commission*, the Value of the 68 Parts cut off, is found by multiplying them by 20, and cutting off 2 Figures as afore, you have 13 *Shillings* and 60 Parts, then multiplying the 60 Parts by 12, and cutting off 2 Figures, you have
7 Pence

7 Pence and 20 Parts; so that the Commission of 256 Pounds, is 7*l.* 13*s.* 7*d.* as in the Example.

In Example 2. The Sum of Money is multiplied by 2, and for the $\frac{1}{4}$ I divide the given Sum of Money by 4; these 2 Sums added together, and 2 Figures off in the Pounds, gives 10 Pounds and 97 Parts; these Parts multiplied by 20, adding 15 Shillings, and cutting off 2 Figures as afore, gives 19 Shillings and 15 Parts; these Parts multiplied by 12, adding 6 Pence, and cutting off 2 Figures, gives 1 Penny and 86 Parts; these Parts multiplied by 4, cutting off 2 Figures, gives 3 Farthings and 44 Parts, so that the Commission of 487*l.* 18*s.* at $2\frac{1}{4}$ per Cent. is 10*l.* 19*s.* 1*d.* $\frac{3}{4}$ as in the Example.

In Example 3. The given Sum is multiplied by 2, and for the $\frac{1}{2}$ per Cent. I divide the given Sum by 2; these 2 Sums added together, and 2 Figures cut off in the Pounds, gives 86 Pounds and 42 Parts; the Value of which Parts is found as afore.

In Example 4. The given Sum is multiplied by 2, and for $\frac{1}{2}$ per Cent. I divide the given Sum by 2, as afore, and for $\frac{1}{4}$ per Cent. I divide the Sum for $\frac{1}{2}$ per Cent. by 2; these 3 Sums added together, and 2 Figures cut off in the Pounds, gives 207 Pounds and 04 Parts; the Value of which Parts is found as afore.

BROKAGE

Is Money paid to *Brokers* for helping *Merchants*, or *Factors* to Persons, to buy or sell them Goods.

At $\frac{1}{2}$ per Cent. what is the Brokage of 567*l.* 12*s.* At $\frac{1}{2}$ per Cent. what is the Brokage of 248*l.* 12*s.* 10*d.*

$$\begin{array}{r} \frac{1}{2} \text{ --- } \text{£. } 2 | 83 \quad 16 \\ \quad \quad \quad 20 \\ \text{Sh. } 16 | 76 \\ \quad \quad \quad 12 \\ \text{d. } 9 | 12 \end{array}$$

$$\begin{array}{r} \frac{1}{2} \text{ --- } \text{£. } 1 | 24 \quad 6 \quad 5 \\ \quad \quad \quad 20 \\ \text{Sh. } 4 | 86 \\ \quad \quad \quad 12 \\ \text{d. } 10 | 37 \\ \quad \quad \quad 4 \\ \text{f. } 1 | 48 \end{array}$$

At $\frac{1}{2}$ per Cent. what is the
 Brokeage of 675*l.* 11*s.* 9*d.*

$\frac{1}{2}$ ---	£. 337	15	10 $\frac{1}{2}$
$\frac{1}{4}$ ---	168	17	11 $\frac{1}{2}$
	5 06	13	9 $\frac{1}{2}$
	20		
	<i>Sh.</i> 1 33		
	12		
	<i>d.</i> 4 05		

At $\frac{1}{2}$ per Cent. what is the
 Brokeage of 345*l.* 12*s.* 10*d.* $\frac{1}{2}$

$\frac{1}{2}$ ---	£. 172	16	5 $\frac{1}{2}$
$\frac{1}{4}$ ---	86	8	2 $\frac{1}{2}$
	£. 2 59	4	7 $\frac{1}{2}$
	20		
	<i>Sh.</i> 11 84		
	12		
	<i>d.</i> 10 15		

The same Method is pursued in these *Examples*,
 as afore in the *Commissions*; and therefore I think
 it needless to say any thing further about them.

PURCHASES.

In purchasing of *Stocks*, I multiply the Sum to
 be *purchased* by the Excess above 100, and the
 Produce of that added to the given Sum, gives the
Purchase thereof, as in the following *Examples*.

At 119 per C. what is the
Purchase of 2756*l.*
Bank Stock. 9 10

£. 27560
24804
£. 523 64
20
<i>Sh.</i> 12 80
12
<i>d.</i> 9 60
4
<i>f.</i> 2 40

£. 2756
523 12 9 $\frac{1}{2}$
£. 3279 12 9 $\frac{1}{2}$

At 108 $\frac{1}{2}$ per C. what is the
Purchase of 3287*l.* 14*s.*
S. Sea Stock. 8

£. 26301	12
$\frac{1}{2}$ ---	821 18 6
$\frac{1}{4}$ ---	410 19 3
£. 275 34	9 9
20	
<i>Sh.</i> 6 89	
12	
<i>d.</i> 10 77	
4	
<i>f.</i> 3 08	

£. 3287 14
275 6 10 $\frac{1}{2}$
£. 3563 0 10 $\frac{1}{2}$

COMMISSIONS.

71

At 124½ per Cent. what is the
Purchase of 758l. 17s. 10d.
India Stock.

	4	10
£. 7588	18	4
		2
£. 15177	16	8
3035	11	4
-----	379	8 11
-----	94	17 2½
£. 186187	14	1½
	20	
Sh. 17154		
	12	
d. 6149		
	4	
f. 1199		
7. 758 17 10		
186 17 6½		
£. 945 15 4½		

At 105½ per Cent. what is the
Purchase of 845l. 19s. 11d.½
Mill. Bank.

	5
£. 4229	19 10½
-----	422 19 11½
-----	211 9 11½
£. 48164	9 10½
	20
Sh. 12189	
	12
d. 10178	
	4
f. 3113	
£. 845 19 11½	
48 12 10½	
£. 894 12 10½	

To Account for the *Interest* of Money, I multiply the given Sum by the Rate *per Cent.* and divide by 100, it gives the *Interest* of the Money for *one Year*, then the *Interest* for the Time required is obtained, as in the *Examples* following.

and for the *Capital* — 1000

At

INTEREST

At 5 per Cent. per Annum
what is the Interest of
69*l.* forborn 3*m.* 18*d.*

l.
69
- 5

£. 3|45
20
Sb. 9|00

<i>m. d.</i>	<i>l. s.</i>
3 18 at 3	9 per Ann.
2 Months 0 11	6
1 Month - - -	5 9
15 Days - - -	2 10½
3 Days - - -	6½
	<u> </u>
	<i>£.</i> 1 0 8½

At 5½ per Cent. per Ann.
what is the Interest of
78*l.* 11*s.* forborn 5*m.*
22 Days.

l. s.
78 11
- 5

£. 392 15
19 12 9

£. 4|12 7 9
20
Sb. 2|47
12
d. 5|73
4
f. 2|92

<i>m. d.</i>	<i>l. s. d.</i>
5 22 at 4	2 5½ per Annum.
4 Months 1	7 5½
1 Month - - -	6 10½
15 Days - - -	3 5
6 Days - - -	1 4½
1 Day - - -	2½
	<u> </u>
	<i>£.</i> 1 19 3½

At

INTEREST.

73.

At 6 per Cent. per Annum
what is the Interest of
562*l.* 14*s.* 5*d.* for-
born 7 mon. 25 Days.

	<i>l.</i>	<i>s.</i>	<i>d.</i>
562	14	5	
			6
<hr/>			
£. 33	76	6	6
		20	
<i>Sb.</i>	15	26	
		12	
<hr/>			
<i>d.</i>	3	18	

m. d. l. s. d.
7 25 at 33 15 3 per
Annum.

6 Mon.	16	17	7½
1 Mon.	2	16	3½
15 Days -	1	8	1½
10 Days - - -	18	9	
<hr/>			
£.	22	0	9½

At 6½ per Cent. per Ann.
what is the Interest of
698*l.* 18*s.* 10*d.*½ for-
born 3 Years.

	<i>l.</i>	<i>s.</i>	<i>d.</i>
698	18	10½	
			6
<hr/>			
£. 41	93	13	3
½ - - - - -	349	9	5½
<hr/>			
£. 45	43	2	8½
		20	
<i>Sb.</i>	8	62	
		12	
<hr/>			
<i>d.</i>	7	52	
		4	
<hr/>			
<i>f.</i>	2	109	

	<i>l.</i>	<i>s.</i>	<i>d.</i>
3 Years at	45	8	7½
per Ann.			3
<hr/>			
£.	136	5	10½

L

At

At 7 per Cent. per Annum,
what is the Interest of
4278*l.* forborn 4 Years,
10 Months.

<i>l.</i>	
4278	
7	
<u>£. 299 46</u>	
20	
<u>S<i>h.</i> 9 20</u>	
12	
<u>d. 2 40</u>	
4	
<u>f. 1 60</u>	

<i>ye. m.</i>	<i>l.</i>	<i>s.</i>	<i>d.</i>
4 10 at 299	9	2½	
per Ann.		4	

<i>£.</i> 1197 16 9
6 Mon. - 149 14 7
4 Mon. - 99 16 4½

£. 1447 7 8½

At 7½ per Cent. per Ann.
what is the Interest of
5284*l.* 16*s.* forborn 5
Years, 11 Mon. 29 Da.

<i>l.</i>	<i>s.</i>
5284	16
7	
<u>£. 36993 12</u>	
½	8
¼	4
<u>£. 409 57 4</u>	
20	
<u>S<i>h.</i> 11 44</u>	
12	
<u>d. 5 28</u>	
4	
<u>f. 1 12</u>	

<i>ye. m. d.</i>	<i>l.</i>	<i>s.</i>	<i>d.</i>
5 11 29 at 409	11	5½	
per Ann.		5	

<i>£.</i> 2047 17 2½
6 Mon. - 204 15 8½
3 Mon. - 102 7 10½
2 Mon. - 68 5 2½
15 Days - 17 1 3½
10 Days - 11 7 6½
3 Days - 3 8 3
1 Day - 1 2 9

£. 2456 5 9½

Though this Method of computing *Interest*, be not strictly and properly exact, yet it may very well serve for common Computations, varying but little from Truth. A more exact Method shall be shewn

shewn in its proper Place, when I come to treat of those Parts of *Arithmetic* which cannot be brought under this general Way of accounting.

To Account for foreign *Exchanges*, it would be necessary to have a true Account at all Times of the just Values of *foreign Coins*, because the *Par of Exchange* rises and falls according as Money is plenty or scarce, or according to the Time allowed for Payment of the Money in *Exchange*. The which is *weekly* published, and to be seen at most of the *Coffee-houses* about the *Royal Exchange*; and because I have often found the same as generally published very unintelligible to many Persons, I have transcribed one with some Additions to render the same more intelligible.

1 £. Ster. at <i>Amsterdam</i> is	34	11 Flemish
Ditto - - - Ditto at Sight	34	8
Ditto - - - <i>Rotterdam</i> - -	35	1
Ditto - - - <i>Antwerp</i> - - -	35	9
Ditto - - - <i>Hamburg</i> - -	33	6
3 1/2 Pence. <i>Paris</i> at 1 Days date	1	Crown of 3 Livres
3 1/2 - - - <i>Bordeaux</i> at 1 1/2 Ufance	Ditto	
4 1/2 - - - <i>Cadiz</i> - - - - -	1	Peso of 8 Reals
4 1/2 - - - <i>Madrid</i> - - - - -	1	Peso of 10 Reals
50 3/4 - - - <i>Leghorn</i> - - - - -	1	Peso of 6 Livres
53 1/2 - - - <i>Genoua</i> - - - - -	1	Peso of 5 Livres
48 1/2 - - - <i>Venice</i> - - - - -	1	Bank Ducat
5 s. 5 d. 5/8 <i>Lisbon</i> - - - - -	1000	Reas
5 s. 5 d. 1/2 <i>Porto</i> - - - - -	1000	Reas
<i>Dublin</i> - - - - -	11 1/8	per Cent.

d.
759 Dollars at $55\frac{1}{2}$ per
Dollar.

$$\begin{array}{r}
 12 \overline{) 55\frac{1}{2}} \\
 \text{Sh. } 4 \quad 7\frac{1}{2} \\
 \underline{9 \quad 10} \\
 \text{£. } 2 \quad 6 \quad 0\frac{1}{2} \\
 \underline{5 \quad 10} \\
 \text{£. } 23 \quad 0 \quad 5 \\
 \underline{7} \\
 \text{£. } 161 \quad 2 \quad 11 \\
 \underline{11 \quad 10 \quad 2\frac{1}{2}} \\
 \text{£. } 2 \quad 1 \quad 5\frac{1}{2} \\
 \underline{ 174 \quad 14 \quad 6\frac{1}{2}}
 \end{array}$$

148 Dollars at $52\frac{1}{2}$ per
Dollar.

$$\begin{array}{r}
 12 \overline{) 52\frac{1}{2}} \\
 \text{Sh. } 4 \quad 4\frac{1}{2} \\
 \underline{8 \quad 10} \\
 \text{£. } 2 \quad 3 \quad 11\frac{1}{2} \\
 \underline{4 \quad 10} \\
 \text{£. } 21 \quad 19 \quad 7 \\
 \underline{8 \quad 15 \quad 10} \\
 \text{£. } 1 \quad 15 \quad 2 \\
 \underline{ 32 \quad 10 \quad 7}
 \end{array}$$

doll. rea. d.
269 7 at $53\frac{1}{2}$ per Dollar

$$\begin{array}{r}
 12 \overline{) 53\frac{1}{2}} \\
 \text{Sh. } 4 \quad 5\frac{1}{2} \\
 \underline{9 \quad 10} \\
 \text{£. } 2 \quad 4 \quad 3\frac{1}{2} \\
 \underline{6 \quad 10} \\
 \text{£. } 22 \quad 2 \quad 8\frac{1}{2} \\
 \underline{2} \\
 \text{£. } 44 \quad 5 \quad 5 \\
 \underline{13 \quad 5 \quad 7\frac{1}{2}} \\
 \text{£. } 1 \quad 19 \quad 10\frac{1}{2} \\
 4 \text{ Reals} \text{ ----- } 2 \quad 2\frac{1}{2} \\
 2 \text{ ----- } 1 \quad 1\frac{1}{2} \\
 1 \text{ ----- } 6\frac{1}{2} \\
 \underline{ 59 \quad 14 \quad 9}
 \end{array}$$

Note, 8 Reals is a Dollar.

doll. rea. d.
387 3 at $54\frac{1}{2}$ per Dollar.

$$\begin{array}{r}
 12 \overline{) 54\frac{1}{2}} \\
 \text{Sh. } 4 \quad 6\frac{1}{2} \\
 \underline{7 \quad 10} \\
 \text{£. } 2 \quad 5 \quad 3\frac{1}{2} \\
 \underline{8 \quad 10} \\
 \text{£. } 22 \quad 13 \quad 1\frac{1}{2} \\
 \underline{3} \\
 \text{£. } 67 \quad 19 \quad 4\frac{1}{2} \\
 \underline{18 \quad 2 \quad 6} \\
 \text{£. } 1 \quad 11 \quad 8\frac{1}{2} \\
 2 \text{ Reals} \text{ ----- } 1 \quad 1\frac{1}{2} \\
 1 \text{ ----- } 6\frac{1}{2} \\
 \underline{ 87 \quad 15 \quad 3\frac{1}{2}}
 \end{array}$$

456 Ducats

E X C H A N G E S. 77

d.
456 Ducats at 51 $\frac{1}{2}$ per Ducat.

$$\begin{array}{r}
 12 \overline{) 51 \frac{1}{2}} \\
 \text{Sp. } 4 \quad 3 \frac{1}{2} \\
 \underline{6 \quad 10} \\
 \text{£. } 2 \quad 3 \quad 0 \frac{3}{4} \\
 \underline{5 \quad 10} \\
 \text{£. } 21 \quad 10 \quad 2 \frac{1}{4} \\
 \underline{4} \\
 \text{£. } 86 \quad 0 \quad 10 \\
 10 \quad 15 \quad 1 \frac{3}{4} \\
 \underline{1 \quad 5 \quad 9 \frac{5}{8}} \\
 \text{£. } 98 \quad 1 \quad 9
 \end{array}$$

d.
573 Ducats at 56 $\frac{1}{2}$ per Ducat.

$$\begin{array}{r}
 12 \overline{) 56 \frac{1}{2}} \\
 \text{Sp. } 4 \quad 8 \frac{1}{2} \\
 \underline{3 \quad 10} \\
 \text{£. } 2 \quad 7 \quad 4 \frac{5}{8} \\
 \underline{7 \quad 10} \\
 \text{£. } 23 \quad 13 \quad 11 \frac{1}{4} \\
 \underline{5} \\
 \text{£. } 118 \quad 9 \quad 9 \frac{1}{4} \\
 16 \quad 11 \quad 9 \frac{3}{4} \\
 \underline{14 \quad 2 \frac{5}{8}} \\
 \text{£. } 135 \quad 15 \quad 9 \frac{1}{4}
 \end{array}$$

mil rea. rea. s. d.
574 396 at 5 5 $\frac{1}{2}$
per Mil Rea. 4 10

$$\begin{array}{r}
 \text{£. } 2 \quad 14 \quad 5 \frac{5}{8} \\
 \underline{7 \quad 10} \\
 \text{£. } 27 \quad 4 \quad 9 \frac{1}{4} \\
 \underline{5} \\
 \text{£. } 136 \quad 3 \quad 11 \frac{1}{4} \\
 19 \quad 1 \quad 4 \frac{3}{8} \\
 \text{Reas } 1 \quad 1 \quad 9 \frac{1}{8} \\
 250 \dots\dots\dots 4 \frac{3}{8} \\
 100 \dots\dots\dots 6 \frac{1}{8} \\
 20 \dots\dots\dots 1 \frac{3}{8} \\
 20 \dots\dots\dots 1 \frac{3}{8} \\
 5 \dots\dots\dots \frac{3}{8} \\
 395 \text{ £. } 156 \quad 9 \quad 2 \frac{1}{2}
 \end{array}$$

mil rea. rea. s. d.
649 629 at 5 7 $\frac{1}{2}$
per Mil Rea. 9 10

$$\begin{array}{r}
 \text{£. } 2 \quad 15 \quad 11 \frac{1}{4} \\
 \underline{4 \quad 10} \\
 \text{£. } 27 \quad 19 \quad 4 \frac{1}{4} \\
 \underline{6} \\
 \text{£. } 167 \quad 16 \quad 3 \\
 11 \quad 3 \quad 9 \\
 \text{Reas. } 2 \quad 10 \quad 4 \frac{1}{4} \\
 500 \dots\dots\dots 2 \quad 9 \frac{1}{4} \\
 100 \dots\dots\dots 6 \frac{1}{4} \\
 20 \dots\dots\dots 1 \frac{3}{8} \\
 5 \dots\dots\dots \frac{3}{8} \\
 4 \dots\dots\dots \frac{3}{8} \\
 629 \text{ £. } 181 \quad 13 \quad 9 \frac{1}{4}
 \end{array}$$

Note, 1000 Reas is a Mil Rea.

PRACTICE.

99

A General Rule.

Divide the Quantity by the even Parts of a Penny, Shilling, or Pound; the Quotient will be the Answer to your Question in Pence, Shillings, or Pounds.

$$\begin{array}{r}
 2479 \text{ Ounces, at } \frac{1}{4} \text{ of a penny per Ounce,} \\
 \hline
 \frac{1}{4} \quad \frac{1}{4} \quad \frac{1}{4} \quad \frac{1}{4} \\
 d. \ 309 \ \frac{1}{4} \\
 Sb. \ 2 \ 15 \ 9 \\
 \hline
 L. \ 1 \ 5 \ 9 \ \frac{1}{4}
 \end{array}$$

$$\begin{array}{r}
 3257 \text{ Ounces, at } \frac{1}{4} \text{ of a penny per Ounce.} \\
 \hline
 \frac{1}{4} \quad \frac{1}{4} \quad \frac{1}{4} \quad \frac{1}{4} \\
 d. \ 814 \ \frac{1}{4} \\
 Sb. \ 6 \ 17 \ 10 \\
 \hline
 L. \ 3 \ 7 \ 10 \ \frac{1}{4}
 \end{array}$$

$$\begin{array}{r}
 6787 \text{ Pounds, at } \frac{1}{4} \text{ of a Penny per Pound.} \\
 \hline
 \frac{1}{4} \quad \frac{1}{4} \quad \frac{1}{4} \quad \frac{1}{4} \\
 d. \ 1696 \ \frac{1}{4} \\
 \frac{1}{4} \quad \frac{1}{4} \quad \frac{1}{4} \quad \frac{1}{4} \\
 d. \ 848 \ \frac{1}{4} \\
 d. \ 2545 \\
 Sb. \ 2 \ 12 \ 1 \\
 \hline
 L. \ 10 \ 12 \ 1
 \end{array}$$

$$\begin{array}{r}
 5324 \text{ Pounds, at } \frac{1}{4} \text{ of a Penny per Pound.} \\
 \hline
 \frac{1}{4} \quad \frac{1}{4} \quad \frac{1}{4} \quad \frac{1}{4} \\
 d. \ 2662 \\
 Sb. \ 22 \ 1 \ 10 \\
 \hline
 L. \ 11 \ 1 \ 10
 \end{array}$$

$$\begin{array}{r}
 5872 \text{ Grains, at } \frac{1}{4} \text{ of a Penny per Grain.} \\
 \hline
 \frac{1}{4} \quad \frac{1}{4} \quad \frac{1}{4} \quad \frac{1}{4} \\
 d. \ 2936 \\
 \frac{1}{4} \quad \frac{1}{4} \quad \frac{1}{4} \quad \frac{1}{4} \\
 d. \ 734 \\
 d. \ 3670 \\
 Sb. \ 30 \ 5 \ 10 \\
 \hline
 L. \ 15 \ 5 \ 10
 \end{array}$$

$$\begin{array}{r}
 3296 \text{ Grains, at } \frac{1}{4} \text{ of a Penny per Grain.} \\
 \hline
 \frac{1}{4} \quad \frac{1}{4} \quad \frac{1}{4} \quad \frac{1}{4} \\
 d. \ 1648 \\
 \frac{1}{4} \quad \frac{1}{4} \quad \frac{1}{4} \quad \frac{1}{4} \\
 d. \ 824 \\
 d. \ 2472 \\
 Sb. \ 20 \ 6 \\
 \hline
 L. \ 10 \ 6
 \end{array}$$

PRACTICE.

4522 Drachms at $\frac{1}{4}$
of a Penny per
Drachm.

4522
d. 2261
1130 $\frac{1}{2}$
565 $\frac{3}{4}$
d. 3956 $\frac{1}{4}$
Sh. 32 9 8
£. 16 9 8 $\frac{1}{4}$

In the first of these *Examples*, I divide the Quantity by 8, the Quotient is 309 $\frac{1}{2}$ *Pence*, which reduced into *Pounds*, gives 1*l.* 5*s.* 9*d.* $\frac{1}{2}$ for the Price of 2479 *Ounces*, at $\frac{1}{8}$ of a *Penny per Ounce*. The like is to be understood of the second and fourth *Examples*.

In *Example 3.* I begin with $\frac{3}{4}$, as $\frac{3}{4}$ of a Penny, which gives 1696 $\frac{1}{4}$ Pence, the Price of the Quantity, at $\frac{3}{4}$ of a Penny per Pound, $\frac{1}{4}$ of which is the Price at $\frac{1}{4}$, these added together gives 2545 Pence, the Price of 6787 Pounds, at $\frac{3}{4}$ of a Penny per Pound, reduced into Pounds gives 10*l.* 12*s.* 1*d.* The like is to be understood of the 5th, 6th, and 7th Examples.

2756 Grains at 1 Penny per Grain.		3521 Gr. at 1 d. $\frac{1}{12}$ per Grain.	
d.	$\frac{1}{12}$	d.	$\frac{1}{12}$
$\begin{array}{r} 2756 \\ \hline \text{Sh. } 22 9 \quad 8 \\ \hline \text{£. } 11 \quad 9 \quad 8 \end{array}$		$\begin{array}{r} 3521 \\ \hline \text{Sh. } 44 0 \quad 1\frac{1}{2} \\ \hline \text{£. } 22 \quad 0 \quad 1\frac{1}{2} \end{array}$	
5672 Pennywt. at 2 d. per Pennywt.		3726 Pennywt. at 3 d. per Pennywt.	
d.	$\frac{1}{6}$	d.	$\frac{1}{4}$
$\begin{array}{r} 5672 \\ \hline \text{Sh. } 94 5 \quad 4 \\ \hline \text{£. } 47 \quad 5 \quad 4 \end{array}$		$\begin{array}{r} 3726 \\ \hline \text{Sh. } 93 1 \quad 6 \\ \hline \text{£. } 46 \quad 11 \quad 6 \end{array}$	

		3794 Ounces, at 4 d. per Ounce.			5648 Ounces, at 6 d. per Ounce.
d.		3794	d.		5648
4	$\frac{1}{12}$	<u>Sb. 126 4 8</u>	6	$\frac{1}{12}$	<u>Sb. 282 4</u>
		£. 63 4 8			£. 141 4

In the *first* of the preceeding *Examples*, because 1 Penny is the $\frac{1}{12}$ of a Shilling, I divide the Quantity by 12, the Quotient is 229 Shillings and 8 Pence; which reduced into Pounds, gives 11 l. 9 s. 8 d. for the Price of 2756 Grains, at 1 Penny per Grain, as in the *Example*.

The like is to be understood of all the rest of the *Examples*.

Note, In the following *Examples* I take first the greatest even Part, and then part of that Part, &c. and add all together; the Sum total is the *Amount* of the *Quantity* at the given *Price*.

		4254 Ounces, at 5 d. per Ounce.			3729 Ounces at 7 d. per Ounce.
d.		4254	d.		3729
4	$\frac{1}{3}$	<u>Sb. 1418</u>	6	$\frac{1}{2}$	<u>Sb. 1864 6</u>
1	$\frac{1}{4}$	<u>354 6</u>	1	$\frac{1}{8}$	<u>310 9</u>
		Sb. 177 2 6			Sb. 217 5 3
		£. 88 12 6			£. 108 15 3

		9271 Pounds at 8 d. per Pound.			6728 Pounds at 9 d. per Pound.
d.		9271	d.		6728
6	$\frac{1}{2}$	<u>Sb. 4635 6</u>	6	$\frac{1}{2}$	<u>Sb. 3364</u>
2	$\frac{1}{3}$	<u>1545 2</u>	3	$\frac{1}{2}$	<u>1682</u>
		Sb. 618 0 8			Sb. 504 6
		£. 309 0 8			£. 252 6

4268 Pounds at 10 d. $\frac{1}{8}$ per Pound.		3926 Pounds at 11 d. $\frac{1}{8}$ per Pound.	
4268		3926	
d.	Sh. 2134	d.	Sh. 1963
6 $\frac{1}{2}$	1067	6 $\frac{1}{2}$	981 6
3 $\frac{1}{2}$	355 8	3 $\frac{1}{2}$	654 4
1 $\frac{1}{3}$	44 $5\frac{1}{2}$	2 $\frac{1}{3}$	163 7
$\frac{1}{8}$	Sh. 360 1 $1\frac{1}{2}$	$\frac{4}{8}$	40 $10\frac{1}{2}$
	£. 180 1 $1\frac{1}{2}$		Sh. 380 3 $3\frac{1}{2}$
			£. 190 3 $3\frac{1}{2}$

4926 $\frac{1}{4}$ Ells at 5 d. $\frac{1}{4}$ per Ell.		5128 $\frac{1}{2}$ Ells at 7 d. $\frac{1}{2}$ per Ell.	
4926		5128	
d.	Sh. 1642	d.	Sh. 2564
4 $\frac{1}{3}$	410 6	6 $\frac{1}{2}$	427 4
1 $\frac{1}{4}$	102 $7\frac{1}{2}$	1 $\frac{1}{2}$	213 8
$\frac{1}{4}$	Sh. 215 5 $2\frac{1}{4}$	$\frac{1}{2}$	Sh. 320 5 $3\frac{1}{2}$
$\frac{1}{4}$	£. 107 15 $2\frac{1}{4}$		£. 160 5 $3\frac{1}{2}$

5719 $\frac{3}{4}$ Stone, at 9 d. $\frac{3}{4}$ per Stone.		6251 $\frac{3}{4}$ Stone, at 11 d. $\frac{1}{4}$ per Stone.	
5719		6251	
d.	Sh. 2859 6	d.	Sh. 3125 6
6 $\frac{1}{2}$	1429 9	6 $\frac{1}{2}$	1562 9
3 $\frac{1}{3}$	238 $3\frac{1}{2}$	3 $\frac{1}{3}$	1041 10
$\frac{1}{2}$	119 $1\frac{3}{4}$	2 $\frac{1}{3}$	260 $5\frac{1}{2}$
$\frac{1}{4}$	4 $2\frac{1}{4}$	$\frac{1}{2}$	130 $2\frac{1}{4}$
$\frac{1}{4}$	2 $\frac{1}{4}$	$\frac{1}{4}$	5 $\frac{1}{4}$
$\frac{1}{4}$	Sh. 464 7 $3\frac{1}{4}$	$\frac{1}{4}$	2 $\frac{1}{4}$
	£. 232 7 $3\frac{1}{4}$		Sh. 612 1 $5\frac{1}{4}$
			£. 306 1 $5\frac{1}{4}$

In the Examples following, the Price being at even Parts of a Pound, I divide the Quantity by that Part; it answers the Question, and needs no further Explication.

PRACTICE.

83

	2837 Dozen at 1 s. 8 d. per Dozen.		4279 Dozen at 2 s. per Dozen.
<i>s.d.</i> 18	$\frac{1}{12}$ $\frac{2837}{\text{£. 236 8 4}}$	<i>S.</i> 2	$\frac{1}{10}$ $\frac{4279}{\text{£. 427 18}}$

	259 Score, at 2 s. 6 d. per Score.		658 Score, at 3 s. 4 d. per Score.
<i>s.d.</i> 26	$\frac{1}{8}$ $\frac{259}{\text{£. 32 7 6}}$	<i>s.d.</i> 34	$\frac{1}{6}$ $\frac{658}{\text{£. 109 13 4}}$

	432 Yards $\frac{1}{2}$ at 4 s. per Yard.		573 $\frac{1}{2}$ Yards at 5 s. per Yard.
<i>S.</i> 4	$\frac{1}{3}$ $\frac{432}{\text{£. 86 8}}$	<i>S.</i> 5	$\frac{1}{4}$ $\frac{573}{\text{£. 143 5 6}}$
$\frac{1}{4}$	$\frac{1}{4}$ $\frac{1}{\text{£. 86 9}}$	$\frac{1}{2}$	$\frac{1}{2}$ $\frac{1}{\text{£. 143 7 6}}$

	doz. lb. 657 11 at 6 s. 8 d. per Dozen.		doz. lb. 783 10 at 10 s. per Dozen.
<i>s.d.</i> 68	$\frac{1}{3}$ $\frac{657}{\text{£. 219}}$	<i>s.</i> 10	$\frac{1}{2}$ $\frac{783}{\text{£. 391 10}}$
$\frac{1}{6}$	$\frac{1}{2}$ $\frac{3 4}{1 8}$	$\frac{1}{6}$	$\frac{1}{2}$ $\frac{5}{2 6}$
3	$\frac{1}{2}$ $\frac{1 1\frac{1}{4}}{1}$	3	$\frac{1}{2}$ $\frac{10}{10}$
2	$\frac{1}{3}$ $\frac{1}{\text{£. 219 6 1}\frac{1}{4}}$	1	$\frac{1}{3}$ $\frac{1}{\text{£. 391 18 4}}$

Note, In the following Examples I take first the greatest even Part, and then part of that Part, &c. and add the Quotients into one Sum.

579 Yards at 6s. per Yard.		654 Yards at 12s. per Yard.	
S.	<u>579</u>	S.	<u>654</u>
5 $\frac{1}{4}$	£. 144 15	10 $\frac{1}{2}$	£. 327
1 $\frac{1}{2}$	28 19	2 $\frac{1}{2}$	65 8
	<u>£. 173 14</u>		<u>£. 392 8</u>
375 Ells at 14s. per Ell.		728 Ells at 18s. per Ell.	
S.	<u>375</u>	S.	<u>728</u>
10 $\frac{1}{2}$	£. 187 10	10 $\frac{1}{2}$	£. 364
2 $\frac{1}{4}$	37 10	5 $\frac{1}{2}$	182
2 D.	37 10	2 $\frac{1}{2}$	72 16
	<u>£. 262 10</u>	1 $\frac{1}{2}$	36 8
			<u>£. 655 4</u>
438 Months at 7s. per Month.		675 Months at 11s. per Month.	
S.	<u>438</u>	S.	<u>675</u>
5 $\frac{1}{4}$	£. 109 10	10 $\frac{1}{2}$	£. 337 10
1 $\frac{1}{2}$	21 18	1 $\frac{1}{8}$	33 15
1 D.	21 18		<u>£. 371 5</u>
	<u>£. 153 6</u>		

		695 Weeks at 17 s. per Week.			848 Weeks at 19 s. per Week.
		695			848
S. 10 5 2	17 1/2	£. 347 10 173 15 69 10 <hr/> £. 590 15	S. 10 5 2	19	£. 424 212 84 16 84 16 <hr/> 805 12

If the *Quantity* be multiplied with half the *Number* of *Shillings*, the *Units* in the *Product* doubled for *Shillings*, the *Residue* will be *Pounds*; and shew the *Amount* of the *Quantity* at the *Price* of that *Number* of *Shillings* you took the *Half* of, and hereby you may somewhat contract the preceeding *Operations*. I shall therefore repeat the same *Examples*.

		579 Yards at 6 s. per Yard.			654 Yards at 12 s. per Yard.
		579			654
S. M 6 3		£. 173 14	S. M 12 6		£. 392 8

In *Example 1*. I multiply the *Quantity* by 3, the *Product* is 1737. I double the *Units* 7 for *Shillings*, which gives 14 *Shillings*, and so have 173 l. 14 s. for the *Price* of 579 *Yards*, at 6 *Shillings* per *Yard*. The like is to be understood in all the rest of the *Examples* ensuing.

		375 Ells at 14 s. per Ell.			728 Ells at 18 s. per Ell.
		375			728
S. M 14 7		£. 262 10	S. M 18 9		£. 655 4

		438 Months at 7 s. per Month.				675 Months at 11 s. per Month.	
S.	M	438		S.	M	675	
6	3	£.	131 8	10	5	£.	337 10
1	$\frac{1}{8}$		21 18	1	$\frac{1}{16}$		33 15
		£.	153 6			£.	371 5

		695 Weeks at 17 s. per Week.				848 Weeks at 19 s. per Week.	
S.	M	695		S.	M	848	
14	7	£.	486 10	16	8	£.	678 8
2	$\frac{1}{7}$		69 10	2	$\frac{1}{8}$		84 16
1	$\frac{1}{2}$		34 15 $0\frac{1}{4}$	1	$\frac{1}{2}$		42 8
		£.	590 15 $0\frac{1}{4}$			£.	805 12

In these two *Examples* preceeding, I begin with 14*l.* and 16*s.* because, should I begin with 16 and 18, the odd *Shilling* would require 16 and 18 for *Divisors*, to find the Value of the *Quantity*, at 1*s.* per *Week*: Which is not convenient.

		⊕ s. d. 56½ at 4 9 per ⊕				⊕ s. d. 75¼ at 6 9 per ⊕	
S.	M	56		S.	M	75	
4	2	£.	11 4	6	3	£.	22 10
6d	$\frac{1}{8}$		1 8	9d	$\frac{1}{8}$		2 16 3
3	$\frac{1}{2}$		14	$\frac{1}{8}$ ⊕	$\frac{1}{4}$		1 8½
$\frac{1}{2}$ ⊕	$\frac{1}{2}$		2 4½			£.	25 7 11½
		£.	13 8 4½				

PRACTICE.

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Y. qu. na. s. d.				Y. qu. na. s. d.			
57 3 3 at 14 7½ per Yard.				84 2 1 at 16 11½ per Yard.			
S.	M	57		S.	M	84	
12	6	£. 34	4	14	7	£. 58	16
2	6 5	14	2	7 8	8
6d	6 1	8 6	6d	6 2	2
1	6 4	9	3	6 1	1
1	6 1	2½	2	6 14	
2q	6 7	3½	2	6 3	6
1	6 3	7½	2q	6 8	5½
2q	6 1	9½	1	6 1	0½
1	6 10½					
		£. 52	6 1			£. 71	14 0½

⊕ qu. lb				⊕ qu. lb			
65 3 27 at 13s. 6d.¼ per ⊕				72 1 19 at 19s. 9d.¼ per ⊕			
S.	M	65		S.	M	72	
12	6	39	0	16	8	£. 57	12
1	6	.. 3	5	2	8 7	4
6d	6	.. 1	12 6	1	8 3	12
1	6 4	0¼	6d	6 1	16
2q	6 6	9¼	3	6 18	
1	6 3	4½	3	6 4	6
16l	6 1	11¼	1q	6 4	11½
8	6 11	1½	16l	6 2	9¼
2	6 2½		2	6 4	
1	6 1¼		1	6 2	
		£. 44	14 11¼			£. 71	14 9

If the *Question* be upon *Pounds*, &c. multiply the *Quantity* by the *Pounds*; and for your *Shillings* and *Pence* (if any) Work by the preceeding Rules, as *Conveniency* may arise.

Tun.

lb.	oz.	dwt.	gr.		lb.	oz.	dwt.	gr.
572	11	5	7	at	653	9	3	5
£. 7	18	9	per lb.		£. 15	16	8	per lb.
572					653			
7					15			
Sb. M. £. 4004					3265			
16	8	..	457	12	653			
2	$\frac{1}{2}$...	57	4				
6 d.	$\frac{1}{8}$...	14	6	Sb. M. £. 9795			
3	$\frac{1}{4}$...	7	3	14	7	...	457 2
6 oz.	$\frac{1}{2}$...	3	19	2	$\frac{1}{7}$...	65 6
3	$\frac{1}{2}$...	1	19	6 d.	$\frac{1}{4}$...	16 6 6
2	$\frac{1}{8}$...	1	6	2	$\frac{1}{3}$...	5 8 10
4 dwt.	$\frac{1}{16}$...	2	7	$\frac{1}{2}$	$\frac{1}{4}$...	1 7 2
1	$\frac{1}{4}$...	7	3	$\frac{1}{8}$	$\frac{1}{8}$...	13 7
6 gr.	$\frac{1}{8}$...	1	3	6 oz.	$\frac{1}{2}$...	7 18 4
1	$\frac{1}{16}$...	1	3	2	$\frac{1}{8}$...	2 12 9
					1	$\frac{1}{2}$...	1 6 4
£. 4547	13	11	1		2 dwt.	$\frac{1}{16}$...	2 7
					1	$\frac{1}{2}$...	1 3
					4 gr.	$\frac{1}{8}$...	2
					1	$\frac{1}{4}$...	1
					£. 10353	5	10	

Thus having finished what I intended upon the five General Parts of *Arithmetic*; I shall now proceed to shew the Use of *Fractions*, both *Vulgar* and *Decimal*; and because the Work of *Decimals* differs not from whole Numbers, but in separating the *Decimals* from the *Integers* (after I have given you the Definition of a *Fraction*) shall begin with *Decimal Arithmetic*.

FRACTIONS are of two Kinds, viz. *Vulgar* and *Decimal*.

A *Vulgar Fraction* is that which represents a Part or Parts of any thing proposed ; and is expressed by two Numbers placed one above the other, with a Line drawn betwixt them ; the upper Number being called the *Numerator*, and the under the *Denominator*.

Numerator $\frac{3}{4}$ thus expressed $\frac{\text{Three}}{\text{Fourths}}$ of an Unit.
Denominator

The *Denominator* denotes the Number of equal Parts that the Thing is divided into, and the *Numerator* denotes the Number of Parts that are contained in the Fraction.

A *Decimal Fraction* is such a one whose *Denominator* is an *Unit*, with as many Cyphers annexed, as there be Figures in the Numerator.

as $\frac{5}{10}$	thus expressed	$\frac{\text{Five}}{\text{Tenths}}$	Parts of an Unit,
$\frac{75}{100}$	$\frac{\text{Seventy-five}}{\text{Hundreds}}$	
$\frac{125}{1000}$	$\frac{\text{One hundred twenty-five}}{\text{Thousands}}$	

DECIMAL ARITHMETIC,

Is an artificial Invention of managing *Fractions*, or broken Numbers, by a much more commodious and easy Way than that of *Vulgar Fractions*; and supposeth every whole *Unit* to be divided into Ten Parts, and therefore they are called *Decimal Fractions*, the Number 10 being always the least *Denominator* ; but because this small Number is not sufficient so plainly and fully to distinguish the Value of all *Fractions*, it is still increased by 10, viz. to 100, 1000, 10000, 100000, or into as many *Tenths* as you please ; but still the *Numerator* and *Denominator* bear the same Value and Proportion

portion to each other, as if they were in lesser Numbers.

The *Denominators* of *Decimal Fractions* are never set down, but only the *Numerators*, which are distinguished from the *Integers*, or Whole Numbers, by a *Period*, or some other Note of Distinction, as in the following Examples.

Thus, 5.7 expressed thus, $5\frac{7}{10}$

25.78 $25\frac{78}{100}$

36.069 $36\frac{69}{1000}$

347.7298 $347\frac{7298}{10000}$

322.007298 $322\frac{7298}{1000000}$

Note : *Cyphers* annexed to a *Decimal*, alters not its Value.

For .5 and .50 or .500 are equal one to another.

But *Cyphers* prefixed, decreases their Value.

As .5 is five Parts of 10

.05 is five Parts of 100

.005 is five Parts of 1000, &c.

Vulgar Fractions are reduced into *Decimal* ones by this General Rule.

R U L E.

Annex *Cyphers* to the *Numerator*, and then divide by the *Denominator* ; the *Quotient* gives the *Decimal* Parts equal to the given Fraction; or at least so near it as may be thought necessary to approach.

N 2

Note :

Note : the *Quotient* must contain as many *Decimal Places*, as there are *Cyphers* annexed, and when wanting, must be supply'd by prefixing *Cyphers*.

Let $\frac{1}{4}$ $\frac{1}{2}$ $\frac{3}{4}$ $\frac{2}{3}$ $\frac{7}{8}$ $\frac{11}{12}$ $\frac{23}{56}$ $\frac{398}{3990}$ and $\frac{21}{2792}$
be reduced into *Decimals*.

$$\begin{array}{r} \frac{1}{4} \quad .25 \\ 4 \overline{) 1.00} \\ \text{Rem. } 0 \end{array} \quad \begin{array}{r} \frac{1}{2} \quad .5 \\ 2 \overline{) 1.0} \\ \text{Rem. } 0 \end{array} \quad \begin{array}{r} \frac{3}{4} \quad .75 \\ 4 \overline{) 3.00} \\ \text{Rem. } 0 \end{array} \quad \begin{array}{r} \frac{2}{3} \quad .6666 \\ 3 \overline{) 2.0000} \\ \text{Rem. } 2 \end{array}$$

$$\begin{array}{r} \frac{7}{8} \quad .875 \\ 8 \overline{) 7.000} \\ \text{Rem. } 0 \end{array} \quad \begin{array}{r} \frac{11}{12} \quad .9166 \\ 12 \overline{) 11.0000} \\ \text{Rem. } 8 \end{array} \quad \begin{array}{r} \frac{23}{56} \quad .410714 \\ 56 \overline{) 23.000000} \\ \text{Rem. } 16 \end{array}$$

$$\begin{array}{r} \frac{398}{3990} \quad .099749 \\ 3990 \overline{) 398.000000} \\ \text{Rem. } 1490 \end{array} \quad \begin{array}{r} \frac{21}{2792} \quad .007521 \\ 2792 \overline{) 21.000000} \\ \text{Rem. } 1368 \end{array}$$

Note : In reducing *Vulgar Fractions* to *Decimals*, if nothing remain, the *Decimal Fraction* is exactly equal to the *Vulgar* : But if there be any Remainder, it is not so ; yet by annexing more *Cyphers*, and continuing the Division, you may bring them very near the Truth ; but 6 or 7 Places will be near enough for most Operations

From hence it will not be difficult to find the *Decimal Parts* of *Money*, *Weight*, or *Measure*, if you first reduce the given Parts of *Money*, *Weight*, or *Measure* into a *Vulgar Fraction*, whose *Denominator* is the Number of those known Parts contained in the *Integer*, and the given Parts its *Numerator* ; as will plainly appear in the following Examples :

Let the following Sums of *Money* be reduced into *Decimals*, a *Pound* being *Integer*, viz. 18 s. and 3 s. 6 d. and 2 s. 5 $\frac{1}{2}$ d. and 11 d. and 4 $\frac{1}{4}$ d. and $\frac{1}{2}$ d. $\frac{1}{4}$ d. and $\frac{1}{8}$ d.

First,

First, 18 Shillings is $\frac{18}{20}$ of a Pound.

Again, 3s. 6d. is 42 Pence, that is $\frac{42}{240}$ of a Pound; and 2s. 5 $\frac{1}{2}$ d. is 118 Farthings, that is, $\frac{118}{960}$ of a Pound; and 11 Pence is $\frac{11}{960}$ of a Pound; and 4 $\frac{1}{2}$ d. is 17 Farthings, that is $\frac{17}{960}$ of a Pound; and $\frac{1}{2}$ d. is $\frac{3}{960}$ of a Pound; and $\frac{1}{4}$ d. is $\frac{2}{960}$ of a Pound.

Therefore these Fractions,

$$\frac{18}{20} \quad \frac{42}{240} \quad \frac{118}{960} \quad \frac{11}{240} \quad \frac{17}{960} \quad \frac{3}{960} \quad \frac{2}{960} \quad \frac{1}{960}$$

must be reduced into Decimals as followeth:

$$\begin{array}{r} 18 \quad .9 \\ 20 \overline{)18.0} \\ \text{Rem. } 0 \end{array} \quad \begin{array}{r} 42 \quad .175 \\ 240 \overline{)42.000} \\ \text{Rem. } 0 \end{array} \quad \begin{array}{r} 118 \quad .122916 \\ 960 \overline{)118.000000} \\ \text{Rem. } 640 \end{array}$$

$$\begin{array}{r} 11 \quad .045833 \\ 240 \overline{)11.000000} \\ \text{Rem. } 80 \end{array} \quad \begin{array}{r} 17 \quad .017708 \\ 960 \overline{)17.000000} \\ \text{Rem. } 320 \end{array} \quad \begin{array}{r} 3 \quad .003125 \\ 960 \overline{)3.000000} \\ \text{Rem. } 0 \end{array}$$

$$\begin{array}{r} 2 \quad .002083 \\ 960 \overline{)2.000000} \\ \text{Rem. } 320 \end{array} \quad \begin{array}{r} 1 \quad .001041 \\ 960 \overline{)1.000000} \\ \text{Rem. } 640 \end{array}$$

Again, Let the following Sums of Money be reduced into Decimals, a Shilling being Integer, viz. 11d. and 8 $\frac{1}{2}$ d. and $\frac{1}{2}$ d. $\frac{1}{2}$ d. and $\frac{1}{4}$ d.

First, 11 Pence is $\frac{11}{20}$ of a Shilling, and 8 $\frac{1}{2}$ d. is 34 Farthings, that is $\frac{34}{240}$ of a Shilling, and $\frac{1}{2}$ d. is $\frac{1}{48}$, and $\frac{1}{4}$ d. is $\frac{1}{96}$, and $\frac{1}{8}$ d. is $\frac{1}{192}$ of a Shilling. Therefore these Fractions $\frac{11}{20}$ $\frac{34}{240}$ $\frac{1}{48}$ $\frac{1}{96}$ and $\frac{1}{192}$ must be reduced into Decimals, as afore.

$$\begin{array}{r} 11 \quad .916666 \\ 12 \overline{) 11.000000} \\ \text{Rem. } 8 \end{array}$$

$$\begin{array}{r} 34 \quad .708333 \\ 48 \overline{) 34.000000} \\ \text{Rem. } 16 \end{array}$$

$$\begin{array}{r} 3 \quad .0625 \\ 48 \overline{) 3.0000} \\ \text{Rem. } 0 \end{array}$$

$$\begin{array}{r} 2 \quad .041666 \\ 48 \overline{) 2.000000} \\ \text{Rem. } 32 \end{array}$$

$$\begin{array}{r} 1 \quad .020833 \\ 48 \overline{) 1.000000} \\ \text{Rem. } 16 \end{array}$$

In like Manner may *Weight*, and *Measure* be reduced into *Decimals*.

Let 3 *qu.* 27 *lb.* 1 *qu.* 19 *lb.* and 17 *lb.* be reduced into *Decimals*, an *Hundred Weight* being *Integer*,

3 *qu.* 27 *lb.* is 111 *lb.* that is, $\frac{111}{112}$ of an C . and 1 *qu.* 19 *lb.* is 47 *lb.* that is, $\frac{47}{112}$ of an C . and 17 *lb.* is $\frac{17}{112}$ of an C . Therefore $\frac{111}{112}$ and $\frac{47}{112}$ and $\frac{17}{112}$ must be reduced into *Decimals*, and they will produce as followeth.

$$\begin{array}{r} 111 \quad .991071 \\ 112 \overline{) 111.000000} \\ \text{Rem. } 48 \end{array}$$

$$\begin{array}{r} 47 \quad .419642 \\ 112 \overline{) 47.000000} \\ \text{Rem. } 96 \end{array}$$

$$\begin{array}{r} 17 \quad .151785 \\ 112 \overline{) 17.000000} \\ \text{Rem. } 80 \end{array}$$

So likewise, if the *Decimal* of 2 *qu.* 3 *na.* be required, because 2 *qu.* 3 *na.* is 11 *Nails*, that is, $\frac{11}{16}$ of a *Yard*; therefore $\frac{11}{16}$ reduced into a *Decimal* is .6875.

Also, if the *Decimal* of 75 *Days* be required, then $\frac{75}{365}$ reduced into a *Decimal*, will produce it.

Having thus shewn from whence *Decimal Fractions* arise; I shall now proceed to the Management of

of them in the Rules of *Addition, Subtraction, Multiplication, and Division.*

ADDITION of DECIMALS.

THE *Periods* or *Decimal Points* must stand in a direct Line one under another, and the Number of *Decimal Parts* in the *Total*, must be as many as are the most in the given Numbers.

£.	lb Troy	Yards
387.29467	5719.2843	67142.9718
267.1973	317.269	3719.7142
492.6835	412.371	462.916
271.9469	57.29	71.937
382.719	54.72	1.84
467.284	6.9	9.7
<hr/>		
£. 2269.12537	lb 6567.8343	ya. 71409.0790
20	12	4
<hr/>		
Sh. 2.50740	oz. 10.0116	qu. .3160
12	20	4
<hr/>		
d. 6.08880	dwt. 0.2320	na. 1.2640
	24	
	9280	
	4640	
	<hr/>	
	gr. 5.5680	

In these *Examples* the *Total* is obtained by adding them together, as in *Addition of Integers*; and for the Value of the *Decimal Fraction*, I multiply it with the Number of *Units* that are contained in the next lower *Denomination*, retaining so many *Decimal Places* in the *Product* as were in the *Total*; and the rest are *Integers* of that *Denomination*, and so proceed to the lowest *Denomination*, as is plain from

from the preceeding *Examples*, and will further appear as we go on.

SUBTRACTION OF DECIMALS.

AS in *Addition*, so in *Subtraction*, the *Period* or *Decimal Points* must stand in a direct Line one under another, and the Number of *Decimals* in the *Remainder*, must be as many are the most in the given Numbers.

	£.	⊕
From	719467.287939	7524.91673
Take	+ 29763.94671	+ 1978.931792
Rem.	£. 689703.341229	⊕ 5545.984938
	20	4
	<hr/> Sh. - - 6.824580	<hr/> qr. 3.939752
	12	28
	<hr/> d. - - - 9.894960	<hr/> 7518016
	4	1879504
	<hr/> f. - - - 3.579840	<hr/> lb 26.313056

Multiplication of Decimals.

THE Number of *Decimal Parts* in the *Multiplicand* and *Multiplier* counted together, must be cut off in the *Product*:

But if it should so happen that the Number of *Figures* in the *Product* be less than the Number of *Decimal Parts* in the *Multiplicand* and *Multiplier* counted together, then so many *Cyphers* must be prefixed

Decimal Arithmetic.

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fixed as will make up the Number of *Decimal Parts* there equal to the Number of *Decimal Parts* in the *Multiplicand* and *Multiplier* counted together.

$$\begin{array}{r} \text{£.} \\ \text{mult. } 7184627.9148 \\ \text{by } .7 \\ \hline \end{array}$$

$$\begin{array}{r} \text{£. } 5029239.54036 \\ 20 \\ \hline \end{array}$$

$$\begin{array}{r} \text{Sb. } - - 10.80726 \\ 12 \\ \hline \end{array}$$

$$\begin{array}{r} \text{d. } - - - 9.68712 \\ 4 \\ \hline \end{array}$$

$$\begin{array}{r} \text{f. } - - - 2.74848 \\ \hline \end{array}$$

$$\begin{array}{r} \text{Feet} \\ \text{mult. } 71946729.3 \\ \text{by } .39 \\ \hline \end{array}$$

$$\begin{array}{r} 6475205637 \\ 2158401879 \\ \hline \end{array}$$

$$\begin{array}{r} \text{Feet } 28059224.427 \\ 12 \\ \hline \end{array}$$

$$\begin{array}{r} \text{Inches } 5.124 \\ \hline \end{array}$$

$$\begin{array}{r} \text{mult. } .1271937146 \\ \text{by } .0124 \\ \hline \end{array}$$

$$\begin{array}{r} .00157720206104 \\ \hline \end{array}$$

$$\begin{array}{r} \text{lb. } \text{Averdupois} \\ \text{mult. } .124765348 \\ \text{by } .5 \\ \hline \end{array}$$

$$\begin{array}{r} \text{lb. } .0623826740 \\ 16 \\ \hline \end{array}$$

$$\begin{array}{r} 3742960440 \\ 623826740 \\ \hline \end{array}$$

$$\begin{array}{r} \text{oz. } 9981227840 \\ 16 \\ \hline \end{array}$$

$$\begin{array}{r} 59887367040 \\ 9981227840 \\ \hline \end{array}$$

$$\begin{array}{r} \text{dr. } 15.9699645440 \\ \hline \end{array}$$

$$\begin{array}{r} \text{Weeks} \\ \text{mult. } 3152.796 \\ \text{by } 48 \\ \hline \end{array}$$

$$\begin{array}{r} 25222368 \\ 12611184 \\ \hline \end{array}$$

$$\begin{array}{r} \text{Weeks } 151334.208 \\ 7 \\ \hline \end{array}$$

$$\begin{array}{r} \text{Days } 1.456 \\ 24 \\ \hline \end{array}$$

$$\begin{array}{r} 1824 \\ 912 \\ \hline \end{array}$$

$$\begin{array}{r} \text{Hours } 10.944 \\ 60 \\ \hline \end{array}$$

$$\begin{array}{r} \text{Min. } 56.640 \\ \hline \end{array}$$

In

In the *Example of Pounds* there are *four Decimals* in the *Multiplicand*, and *one* in the *Multiplier*, therefore I cut off *five* in the *Product*.

In the *Example of Pounds Averdupoize* there is *nine Decimals* in the *Multiplicand*, and *one* in the *Multiplier*, which is *ten*, to be cut off in the *Product*; but there are but *nine Figures* in the *Product*; therefore *one Cypher* is prefixed to make *ten*.

Understand the like in the rest of the *Examples*.

DIVISION OF DECIMALS.

FROM the Number of *Decimal Parts* in the *Dividend*, take the Number of *Decimal Parts* in the *Divisor*, the *Remainder* is the Number of *Decimal Parts* that must be cut off in the *Quotient*.

If the *Dividend* have no *Decimal Parts*, or a less Number of *Decimal Parts* than are in the *Divisor*, then so many *Cyphers* at least, must be annexed to the *Dividend*, as will make the Number of *Decimals* there equal to the Number of *Decimal Parts* in the *Divisor*.

And if it should so happen, that the Number of *Figures* in the *Quotient*, be less than the *Difference* between the *Dividend* and *Divisor*, then so many *Cyphers* must be prefixed as will make the Number of *Decimal Parts* there, equal to the *Difference* between the *Dividend* and *Divisor*.

$$\begin{array}{r} 4181 \quad 742934 \\ .5 \overline{) 371467.2} \\ \text{Rem. } 2 \end{array}$$

$$\begin{array}{r} 5313.5245 \\ .7 \overline{) 3719.46719} \\ \text{Rem. } 4 \end{array}$$

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3021630

5196612

9)2719467.0

.08)415729.00

Rem. 0

Rem. 4

1928

.00725

37-2946)71927.2100

374-6)2.719462

Rem. 232212

Rem. 3612

In the first *Example*, there being *one Decimal* in the *Dividend*, and *one* in the *Divisor*, there remains none to be cut off in the *Quotient*, so that the *Quotient* is all *Integers*.

Example 2. There are *five Decimals* in the *Dividend*, and *one* in the *Divisor*, therefore *one* from *five* remains *four Decimals*, to be cut off in the *Quotient*.

Example 3. There being no *Decimals* in the *Dividend*, and *one* in the *Divisor*, *one Cypher* is annexed to the *Dividend*, and the *Quotient* is all *Integers*.

Example 4. There being no *Decimals* in the *Dividend*, and *two* in the *Divisor*, *two Cyphers* are annexed to the *Dividend*, and the *Quotient* is all *Integers*.

Example 5. There being *two Decimals* in the *Dividend*, and *four* in the *Divisor*, *two Cyphers* are annexed to the *Dividend*, and the *Quotient* is all *Integers*.

Example 6. There being *six Decimals* in the *Dividend*, and *one* in the *Divisor*, *one* from *six* remains *five*, to be cut off in the *Quotient*; but the *Quotient* having but *three Figures*, *two Cyphers* are prefixed.

Note, In those *Examples* where *Cyphers* are annexed, but so many are annexed, as makes the *Decimal Parts* equal to the Number of *Decimal Parts* in the *Divisor*, you may annex as many more as you please, and then the *Quotient* will consist of some *Decimal Parts* besides the *Integers*.

The Computation of the Duties payable in the *Custom-house* upon Goods imported or exported, requiring some Skill in *Decimals*; I shall here add some *Examples* thereupon,

1. What is the *Duty* of 96 *Dozen*, at
Sb. 1 : 8.52 *per Dozen*.

$$\begin{array}{r} 6 \ 10 \\ \hline \end{array}$$

$$\begin{array}{r} \text{Sb. } 17 : 1.20 \\ \hline \end{array}$$

$$\begin{array}{r} 9 \\ \hline \end{array}$$

$$\begin{array}{r} \text{£. } 7 : 13 : 10.80 \\ \hline \end{array}$$

$$\begin{array}{r} 10 : 3.12 \\ \hline \end{array}$$

$$\text{£. } 8 : 4 : 1.92 \text{ Duty.}$$

Note, the *Penny*
 is divided into
 100 *Parts*.

2. What is the *Duty* of 108 *Gross*, at
Sb. 2 : 6.49½ *per Gross*.

$$\begin{array}{r} 8 \ 10 \\ \hline \end{array}$$

$$\begin{array}{r} \text{£. } 5 : 5 : 4.95 \\ \hline \end{array}$$

$$\begin{array}{r} 10 \\ \hline \end{array}$$

$$\begin{array}{r} \text{£. } 12 : 14 : 1.50 \\ \hline \end{array}$$

$$\begin{array}{r} 1 : 0 : 3.96 \\ \hline \end{array}$$

$$\text{£. } 13 : 14 : 5.46 \text{ Duty.}$$

3. What

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3. What is the *Duty* of 496 Pounds, at
d. 1.03½ per Pound.

$$\begin{array}{r}
 6 \text{ } 10 \\
 \hline
 \text{d. } 10.31\frac{1}{2} \\
 9 \text{ } 10 \\
 \hline
 \text{Sb. } 8 : 7.12\frac{1}{2} \\
 4 \\
 \hline
 \text{£. } 1:14 : 4.50 \\
 7 : 8.81\frac{1}{2} \\
 6.18\frac{1}{2} \\
 \hline
 \text{£. } 2 : 2 : 7.50 \text{ Duty.}
 \end{array}$$

4. What is the *Duty* of 296 Pieces, at
Sb. 1:10.72½ per Piece.

$$\begin{array}{r}
 6 \text{ } 10 \\
 \hline
 \text{Sb. } 18:11.22\frac{1}{2} \\
 9 \text{ } 10 \\
 \hline
 \text{£. } 9 : 9 : 6.25 \\
 2 \\
 \hline
 \text{£. } 18:19 : 0.50 \\
 8:10 : 5 .2\frac{1}{2} \\
 11 : 4.33\frac{1}{2} \\
 \hline
 \text{£. } 28 : 0 : 9.86 \text{ Duty.}
 \end{array}$$

5. What is the *Duty* of 729 Skins, at
d. 1.89½ per Skin.

$$\begin{array}{r}
 9 \text{ } 10 \\
 \hline
 \text{Sb. } 1 : 6.93\frac{1}{2} \\
 2 \text{ } 10 \\
 \hline
 \text{Sb. } 15 : 9.37\frac{1}{2} \\
 7 \\
 \hline
 \text{£. } 5:10 : 5.62\frac{1}{2} \\
 3 : 1.87\frac{1}{2} \\
 1 : 5 .4\frac{1}{2} \\
 \hline
 \text{£. } 5:15 : 0.54\frac{1}{2} \text{ Duty.}
 \end{array}$$

6. What

102 CUSTOM DUTIES

6. What is the Duty of 649 Skins, at
d. 0.65¢ per Skin.

$$\begin{array}{r} 9 \text{ } 10 \\ \hline d. 6.56\frac{1}{2} \\ 4 \text{ } 10 \\ \hline \text{Sh. } 5 : 5.62\frac{1}{2} \\ 6 \end{array}$$

$$\begin{array}{r} \text{£. } 1:12 : 9.75 \\ 2 : 2.25 \\ \hline 5.90\frac{1}{2} \end{array}$$

£. 1:15 : 5.90¢ Duty.

7. What is the Duty of 127½ Dozens, at
Sh. 1 : 3.28 ¢ per Dozen.

$$\begin{array}{r} 7 \text{ } 10 \\ \hline \text{Sh. } 12 : 8.88 \frac{1}{2} \\ 2 \text{ } 10 \\ \hline \text{£. } 6 : 7 : 4.87 \frac{1}{2} \\ 1 : 5 : 5.77 \frac{1}{2} \\ 8 : 11 .2 \frac{1}{2} \\ \frac{1}{2} \text{ doz. } - - - - - 7.64 \frac{1}{2} \frac{1}{2} \\ \frac{1}{4} \text{ doz. } - - - - - 3.82 \frac{1}{4} \frac{1}{4} \frac{1}{4} \\ \hline \text{£. } 8 : 2 : 9.13 \frac{1}{4} \frac{1}{4} \text{ Duty.} \end{array}$$

8. What

CUSTOM DUTIES. 393

Ⓕ qu. lb

8. What is the Duty of 479:3:26 at

Sb. 3:3.63 1/2 per Ⓕ

9 10

£. 1:13:0.37 1/2
7 10

£. 16:10:3.75
4

£. 66:01:3.00

11:11:2.62 1/2

qu. 1:09:8.73 1/2

2 1/2 --- 1:7.81 1/2

1 1/2 --- 9.90 1/2

lb. 16 1/2 --- 5.56 1/2

8 1/2 --- 2.83 1/2

2 1/2 --- .70 1/2

£. 79:5:5.29 1/2

9. What

164 CUSTOM DUTIES.

9. What is the Duty of 22:1:15 at 6:8 per Pound, paying 5 per Cent. and 5 per Cent. off.

qu. lb.

22:1:15

4

89

28

727

178

2507 lb.

s. d.

6:8 $\frac{1}{2}$ £. 835:13:4 at 5 per Cent.

5

£. 41|78:06:8 £. 41:15:8 at 5 per Ct.

20

5

Sb. 15|66

£. 2|08:18:4

12

20

d. 8|00

Sb. 1|78

12

d. 9.40

l. s. d.

Duty 41:15:8

5 per Cent. off. 2:1:9.40

neat Duty. 39:13:10.60

10. What

CUSTOM DUTIES. 105

10. What is the *neat Duty* of $96\frac{1}{2}$ £ at 2 l. per £ paying *Subsidy* at 5 per Cent. and 5 per Cent. off, *Impost* at 2 s. 6 d. per £ and $6\frac{1}{2}$ per Cent. off.

£ $96\frac{1}{2}$

$\frac{2}{\text{£}. 193}$ at 5 per Cent.

$\frac{5}{\text{£}. 9|65}$

$\frac{20}{\text{Sb. } 13|00}$

l. s.

9:13 at 5 per Cent.

$\frac{5}{148:05}$

20

$\text{Sb. } 9|65$

12

s. d.

d. 7:80

2: 6 $\frac{1}{4}$ - - - $96\frac{1}{2}$

12

$\frac{1}{2}$ £ - - - 0:1:3

$\text{£}. 12:1:3$ at $6\frac{1}{4}$ per Cent.

6

$\text{£}. 72:7:6$

$\frac{3}{4}$ - - - - - 3:0:3 $\frac{1}{2}$

175:7:9 $\frac{1}{4}$

20

$\text{Sb. } 15|07$

12

193 $\frac{1}{2}$

Subsidy L. 9:13:0

5 per Cent. off - - 9:7.80

Neat Subsidy $\text{£}. 9:3:4.20$

Impost $\text{£}. 12:1:3$

$6\frac{1}{4}$ per Cent. off - - - 15:0.93 $\frac{1}{2}$

Neat Impost $\text{£}. 11:6:2.06\frac{1}{4}$

Neat Subsidy 9:3:4.20

Neat Duty $\text{£}. 20:9:6.26\frac{1}{4}$

P

What

106 CUSTOM DUTIES.

11. What is the *Neat Duty* of 35 qrs. 24 *lb.* at 3 *s.* 4 *d.* per *lb.* paying *Subsidy* at 5 per Cent. and 5 per Cent. off, *Impost* at 10 per Cent. and 6 $\frac{1}{4}$ per Cent. off.

qrs. 35:3:24

4

143

28

1148

288

s. d. 4028 *lb.* at 3 *s.* 4 *d.* per *lb.*

3:4 $\frac{1}{4}$ - 1671: 6:8 at 5 per Cent.

5

£. 33|56:13:4 £. 33:11:4 at 5 per Ct.

20

5

Sh. 11|33

£. 1|67:16:8

12

20

d. 4|00

Sh. 13|56

12

d. 6|80

£. 671:6:8 at 10 per Cent.

10

£. 67|13:6:8 £. 67: 2:8 at 6 $\frac{1}{4}$ per Cent.

20

6

Sh. 2|66

£. 402:16:0

12

$\frac{1}{4}$ - - - 16:15:8

d. 8|00

£. 4|19:11:8

20

Sh. 3|91

12

d. 11|00

CUSTOM DUTIES. 107

	l.	s.	d.
Subsidy	33	11	4
5 per Cent. off	1	13	6 . 80

Neat Subsidy £. 31:17: 9 . 20

Impost	£. 67	2	8
6 $\frac{1}{4}$ per Cent. off	4	3	11

Neat Impost	£. 62	18	9
Neat Subsidy	31	17	9 . 20

Neat Duty £. 94:16: 6 . 20

12. What is the *Neat Duty* of 976 Dozen, at 6l. per Dozen, paying *old Subsidy*, and *new Subsidy*, each 5 per Cent. and 5 per Cent. off, and $\frac{1}{2}$ Subsidy at 1l. 13s. 4d. per Cent. and 5 per Cent. off.

976 Dozen, at 6l. per Dozen.

6

5856l. at 5 per Cent.

5
£. 292 80
20
Sb. 16 00

	l.	s.
Old Subsidy	292	16
New Subsidy	292	16
$\frac{1}{2}$ Subsidy . .	97	12

Total Subsidy £. 683 : 4

£. 683 : 4 at 5 per Cent.

5
£. 34 16 : 0
20

Sb. 3 20
12

d. 2 | 40

	l.	s.	d.
Subsidies	683	4	0
5 per Cent. off	34	3	2 . 40
Neat Duty	£. 649	0	9 . 60

108 CUSTOM DUTIES.

13. What is the *Neat Duty* of 754 $\frac{1}{2}$ hundred Dozen, at 13 s. 4 d. per hundred Dozen; paying Old Subsidy, New Subsidy, and $\frac{1}{3}$ and $\frac{2}{3}$ Subsidy, each 5 per Cent. and 5 per Cent. off.

Note, $\frac{1}{3}$ and $\frac{2}{3}$ Subsidy, make an whole Subsidy.

s. d. 754 $\frac{1}{2}$ at 6 s. 8 d. per hundred Dozen.
 6:8 $\frac{1}{2}$ 251: 6: 8
 6:8 d^o 251: 6: 8
 $\frac{1}{2}$. . . 6: 8

£. 503: 0: 0 Old Subsidy.
 503: 0: 0 New Subsidy.
 503: 0: 0 $\frac{1}{3}$ and $\frac{2}{3}$ Subsidy.

£. 1509: 0: 0 at 5 per Cent.

 5
 £. 75|45: 0: 0
 20

Sb. 9|00

l. s.
 75: 9 at 5 per Cent.
 5

£. 3|77: 5
 20

Sb. 15|45

 12
a. 5|40

Booc

Subsidies £. 75: 9: 0
 5 per Cent. off 3: 15: 5. 40
 Neat Duty £. 71: 13: 6. 60

SQUARE

SQUARE *and* CUBE ROOT.

I Shall in this Place fix the Extraction of the *Square* and *Cube* Roots ; because in Practice it very seldom happens, that the Number of which the *Square* and *Cube* Root is required, proves an exact *Square* or *Cube*, and therefore the *Root* is imperfect ; but by the Use of a *Decimal*, we may approach as near thereunto, so as to want so little of Exactness, as may be very well deemed as nothing.

The following Table gives you the true *Square* and *Cube*, from an *Unit* to *Nine*, which is all that need be committed to the Memory.

Roots.	Squares.	Cubes.
1	1	1
2	4	8
3	9	27
4	16	64
5	25	125
6	36	216
7	49	343
8	64	512
9	81	729

To extract the *Square* Root of any Sum or Number, the first Thing to be done, is to place a *Period*, or any other Note of Distinction, over the last Figure ; then leaving *one* Figure, place another over the next, and so proceed, leaving *one* Figure, till you have gone through the whole, the Number of these *Periods* shew you of how many Figures the *Root* consists. Then seek the nearest *Square* to the *first Period*, and subtract it therefrom, and bring down the next two Figures thereto, which gives a *Dividend*. Now to find a *Divisor* thereto, you

you must multiply the *Root* by 2, then see how often that *Divisor* is contained in the *Dividend*, without regarding the *Units* thereof, and place the same to the *Root*. To find the *Subtrahend*, multiply the *Divisor* with the *last* Figure in the *Root* annexed, by the *last* Figure in the *Root*, and bring down the *two* next Figures to the *Remainder*, which will give a new *Dividend*, then proceed as before, till the *Root* is finished. An *Example* or two will render these Instructions more plain.

Note: When your Work is ended, if there be any *Remainder*, by the bringing down *two Cyphers*, and proceeding thereupon, you will have a *true Root* to the *tenth* Part of an *Unit*, and if you proceed to bring down *two Cyphers* more, it will be a *true Root* to the *hundredth* Part of an *Unit*, and so on, you may have a *true Root* to the *thousandth* and *ten thousandth* Part of an *Unit*, or to what *Exactness* you please.

$$\begin{array}{r} 2 \quad 3 \quad 7 \quad 2 \quad 8 \\ \sqrt{13897984} \\ 9 \end{array}$$

$$\begin{array}{r} 67|489 \\ 469 \end{array}$$

$$\begin{array}{r} 742|2079 \\ 1484 \end{array}$$

$$\begin{array}{r} 7448|59584 \\ 59584 \\ \hline 0 \end{array}$$

$$\begin{array}{r} 2 \quad 1 \quad 3 \quad 3 \quad 8 \quad 9 \quad 7 \quad 2 \\ \sqrt{17928467958.00} \\ 1 \end{array}$$

$$\begin{array}{r} 23|79 \\ 69 \end{array}$$

$$\begin{array}{r} 263|1028 \\ 789 \end{array}$$

$$\begin{array}{r} 2668|23946 \\ 21344 \end{array}$$

$$\begin{array}{r} 26769|260279 \\ 240921 \end{array}$$

$$\begin{array}{r} 267787|1935858 \\ 1874509 \end{array}$$

$$\begin{array}{r} 2677942|6134900 \\ 5355884 \\ \hline 779116 \end{array}$$

SQUARE ROOT.

III

$$\begin{array}{r}
 \sqrt{598855.085} \\
 \underline{358627412975.000000} \\
 25 \\
 109 \overline{)1086} \\
 \underline{981} \\
 1188 \overline{)10527} \\
 \underline{9504} \\
 11968 \overline{)102341} \\
 \underline{95744} \\
 119765 \overline{)0659729} \\
 \underline{598825} \\
 1197705 \overline{)6090475} \\
 \underline{5988525} \\
 119771008 \overline{)1019500000} \\
 \underline{958168064} \\
 1197710165 \overline{)6133193600} \\
 \underline{5988550825} \\
 144642775
 \end{array}$$

The *Root* squared with the *Remainder*, if any, added thereunto, proves the Truth of the Operation, as exemplified in this last *Example*.

$$\begin{array}{r}
 598855.085 \\
 \underline{598855.085} \\
 2994275425 \\
 4790840680 \\
 29942754250 \\
 2994275425 \\
 4790840680 \\
 4790840680 \\
 5389695765 \\
 2994275425 \\
 \text{Rem.} \dots 144642775 \\
 \hline
 \text{Square, } 358627412975.000000
 \end{array}$$

To extract the *Cube Root* of any Sum or Number. First, place a *Period* over the *last* Figure; then leaving *two* Figures, point the next, and so on, to the *End*; then seek the nearest *Cube*, to the first Point, and subtract it therefrom; bring down the next *three* Figures to the *Remainder*, it gives you a *Dividend*.

To find a *Divisor* thereto. Multiply the *Square* of the *Root* by 3; then see how often the *Divisor* is contained in the *Dividend*, when two Figures, *viz.* the *Units* and *Tens*, are rejected, and place the same in the *Root*.

To find the *Subtrahend*.

1. *Cube* the last Figure placed in the *Root*.
2. Multiply the *Triple* of all the Figures, except the *last*, by the *Square* of the last.
3. Multiply the *Divisor* by the *last* Figure.

These three *Products*, rightly placed, and added together, gives the *Subtrahend*.

Note, By adding three *Cyphers* to the *Remainder*, you may proceed, and obtain a true *Root* to the 10th, 100th, 1000th Part of an *Unit*, or nearer, if you please.

*Note by adding three Cyphers
to the Remainder is*

CUBE ROOT.

113

Near 1st Cube.

$$\begin{array}{r} 8 \ 3 \ 6 \ 9 \\ \sqrt{586166107409} \\ 512 \end{array}$$

$$\begin{array}{r} 192 \overline{) 74166} \\ \hline \end{array}$$

Is the 1st 64 is 8 Root

27

216

576

59787

$$\begin{array}{r} 20667 \overline{) 14379107} \\ \hline \end{array}$$

216

8964

124002

12490056

$$\begin{array}{r} 2096688 \overline{) 1889051409} \\ \hline \end{array}$$

729

203148

18870192

1889051409

0

Q

4 2

CUBE ROOT.

$$\begin{array}{r} 4 \quad 2 \quad 0 \quad 3 \quad 3 \cdot 9 \\ \sqrt[3]{74267948297643.000} \\ 64 \end{array}$$

$$48 \overline{)10267}$$

08

48

96

10088

$$529200 \overline{)179948297}$$

27

11340

1587600

158873427

$$52996627 \overline{)21074870643}$$

27

113481

158986881

15899822937

$$5300319267 \overline{)5175047706000}$$

729

10214019

47702873403

4770389481219

404658224781

$$\begin{array}{r}
 \begin{array}{r}
 \overset{2}{1} \overset{3}{6} \overset{0}{0} \overset{5}{5} \overset{7}{7} \\
 \sqrt{9746285197625.000000} \\
 8 \\
 \hline
 12 \overline{) 1746} \\
 \underline{01} \\
 06 \\
 \underline{12} \\
 1261 \\
 \hline
 1323 \overline{) 485285} \\
 \underline{27} \\
 567 \\
 \underline{3969} \\
 402597 \\
 \hline
 136107 \overline{) 82688197} \\
 \underline{216} \\
 23004 \\
 \underline{816642} \\
 81894456 \\
 \hline
 1368748800 \overline{) 793741625000} \\
 \underline{125} \\
 1602000 \\
 \underline{6843744000} \\
 684390420125 \\
 \hline
 136881288075 \overline{) 109351204875000} \\
 \underline{343} \\
 31399935 \\
 \underline{958169016525} \\
 95817215652193 \\
 \hline
 13533989222807
 \end{array}
 \end{array}$$

The Root *cubed*, with the *Remainder*, if any, added thereunto, proves the Truth of the Operation, as exemplified in this last Example.

$$\begin{array}{r}
 21360.57 \\
 21360.57 \\
 \hline
 14952399 \\
 10680285 \\
 128163420 \\
 6408171 \\
 2136057 \\
 4272114 \\
 \hline
 \text{Square } 456273950.7249 \\
 21360.57 \\
 \hline
 31939176550743 \\
 22813697536245 \\
 273764370434940 \\
 13688218521747 \\
 4562739507249 \\
 9125479014498 \\
 \text{Remainder } 13533989222807 \\
 \hline
 \text{Cube } 9746285197625.000000
 \end{array}$$

If the *Biquadratic Root* of any Number be required ; first extract the *Square Root* of that Number, and then extract the *Square Root* of that *Root*, it will give the *Biquadratic Root* of the given Number.

I shall leave the Reader to exemplify it.

Thus having briefly run through as much as is necessary in *Decimal Fractions*, I shall proceed to *Vulgar Fractions*, wherein the Examples in *Addition*, &c. shall be *decimally* performed, which will be of great Advantage to the Learner, to lead him into a right Understanding, and true Knowledge of this excellent Rule of *Decimal Arithmetic*.

VULGAR

VULGAR FRACTIONS

ARE of three Kinds, viz. *Proper*, *Improper*, and *Compound*.

A *Proper Fraction* is that which is less than an *Unit*, and therefore its *Numerator* is less than its *Denominator*. As $\frac{3}{4}$ $\frac{4}{5}$ $\frac{7}{8}$ are *Proper Fractions*.

An *Improper Fraction* is that which is greater than an *Unit*, and therefore its *Numerator* is greater than its *Denominator*. As $\frac{4}{3}$ $\frac{5}{4}$ $\frac{8}{7}$ are *Improper Fractions*.

A *Compound Fraction* is a *Fraction* of a *Fraction*, consisting of several *Numerators* and *Denominators*, connected together with the Word *of* between them. As $\frac{2}{3}$ of $\frac{3}{4}$ and $\frac{4}{5}$ of $\frac{5}{6}$ of $\frac{11}{12}$ are *Compound Fractions*.

Note : If the *Numerator* and *Denominator* of a *Fraction* be equal, then that *Fraction* is equal to *Unity*. As $\frac{5}{5}$ $\frac{3}{3}$ $\frac{6}{6}$ are each equal to an *Unit*.

REDUCTION of FRACTIONS

Discovereth the principal Knowledge of *Fractions*, and is contained under five several Heads, viz.

1. To reduce *Improper Fractions* into *Integers*.
2. To reduce *Compound Fractions* to *Proper Fractions*.
3. To reduce *Integers* into the Form of *Fractions*.
4. To reduce *Fractions* into their *least Terms*.
5. To reduce *Fractions* of different *Denominators*, into *Fractions* of the same *Denominator*.

To reduce Improper Fractions into Integers.

R U L E.

Divide the *Numerator* by the *Denominator*, the *Quotient* gives the *Integers*.

Example.

Reduce $\frac{15}{4}$ $\frac{108}{3}$ $\frac{379}{22}$ $\frac{4796}{119}$ into Integers.

$$\begin{array}{r} 15 \quad 3\frac{3}{4} \quad 108 \quad 36 \quad 379 \quad 17\frac{17}{22} \quad 4796 \quad 40\frac{116}{119} \\ 4 \overline{) 15} \quad 3 \overline{) 108} \quad 22 \overline{) 379} \quad 119 \overline{) 4796} \\ \underline{4} \quad \underline{9} \quad \underline{66} \quad \underline{258} \quad \underline{253} \quad \underline{4752} \\ 3 \quad 0 \quad 5 \quad 36 \end{array}$$

Thus $\frac{15}{4}$ reduced is $3\frac{3}{4}$. $\frac{108}{3}$ is 36. $\frac{379}{22}$ is $17\frac{17}{22}$ &c.

To reduce Compound Fractions to Proper Fractions.

R U L E.

Multiply all the *Numerators* together for a new *Numerator*, and all the *Denominators* together for a new *Denominator*.

Example.

Reduce $\frac{2}{3}$ of $\frac{3}{4}$ $\frac{7}{8}$ of $\frac{8}{9}$ $\frac{3}{5}$ of $\frac{2}{3}$ of $\frac{11}{12}$

and $\frac{1}{2}$ of $\frac{3}{4}$ of $\frac{5}{9}$ of $\frac{2}{11}$ into *Proper Fractions*.

$$\frac{\frac{2}{3} \text{ of } \frac{3}{4}}{12}$$

$$\frac{\frac{7}{8} \text{ of } \frac{8}{9}}{72}$$

$$\frac{\frac{3}{5} \text{ of } \frac{2}{3} \text{ of } \frac{11}{12}}{180}$$

$$\frac{\frac{1}{2} \text{ of } \frac{3}{4} \text{ of } \frac{5}{9} \text{ of } \frac{2}{11}}{792}$$

Thus $\frac{2}{3}$ of $\frac{3}{4}$ is $\frac{6}{12}$ and $\frac{7}{8}$ of $\frac{8}{9}$ is $\frac{56}{72}$ &c.

To reduce Integers into the Form of FRACTIONS.

UNDER this Head there are three Cases.

1. If the Integer have no assigned Denominator.
2. If the Integer have an assigned Denominator.
3. If the Integer have a Fraction annexed.

Case 1. If the Integer have no assigned Denominator.

R U L E.

An Unit subscribed must be the Denominator.

Example. Reduce 7 : 12 : 56 : 248 : into the Form of Fractions,

$$\text{Thus, } \frac{7}{1} \quad \frac{12}{1} \quad \frac{56}{1} \quad \frac{248}{1}.$$

Case 2. If the Integer have an assigned Denominator.

R U L E.

Multiply the Integer by the assigned Denominator, the Product is the Numerator to the assigned Denominator.

Example 1. Reduce 9 into the Form of a Fraction whose Denominator shall be 8.

Example 2.

Example 2. Reduce 19, whose *Denominator* shall be 11.

Example 3. Reduce 21, whose *Denominator* shall be 12.

9 multiplied by 8 is 72; that is, $\frac{72}{8}$.

19 multiplied by 11 is 209; that is, $\frac{209}{11}$.

21 multiplied by 12 is 252; that is, $\frac{252}{12}$.

Now if these *Improper Fractions* $\frac{72}{8}$ $\frac{209}{11}$ $\frac{252}{12}$ be reduced into *Integers*, they will produce 9. 19. 21, and prove the Truth of the Operations.

Case 3. If the *Integer* have a *Fraction* annexed.

R U L E.

Multiply the *Integer* by the *Denominator*, and add thereto the *Numerator*. The Sum is the *Numerator* to the *Denominator* of the annexed *Fraction*.

Example. Reduce $7\frac{7}{8}$ $21\frac{19}{27}$ $119\frac{35}{38}$ into *Improper Fractions*.

$$\begin{array}{r} 63 \\ \hline 7\frac{7}{8} \\ \hline 8 \end{array}$$

$$\begin{array}{r} 586 \\ \hline 21\frac{19}{27} \\ \hline 27 \end{array}$$

$$\begin{array}{r} 4557 \\ \hline 119\frac{35}{38} \\ \hline 38 \end{array}$$

Reduce $\frac{63}{8}$ $\frac{586}{27}$ $\frac{4557}{38}$ into *Integers*, it will prove the Truth of the Operations.

To reduce FRACTIONS into their least Terms.

THIS by some is called *Abbreviation of Fractions*, and is best performed by the *greatest Common Measure*.

The *greatest Common Measure* is a certain Number, that will divide both *Numerator* and *Denominator* without any *Remainder*, and by that Means reduce the *Fraction* into its *lowest Terms* at once, and is found by the following Rule.

R U L E.

Divide the *Denominator* by the *Numerator*, and then the *Numerator* by the *Remainder*, and so proceed, dividing the *last Divisor* by its *Remainder*, till either *Nothing* or an *Unit* remains. If *Nothing* remain, the *last Divisor* is the *greatest Common Measure*; but if an *Unit*, the *Fraction* is in its *lowest Terms* already.

Example. Reduce $\frac{1080}{1944}$ $\frac{424}{536}$ $\frac{129}{3271}$ into their lowest Terms.

$$\begin{array}{r} 1080 \\ \underline{1080} \quad | 1944 \quad | 1 \\ 864 \quad | 1080 \quad | 1 \end{array}$$

The greatest
Common
Measure

$$\begin{array}{r} 216 \overline{) 1080} 5 \\ \underline{1080} \\ 216 \overline{) 1944} 9 \\ \underline{1944} \\ 0 \end{array}$$

Therefore $\frac{1080}{1944}$ reduced into its lowest Terms is $\frac{5}{9}$

$$\begin{array}{r} 424 \\ \hline 424 \overline{) 536} | 1 \end{array}$$

$$112 \overline{) 424} | 3$$

$$88 \overline{) 112} | 1$$

$$24 \overline{) 88} | 3$$

$$16 \overline{) 24} | 1$$

The greatest Com. Mea. $8 \overline{) 16} | 2$

0

Therefore $\frac{424}{536}$ reduced into its lowest Terms is $\frac{11}{14}$.

$$\begin{array}{r} 129 \\ \hline 129 \overline{) 3271} | 25 \end{array}$$

$$46 \overline{) 129} | 2$$

$$37 \overline{) 46} | 1$$

$$9 \overline{) 37} | 4$$

1

Therefore $\frac{129}{3271}$ is in its lowest Terms already.

Note: If *Cyphers* are annexed to the *Numerator* and *Denominator*, by cutting off a like Number from both, the *Fraction* will be abbreviated thereby:

As $\frac{2}{3} \frac{0}{0} \frac{0}{0}$ will be $\frac{2}{3}$, and $\frac{4}{4} \frac{2}{0} \frac{0}{0}$ will be $\frac{4}{4} \frac{2}{0}$, &c.

Also *Fractions* may be abbreviated, by dividing them by a *Common Measure*, again and again, till they are reduced into their lowest Terms.

$$\begin{array}{r} \frac{2}{1080} \quad \frac{2}{540} \quad \frac{2}{270} \quad \frac{3}{135} \quad \frac{9}{45} \quad \frac{9}{5} \\ \hline 1944 \overline{) 972} | 486 \overline{) 243} | 81 \overline{) 9} \end{array}$$

To

To reduce FRACTIONS of different Denominators into FRACTIONS of the same Denominator.

R U L E.

Multiply all the *Denominators* together for a *Common Denominator*, and each *Numerator* into every *Denominator*, except its own, for *New Numerators*.

1. Reduce $\frac{7}{8}$ and $\frac{5}{6}$ into *Fractions* of the same *Denominator*.

$$\begin{array}{r} 42 \quad 40 \\ \hline \frac{7}{8} \quad \frac{5}{6} \\ \hline 48 \end{array}$$

First. The two *Denominators* 8 and 6, multiplied together, gives 48 for a *Common Denominator*, and then the *Numerator* 7 by the *Denominator* 6 for a *New Numerator* to the first *Fraction*, and the *Numerator* 5 by the *Denominator* 8 for a *New Numerator* to the second *Fraction*; so that the *Fractions* $\frac{7}{8}$ and $\frac{5}{6}$, reduced as was required, becomes $\frac{42}{48}$ and $\frac{40}{48}$, which are equal to $\frac{7}{8}$ and $\frac{5}{6}$.

From hence, if it be necessary to know which of *two* or more *Fractions* is the greatest, let them be reduced into *Fractions* of the same *Denominator*, and that *Fraction* which hath the greatest *Numerator* is the greatest *Fraction*.

2. Reduce $\frac{2}{3}$ $\frac{3}{4}$ $\frac{4}{5}$ $\frac{5}{6}$ $\frac{7}{8}$ into *Fractions* of the same *Denominator*.

1920	2160	2304	2400	2520
$\frac{2}{3}$	$\frac{3}{4}$	$\frac{4}{5}$	$\frac{5}{6}$	$\frac{7}{8}$
2880				
R 2				

			3
		2	4
	3	4	<u>12</u>
4	<u>3</u>	8	5
3	9	5	<u>60</u>
<u>12</u>	5	40	6
4	<u>45</u>	6	<u>360</u>
<u>48</u>	6	<u>240</u>	8
6	<u>270</u>	8	<u>2880</u>
288	8		<i>Common Denominator.</i>
8		1920	<i>Numerator to the first Fraction.</i>
	2160		<i>Numerator to the second Fraction.</i>
2304			<i>Numerator to the third Fraction.</i>

	5
7	3
<u>3</u>	<u>15</u>
21	4
<u>4</u>	<u>60</u>
84	5
<u>5</u>	<u>300</u>
420	8
<u>6</u>	<u>2400</u>
	<i>Numerator to the fourth Fraction.</i>
2520	<i>Numerator to the fifth Fraction.</i>

If $\frac{1920}{2880}$ $\frac{2160}{2880}$ $\frac{2304}{2880}$ $\frac{2400}{2880}$ $\frac{2520}{2880}$ be reduced into their *least Terms*, they will produce $\frac{2}{3}$ $\frac{3}{4}$ $\frac{4}{5}$ $\frac{5}{6}$ $\frac{7}{8}$, and prove the Truth of the Operation.

3. Reduce $\frac{2}{3}$ of $\frac{3}{4}$ $\frac{5}{6}$ of $\frac{7}{8}$ and $\frac{2}{9}$ of $\frac{11}{12}$ into *Fractions* of the same *Denominator*.

$$\begin{array}{r} \frac{6}{\frac{2}{3} \text{ of } \frac{3}{4}} \\ \hline 12 \end{array} \quad \begin{array}{r} \frac{35}{\frac{5}{6} \text{ of } \frac{7}{8}} \\ \hline 48 \end{array} \quad \begin{array}{r} \frac{22}{\frac{2}{9} \text{ of } \frac{11}{12}} \\ \hline 108 \end{array}$$

$$\begin{array}{r} \frac{2592}{\frac{1}{2}} \quad \frac{3780}{\frac{35}{48}} \quad \frac{1056}{\frac{11}{54}} \\ \hline 5184 \end{array}$$

Having first reduced the *Compound Fractions* to *Proper Fractions*, they produce $\frac{6}{12}$ $\frac{35}{48}$ and $\frac{22}{108}$.

Then $\frac{6}{12}$ and $\frac{22}{108}$ being reduced into their *least*

Terms, produce $\frac{1}{2}$ and $\frac{11}{54}$. Then $\frac{1}{2}$ $\frac{35}{48}$ and $\frac{11}{54}$

reduced into *Fractions* of the same *Denominator*,

produce $\frac{2592}{5184}$ $\frac{3780}{5184}$ $\frac{1056}{5184}$ as above.

4. Reduce $3\frac{1}{4}$ $7\frac{2}{8}$ $9\frac{5}{6}$ into *Fractions* of the same *Denominator*.

$$\begin{array}{r} \frac{15}{3\frac{1}{4}} \\ \hline 4 \end{array} \quad \begin{array}{r} \frac{63}{7\frac{2}{8}} \\ \hline 8 \end{array} \quad \begin{array}{r} \frac{59}{9\frac{5}{6}} \\ \hline 6 \end{array}$$

$$\begin{array}{r} \frac{720}{\frac{15}{4}} \quad \frac{1512}{\frac{63}{8}} \quad \frac{1888}{\frac{59}{6}} \\ \hline 192 \end{array}$$

In this *Example*, the *Integers* with the *Fractions* annexed are first reduced into *Improper Fractions*,
and

and then the *Improper Fractions* reduced into *Fractions* of the same *Denominator*, as the *Example* plainly sheweth.

5. Reduce $7\frac{7}{8}$ $\frac{2}{3}$ of $\frac{5}{9}$ $\frac{11}{12}$ and 9 into *Fractions* of the same *Denominator*.

$\frac{63}{7\frac{7}{8}}$	$\frac{10}{\frac{2}{3} \text{ of } \frac{5}{9}}$		
$\frac{8}{8}$	$\frac{27}{27}$		
20412	960	2376	23328
$\frac{63}{8}$	$\frac{10}{27}$	$\frac{11}{12}$	$\frac{9}{1}$
2592			

What hath hitherto been done in Reduction of *Fractions*, is chiefly to fit and prepare *Fractions* for *Addition*, *Subtraction*, *Multiplication*, and *Division*, as Occasion requires, and will, as we go on, plainly appear.

ADDITION OF FRACTIONS.

R U L E.

IF the *Denominators* are equal, add the *Numerators* together; but if the *Denominators* be unequal, reduce them into *Fractions* of the same *Denominator*, and add the *New Numerators* together.

1. Add

Vulgar Fractions.

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1. Add $\frac{2}{19}$ $\frac{3}{19}$ $\frac{4}{19}$ and $\frac{7}{19}$ together.

$$\begin{array}{r} 16 \\ \hline \frac{2}{19} \quad \frac{3}{19} \quad \frac{4}{19} \quad \frac{7}{19} \\ \hline 19 \end{array} \quad \text{The Total is } \frac{16}{19}.$$

2. Add $\frac{1}{5}$ $\frac{2}{5}$ $\frac{3}{5}$ and $\frac{4}{5}$ together.

$$\begin{array}{r} 10 \\ \hline \frac{1}{5} \quad \frac{2}{5} \quad \frac{3}{5} \quad \frac{4}{5} \\ \hline 5 \overline{) 10} 2 \\ 0 \end{array}$$

The Total is $\frac{10}{5}$, which reduced into *Integers*, is 2.

3. Add $\frac{3}{13}$ $\frac{4}{13}$ $\frac{11}{13}$ and $\frac{12}{13}$ together.

$$\begin{array}{r} 30 \\ \hline \frac{3}{13} \quad \frac{4}{13} \quad \frac{11}{13} \quad \frac{12}{13} \\ \hline 13 \overline{) 30} 2 \frac{4}{13} \\ 26 \\ \hline 4 \end{array}$$

The Total is $\frac{30}{13}$, which reduced into *Integers*, is $2 \frac{4}{13}$.

4. Add $\frac{7}{8}$ $\frac{8}{9}$ $\frac{3}{4}$ $\frac{5}{6}$ together *Fractionally* and *Decimally*.

				5784	
1512	1536	1296	1440		1512
					1536
					1296
$\frac{7}{8}$	$\frac{8}{9}$	$\frac{3}{4}$	$\frac{5}{6}$		1440
					<hr/>
					5784
				1728	5784
					3 $\frac{21}{72}$
				5184	
				<hr/>	
				600	

In this *Example* the given *Fractions* having *unequal* Denominators, are first reduced into *Fractions* of the *same* Denomination, and the *new* Numerators added together makes the *total* Sum of those *Fractions* to be $\frac{17\frac{2}{3}}{28}$ an *improper* Fraction, which being reduced into *Integers*, gives $3\frac{1}{2}$.

Note : $\frac{2}{72}$ comes of $\frac{400}{728}$ reduced into its least Terms.

The preceeding *Example* wrought *Decimally*.

$$\begin{array}{r} 7 \quad .875 \\ 8 \overline{) 7.000} \end{array}$$

$$\begin{array}{r} 8 \quad .888888 \\ 9 \overline{) 8.000000} \\ \text{Rem. } 8 \end{array}$$

$$\begin{array}{r} 5 \quad .833333 \\ 6 \overline{) 5.000000} \\ \text{Rem. } 2 \end{array}$$

$$\begin{array}{r} 3 \quad .75 \\ 4 \overline{) 3.00} \end{array}$$

$$\begin{array}{r} .875 \\ .888888 \\ .75 \\ .833333 \end{array}$$

$$\begin{array}{r} 25 \quad 347222 \\ 72 \overline{) 25.000000} \\ \text{Rem. } 16 \end{array}$$

Total 3.347221 which agrees with the Total *fractionally*, as appears by the Operation of $\frac{2}{72}$ reduced into a *Decimal*.

5. Add $\frac{2}{3}$ of $\frac{3}{4}$ $\frac{4}{5}$ of $\frac{5}{9}$ $\frac{7}{8}$ of $\frac{11}{12}$ together *Fractionally* and *Decimally*.

$$\begin{array}{r} 6 \\ \frac{2}{3} \text{ of } \frac{3}{4} = \frac{1}{2} \\ 12 \end{array}$$

$$\begin{array}{r} 20 \\ \frac{4}{5} \text{ of } \frac{5}{9} = \frac{4}{9} \\ 45 \end{array}$$

$$\begin{array}{r} 77 \\ \frac{7}{8} \text{ of } \frac{11}{12} \\ 96 \end{array}$$

$$\begin{array}{r} 864 \\ 768 \\ 1386 \\ 3018 \\ \hline 1728 \overline{) 3018} \\ 1728 \\ \hline 1290 \\ 1728 \\ \hline 215 \\ 288 \end{array}$$

Decimally.

Decimally,

$$\begin{array}{r}
 \frac{1}{2} \cdot 5 \\
 \hline
 1.0
 \end{array}
 \quad
 \begin{array}{r}
 \frac{4}{9} \cdot 444444 \\
 \hline
 4.000000 \\
 \text{Rem. } 4
 \end{array}
 \quad
 \begin{array}{r}
 \frac{77}{96} \cdot 802083 \\
 \hline
 77.000000 \\
 \text{Rem. } 32
 \end{array}$$

$$\begin{array}{r}
 .5 \\
 .444444 \\
 .802083 \\
 \hline
 \text{Total } 1.746527
 \end{array}
 \quad
 \begin{array}{r}
 215 \cdot 746527 \\
 \hline
 288 \mid 215.000000 \\
 \text{Rem. } 224
 \end{array}$$

6. Add $7\frac{7}{8}$ $3\frac{3}{4}$ $5\frac{1}{5}$ together *Fractionally* and *Decimally*.

$$\begin{array}{r}
 \frac{63}{7\frac{7}{8}} \quad \frac{15}{3\frac{3}{4}} \quad \frac{26}{5\frac{1}{5}} \\
 \hline
 8 \quad 4 \quad 5
 \end{array}
 \quad
 \begin{array}{r}
 1260 \\
 600 \\
 832 \\
 \hline
 2692
 \end{array}$$

$$\begin{array}{r}
 1260 \quad 600 \quad 832 \\
 \hline
 \frac{63}{8} \quad \frac{15}{4} \quad \frac{26}{5} \\
 \hline
 160 \mid 2692 \quad (16 \frac{132}{160} = \frac{33}{40}) \\
 \text{Rem. } 132
 \end{array}$$

Decimally.

$$\begin{array}{r}
 \frac{7}{8} \cdot 875 \\
 \hline
 7.875
 \end{array}
 \quad
 \begin{array}{r}
 \frac{3}{4} \cdot 75 \\
 \hline
 3.75
 \end{array}
 \quad
 \begin{array}{r}
 \frac{1}{5} \cdot 2 \\
 \hline
 5 \mid 1.0 \\
 5.2
 \end{array}$$

$$\begin{array}{r}
 7.875 \\
 3.75 \\
 5.2 \\
 \hline
 \text{Total } 16.825
 \end{array}
 \quad
 \begin{array}{r}
 33 \cdot 825 \\
 \hline
 40 \mid 33.000
 \end{array}$$

Note : This Example *Fractionally*, might have been wrought without reducing the *Integers*, with their *Fractions* annexed, into *Improper Fractions* ; by adding only the *Fractions* together by themselves, and also the *Integers*, as you may plainly see by the following Operation.

$$\begin{array}{r}
 292 \\
 \hline
 140 \quad 120 \quad 32 \\
 \hline
 7 \quad 3 \quad 1 \\
 8 \quad 4 \quad 5 \\
 \hline
 140 \\
 120 \\
 32 \\
 \hline
 292
 \end{array}
 \quad
 \begin{array}{r}
 160 \quad 292 \quad (1 \quad \frac{132}{160} = \frac{33}{40} \\
 132 \\
 \hline
 160 \\
 \hline
 16 \quad \frac{33}{40}
 \end{array}$$

7. To $7 \frac{7}{8}$ more $\frac{3}{80}$, add $9 \frac{3}{4}$ more $\frac{2}{3}$ of $\frac{7}{8}$
Fractionally and Decimally.

$$\begin{array}{r}
 63 \\
 7 \frac{7}{8} \\
 \hline
 8
 \end{array}
 \quad
 \begin{array}{r}
 39 \\
 9 \frac{3}{4} \\
 \hline
 4
 \end{array}
 \quad
 \begin{array}{r}
 14 \\
 \frac{2}{3} \text{ of } \frac{7}{8} = \frac{7}{12} \\
 \hline
 24
 \end{array}$$

$$\begin{array}{r}
 560512 \\
 \hline
 241920 \quad 1152 \quad 299520 \quad 17920 \\
 \hline
 63 \quad 3 \quad 39 \quad 7 \\
 8 \quad 80 \quad 4 \quad 12 \\
 \hline
 30720 \quad 560512 \quad (18 \quad \frac{59}{240} \\
 7552 \\
 241920 \\
 1152 \\
 299520 \\
 17920 \\
 \hline
 560512 \quad \frac{7552}{30720} = \frac{59}{40}
 \end{array}$$

Decimally.

$$\begin{array}{r}
 7 \quad .875 \\
 8 \overline{) 7.000} \\
 \hline
 7 \quad .583333 \\
 12 \overline{) 7.000000} \\
 \hline
 \text{Rem. } 4 \\
 59 \quad .245833 \\
 240 \overline{) 59.000000} \\
 \hline
 80
 \end{array}
 \quad
 \begin{array}{r}
 3 \quad .0375 \\
 80 \overline{) 3.0000} \\
 \hline
 7.875 \\
 .0375 \\
 9.75 \\
 .583333 \\
 \hline
 18.245833
 \end{array}$$

This Sum *Fractionally*, might have been wrought by reducing the *Integers* into the Form of *Fractions*, subscribing an *Unit* for their Denominators, and then the *Fractions* to be added together, would have been $\frac{7}{1} \frac{7}{8} \frac{3}{80} \frac{9}{1} \frac{3}{4} \frac{7}{12}$ as followeth.

$$\begin{array}{r}
 560512 \\
 \hline
 215040 \quad 26880 \quad 1152 \quad 276480 \quad 23040 \quad 17920 \\
 \hline
 \frac{7}{1} \quad \frac{7}{8} \quad \frac{3}{80} \quad \frac{9}{1} \quad \frac{3}{4} \quad \frac{7}{12} \\
 \hline
 \begin{array}{r}
 215040 \\
 26880 \\
 1152 \\
 276480 \\
 23040 \\
 17920 \\
 \hline
 560512
 \end{array}
 \end{array}$$

$$30720 \mid 560512 \mid 18 \frac{59}{240}$$

8. To $3\frac{1}{4}l.$ and $\frac{7}{8}$ of a *Shilling*, add $9\frac{2}{9}l.$ and $\frac{1}{3}$ of a *Penny*, *Fractionally* and *Decimally*.

$$\begin{array}{r}
 \frac{7}{8} \text{ of } \frac{1}{20} \\
 \hline
 160
 \end{array}
 \quad
 \begin{array}{r}
 \frac{1}{3} \text{ of } \frac{1}{240} \\
 \hline
 720
 \end{array}
 \quad
 \begin{array}{r}
 \frac{15}{3} \frac{83}{4} \\
 \hline
 4
 \end{array}
 \quad
 \begin{array}{r}
 \frac{83}{9} \frac{2}{9} \\
 \hline
 9
 \end{array}$$

$$\begin{array}{r}
 53985600 \\
 \hline
 15552000 \quad 181440 \quad 38246400 \quad 5760 \\
 \hline
 \frac{15}{4} \quad \frac{7}{160} \quad \frac{83}{9} \quad \frac{1}{720} \\
 \hline
 4147200 \mid 53985600 \mid 13l.
 \end{array}$$

$$\begin{array}{r}
 15552000 \\
 181440 \\
 38246400 \\
 5760 \\
 \hline
 53985600
 \end{array}$$

$$\begin{array}{r}
 4147200 \\
 \hline
 12513600 \\
 12441600 \\
 \hline
 72000
 \end{array}$$

$$\begin{array}{r}
 20 \\
 \hline
 1440000 \\
 \hline
 12d.
 \end{array}$$

$$\frac{691200}{4147200} = \frac{1}{6}$$

$$\begin{array}{r}
 4147200 \mid 17280000 \mid 4\frac{1}{2} \\
 \hline
 16588800 \\
 \hline
 691200
 \end{array}$$

Decimally.

$$\begin{array}{r} 3 \ .75 \\ 4 \overline{) 3.00} \end{array}$$

$$\begin{array}{r} 7 \ .04375 \\ 160 \overline{) 7.00000} \end{array}$$

$$\begin{array}{r} 2 \ .222222 \\ 9 \overline{) 2.000000} \end{array}$$

$$\begin{array}{r} 1 \ .001388 \\ 720 \overline{) 1.000000} \\ \text{Rem. } 640 \end{array}$$

£. Rem. 2

3.75

.04375

9.222222

.001388

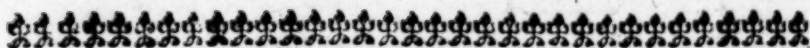
£. 13.017360

20

Sh. .0347200

12

Pence 4.166400



SUBTRACTION of FRACTIONS.

R U L E.

IF the *Denominators* be equal, subtract the Lesser *Numerator* from the Greater, so shall you have the *Remainder* or Difference. But if the *Denominators* be unequal, reduce them into *Fractions* of the same *Denominator*, and subtract the Lesser new *Numerator* from the Greater.

1. From $\frac{6}{9}$ take $\frac{2}{9}$ the Remainder is $\frac{4}{9}$

2. From $\frac{11}{12}$ take $\frac{7}{12}$ the Remainder is $\frac{4}{12} = \frac{1}{3}$

3. From $\frac{7}{8}$ take $\frac{3}{4}$ *Fractionally* and *Decimally*.

$$\begin{array}{r} 4 \\ 28 \quad 24 \\ \hline \end{array}$$

From $\frac{7}{8}$ take $\frac{3}{4}$ The Remainder is $\frac{4}{32} = \frac{1}{8}$

32

Decimally.

Vulgar Fractions.

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Decimally.

$$\begin{array}{r} 7 \text{ .875} \\ 8 \overline{) 7.000} \end{array}$$

$$\begin{array}{r} 3 \text{ .75} \\ 4 \overline{) 3.00} \end{array}$$

From .875

Take .75

Rem. .125

$$1. \text{ .125}$$

$$8 \overline{) 1.000}$$

4. From $\frac{7}{8}$ of $\frac{8}{9}$ take $\frac{2}{3}$ of $\frac{1}{8}$ Fractionally and Decimally.

$$\begin{array}{r} 56 \\ \hline \frac{7}{8} \text{ of } \frac{8}{9} = \frac{7}{9} \\ \hline 72 \end{array}$$

$$\begin{array}{r} 6 \\ \hline \frac{2}{3} \text{ of } \frac{3}{8} = \frac{1}{4} \\ \hline 24 \end{array}$$

$$\begin{array}{r} 19 \\ \hline 28 \quad 9 \\ \hline \text{From } \frac{7}{9} \text{ take } \frac{1}{4} \\ \hline 36 \end{array}$$

Decimally.

$$\begin{array}{r} 7 \text{ .7777777} \\ 9 \overline{) 7.0000000} \\ \text{Rem. 7} \end{array}$$

$$\begin{array}{r} 1 \text{ .25} \\ 4 \overline{) 1.00} \end{array}$$

From .777777

Take .25

Rem. .527777

$$\begin{array}{r} 19 \text{ .527777} \\ 36 \overline{) 19.000000} \end{array}$$

5. From $19\frac{11}{12}$ take $7\frac{3}{80}$ Fractionally and Decimally.

$$\begin{array}{r} 12364 \\ \hline 19120 \quad 6756 \\ \hline 239 \quad 563 \end{array}$$

From 19120

Take 6756

Rem. 12364

From $19\frac{11}{12}$ take $7\frac{3}{80}$

$$\begin{array}{r} 960 \overline{) 12364} \quad 12 \frac{211}{240} \\ \hline 844 = \frac{211}{240} \text{ Rem. 844} \end{array}$$

*Vulgar Fractions.**Decimally.*

$$\begin{array}{r} 11 \quad .916666 \\ 12 \overline{) 11.000000} \\ \text{Rem. } 8 \end{array}$$

$$\begin{array}{r} 3 \quad .0375 \\ 80 \overline{) 3.0000} \end{array}$$

$$\begin{array}{r} \text{From } 19.916666 \\ \text{Take } 7.0375 \\ \hline \text{Rem. } 12.879166 \end{array}$$

$$\begin{array}{r} 211 \quad .879166 \\ 240 \overline{) 211.000000} \\ \text{Rem. } 160 \end{array}$$

6. From $5\frac{3}{4}$ more $\frac{3}{80}$ take $2\frac{5}{9}$ more $\frac{2}{3}$ of $\frac{3}{4}$
Fractionally and Decimally.

$$\begin{array}{r} 252 \\ 240 \quad 12 \\ \hline \frac{3}{4} \text{ add } \frac{3}{80} = \frac{63}{80} \\ \hline 320 \end{array}$$

$$\begin{array}{r} 6 \\ \hline \frac{2}{3} \text{ of } \frac{3}{4} = \frac{1}{2} \\ \hline 12 \end{array}$$

$$\begin{array}{r} 19 \\ \hline 10 \quad 9 \\ \hline \frac{5}{9} \text{ add } \frac{1}{2} \end{array}$$

$$\begin{array}{r} 3934 \\ \hline 8334 \quad 4400 \\ 463 \quad 55 \\ \hline 18 \overline{) 19} \quad 1 \frac{1}{18} \end{array}$$

$$\text{From } 5\frac{63}{80} \text{ take } 3\frac{1}{18}$$

$$\begin{array}{r} \text{From } 8334 \\ \text{Take } 4400 \\ \hline \end{array}$$

$$\text{Rem. } 3934$$

$$\begin{array}{r} 1440 \overline{) 3934} \quad 2 \quad \frac{1054}{1440} = \frac{527}{720} \end{array}$$

Decimally.

$$\begin{array}{r} 3 \quad .75 \\ 4 \overline{) 3.00} \end{array}$$

$$\begin{array}{r} 3 \quad .0375 \\ 80 \overline{) 3.0000} \end{array}$$

$$\begin{array}{r} .75 \\ .0375 \\ \hline .7875 \end{array}$$

$$\begin{array}{r} 5 \quad .555555 \\ 9 \overline{) 5.000000} \\ \text{Rem. } 5 \end{array}$$

$$\begin{array}{r} 1 \quad .5 \\ 2 \overline{) 1.0} \\ \hline 1.055555 \end{array}$$

$$\begin{array}{r} \text{From } 5.7875 \\ \text{Take } 3.055555 \\ \hline \text{Rem. } 2.731945 \end{array}$$

$$\begin{array}{r} 527 \quad .731944 \\ 720 \overline{) 527.000000} \\ \text{Rem. } 320 \end{array}$$

7. From

Vulgar Fractions.

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7. From 21 l. $\frac{1}{2}$ and $\frac{2}{3}$ of $\frac{1}{4}$ of a Penny take $9\frac{1}{2}$ l. and $\frac{1}{2}$ of a Farthing, Fractionally and Decimally.

$$\frac{\frac{2}{3} \text{ of } \frac{3}{4} \text{ of } \frac{1}{240}}{2880} = \frac{1}{480}$$

$$\frac{\frac{1}{2} \text{ of } \frac{1}{960}}{1920}$$

$$\begin{array}{r} 2406 \\ 2400 \quad 6 \\ \hline \frac{5}{6} \text{ add } \frac{1}{480} = \frac{401}{480} \\ \hline 2880 \end{array}$$

$$\begin{array}{r} 13448 \\ 13440 \quad 8 \\ \hline \frac{7}{8} \text{ add } \frac{1}{1920} = \frac{1681}{1920} \\ \hline 15360 \end{array}$$

11022240

$$\begin{array}{r} 20123520 \\ 10481 \\ \hline \end{array} \quad \begin{array}{r} 9101280 \\ 18961 \\ \hline \end{array}$$

From 21 $\frac{401}{480}$ take 9 $\frac{1681}{1920}$

$$92160|0|1102224|0|11 \text{ £.}$$

From 20123520
Take 9101280
Rem. 11022240

$$\begin{array}{r} 88464 \\ 20 \\ \hline \end{array}$$

9216|0|176928|0|19 Sh.

$$\begin{array}{r} 1824 \\ 12 \\ \hline \end{array}$$

9216|21888|2 d.

$$\begin{array}{r} 3456 \\ 4 \\ \hline \end{array}$$

$$9216|13824|1 \quad \frac{4608}{9216} = \frac{1}{2}$$

Decimally.

$$\begin{array}{r} 5 \text{ .}833333 \\ 6|5.000000 \\ \hline \text{Rem. } 2 \end{array}$$

$$\begin{array}{r} 1 \text{ .}002083 \\ 480|1.000000 \\ \hline \text{Rem. } 160 \end{array}$$

$$\begin{array}{r} 7 \text{ .}875 \\ 8|7.000 \\ \hline \end{array}$$

$$\begin{array}{r} 1 \text{ .}000520833 \\ 1920|1.000000000 \\ \hline \text{Rem. } 640 \end{array}$$

$$\begin{array}{r} .833333333 \\ .002803333 \\ \hline .835416666 \end{array}$$

$$\begin{array}{r}
 .875 \\
 .000520833 \\
 \hline
 .875530833
 \end{array}
 \begin{array}{l}
 \text{From } £. 21.835416666 \\
 \text{Take } 9.875520833 \\
 \hline
 £. 11.959895833 \\
 \quad 20 \\
 \hline
 Sb. 19.197916660 \\
 \quad 12 \\
 \hline
 d. 2.374999920 \\
 \quad 4 \\
 \hline
 1.499999680
 \end{array}$$



Multiplication of FRACTIONS.

R U L E.

Multiply the *Numerators* together for the *Numerator* of the *Product*, and the *Denominators* together for the *Denominator* of the *Product*.

1. Multiply $\frac{7}{8}$ by $\frac{3}{4}$ *Fractionally* and *Decimally*.

$$\begin{array}{r}
 21 \\
 \hline
 \frac{7}{8} \quad \frac{3}{4} \\
 \hline
 32
 \end{array}
 \begin{array}{l}
 \text{the Product is } \frac{21}{32}
 \end{array}$$

Decimally.

$$\begin{array}{r}
 7 \quad .875 \\
 8 \overline{) 7.000}
 \end{array}$$

$$\begin{array}{r}
 3 \quad .75 \\
 4 \overline{) 3.00}
 \end{array}$$

$$\begin{array}{r}
 .875 \\
 .75 \\
 \hline
 4375 \\
 6125 \\
 \hline
 .65625
 \end{array}$$

$$\begin{array}{r}
 21 \quad .65625 \\
 32 \overline{) 21.00000}
 \end{array}$$

2. Mul-

2. Multiply $\frac{2}{3}$ of $\frac{3}{4}$ by $\frac{1}{2}$ of $\frac{5}{8}$, Fractionally and Decimally.

$$\frac{\frac{2}{3} \text{ of } \frac{3}{4}}{12} = \frac{1}{2}$$

$$\frac{\frac{7}{8} \text{ of } \frac{5}{9}}{72}$$

Mult. $\frac{1}{2}$ by $\frac{35}{72}$ the Product is $\frac{35}{144}$

Decimally.

$$\begin{array}{r} 1 \ .5 \\ 2 \overline{) 1.0} \end{array}$$

$$\begin{array}{r} 35 \ .486111 \\ 72 \overline{) 35.000000} \\ \underline{8} \end{array}$$

Mult. .486111
by .5
 .2430555

$$\begin{array}{r} 35 \ .2430555 \\ 144 \overline{) 35.000000} \\ \text{Rem. } 80 \end{array}$$

3. Multiply $7\frac{7}{8}$ by $3\frac{3}{4}$ Fractionally and Decimally.

$$\begin{array}{r} 945 \\ 63 \quad 15 \\ \hline \text{Mult. } 7\frac{7}{8} \text{ by } 3\frac{3}{4} \\ \hline 32 \overline{) 945 } 29 \frac{17}{32} \text{ Product.} \\ 17 \end{array}$$

Decimally.

$$\begin{array}{r} 7 \ .875 \\ 8 \overline{) 7.000} \end{array}$$

$$\begin{array}{r} 3 \ .75 \\ 4 \overline{) 3.00} \end{array}$$

Mult. 7.875
by 3.75

Product 29.53125

$$\begin{array}{r} 17 \ .53125 \\ 32 \overline{) 17.00000} \end{array}$$

T

4. Mul-

4. Multiply $7\frac{7}{8}$ more $\frac{2}{3}$ of $\frac{5}{9}$ by $9\frac{11}{12}$ more $\frac{3}{80}$ Fractionally and Decimally.

$$\begin{array}{r} 10 \\ \hline \frac{2}{3} \text{ of } \frac{5}{9} \\ \hline 27 \end{array}$$

$$\begin{array}{r} 269 \\ \hline 189 \quad 80 \\ \hline \frac{7}{8} \text{ add } \frac{10}{27} \\ \hline 216 \overline{)269} \text{ I } \frac{53}{216} \\ \hline 53 \end{array}$$

$$\begin{array}{r} 916 \\ \hline 880 \quad 36 \\ \hline \frac{11}{12} \text{ add } \frac{3}{80} = \frac{229}{240} \\ \hline 960 \end{array}$$

$$\begin{array}{r} 4254809 \\ \hline 1781 \times 2839 \\ \hline \text{Mult. } 8 \frac{53}{216} \text{ by } 9 \frac{229}{240} \\ \hline 51840 \overline{)4254809} 82 \frac{3929}{51840} \\ \hline 3929 \end{array}$$

Decimally.

$$\begin{array}{r} 7 \quad .875 \\ 8 \overline{)7.000} \end{array}$$

$$\begin{array}{r} 10 \quad .370370 \\ 27 \overline{)10.000000} \\ \text{Rem. } 100 \end{array}$$

$$\begin{array}{r} .875 \\ .37037 \\ \hline 1.24537 \end{array}$$

$$\begin{array}{r} 11 \quad .916666 \\ 12 \overline{)11.000000} \\ \text{Rem. } 8 \end{array}$$

$$\begin{array}{r} 3 \quad .0375 \\ 80 \overline{)3.0000} \end{array}$$

$$\begin{array}{r} .916666 \\ .0375 \\ \hline .954166 \end{array}$$

$$\begin{array}{r} \text{Mult. } 9.954166 \\ \text{by } - - - 8.24537 \\ \hline 82.07578658642 \end{array}$$

$$\begin{array}{r} 3929 \quad .07579 \\ 51840 \overline{)3929.00000} \end{array}$$

5. Multiply $7\frac{1}{2}$ d. by $5\frac{1}{4}$ d. Fractionally and Decimally.

Note: This Example admits of three different Solutions, because either a Penny, a Shilling, or a Pound, may be the Integer, as will appear in the following Operations.

First,

First, Let a Penny be Integer.

$$\begin{array}{r}
 315 \\
 \hline
 15 \quad 21 \\
 \hline
 \text{Mult. } 7\frac{1}{2} \text{ by } 5\frac{1}{4} \\
 \hline
 8 \overline{) 315} 39 \text{ Pence.} \\
 \quad 3 \\
 \quad 4 \text{ Farthings.} \\
 8 \overline{) 12} 1\frac{4}{8} = \frac{1}{2} \\
 \quad 4
 \end{array}$$

	<i>Decimally.</i>	<i>Pence.</i>
$\frac{1}{2} \overline{) .5}$	$\frac{1}{4} \overline{) .25}$	Mult. 5.25
		by - - 7.5
		d. 39.375
		$\frac{4}{f. 1.500}$

Secondly, Let a Shilling be Integer.

$7\frac{1}{2}d.$ is 30 Farthings; that is, $\frac{30}{48}$ or $\frac{5}{8}$ of a Shilling.

$5\frac{1}{4}d.$ is 21 Farthings; that is, $\frac{21}{48}$ or $\frac{7}{16}$ of a Shilling.

$$\begin{array}{r}
 35 \\
 \hline
 \text{Mult. } \frac{5}{8} \text{ by } \frac{7}{16} \\
 \hline
 128 \overline{) 35} \\
 \quad 12 \\
 128 \overline{) 420} 3 \text{ Pence.} \\
 \quad 36 \\
 \quad 4 \text{ Farthings.} \\
 128 \overline{) 144} 1\frac{16}{128} = \frac{1}{8} \\
 \quad 16
 \end{array}$$

Vulgar Fractions.

$$\begin{array}{r} 5 \ .625 \\ 8 \overline{) 5.000} \end{array}$$

$$\begin{array}{r} 1 \ .125 \\ 8 \overline{) 1.000} \end{array}$$

$$\begin{array}{r} \text{Decimally.} \\ 7 \ .4375 \\ 16 \overline{) 7.0000} \end{array}$$

$$\begin{array}{r} \text{Sh.} \\ \text{Mult. } .4375 \\ \text{by } -.625 \\ \hline .2734375 \\ \hline 12 \end{array}$$

$$\text{Pence. } 3.2812500$$

$$\begin{array}{r} 4 \\ \hline \text{Farth. } 1.1250000 \end{array}$$

Thirdly, Let a Pound be the Integer.

30 Farthings is $\frac{30}{960}$ or $\frac{1}{32}$ of a Pound.

21 Farthings is $\frac{21}{960}$ or $\frac{7}{320}$ of a Pound.

$$\begin{array}{r} 7 \\ \hline \text{Mult. } \frac{1}{32} \text{ by } \frac{7}{320} \\ \hline 10240 \overline{) 7} \\ \hline 20 \\ \hline 140 \\ \hline 12 \\ \hline 1680 \\ \hline 4 \\ \hline 6720 \end{array}$$

The Product is $\frac{7}{10240}$ of a Pound, which is $\frac{6720}{10240}$ or $\frac{21}{32}$ of a Farthing.

Decimally.

Decimally.

$$\begin{array}{r}
 1 \quad .03125 \\
 \hline
 32 \overline{) 1.00000} \\
 \\
 21 \quad .65625 \\
 \hline
 23 \overline{) 21.00000}
 \end{array}
 \qquad
 \begin{array}{r}
 7 \quad .021875 \\
 \hline
 320 \overline{) 7.000000} \\
 \text{Mult. } .021875 \\
 \text{by } -.03125 \\
 \hline
 .00068359375 \\
 \hline
 20 \\
 \hline
 .01367187500 \\
 \hline
 12 \\
 \hline
 .16406250000 \\
 \hline
 4 \\
 \hline
 .65625000000
 \end{array}$$

From hence you see the three different Solutions of this *Example*.

	d.	f.
If a Penny be Integer, the Product is	39	$1\frac{1}{8}$
If a Shilling be Integer - - - - -	3	$1\frac{1}{8}$
If a Pound be Integer - - - - -	0	$\frac{21}{32}$

6. Multiply 6s. $7\frac{1}{2}$ d. by 9s. $4\frac{1}{2}$ d. Fractionally and Decimally.

This *Example* admits of two different Solutions ; either a Shilling or a Pound being Integer.

First, Let a Shilling be Integer.

$7\frac{3}{4}$ d. is 31 Farthings ; that is, $\frac{31}{48}$ of a Shilling.
 $4\frac{1}{2}$ d. is 18 Farthings ; that is, $\frac{18}{48}$ or $\frac{3}{8}$ of a Shilling.

23925

23925

319 X 75

Mult. $6\frac{31}{48}$ by $9\frac{3}{8}$

384|23925|62 Sb.

117

12

384|1404|3 d.

252

4 f.

384|1008|2 $\frac{44}{384} = \frac{1}{3}$

240

Decimally.

31 .645833

48|31.000000

3 .375

8|3.000

Sb.

Mult. 6.645833

by - - - 9.375

Sb. 62.304684375

12

d. 3.656212500

4

f. 2.624850000

Secondly, Let a Pound be Integer.

6 s. $7\frac{1}{4}$ is 319 Farthings; that is, $\frac{319}{960}$ of a £.9 s. $4\frac{1}{2}$ d. is 450 Farthings; that is, $\frac{450}{960}$ or $\frac{15}{32}$ of a £.

Vulgar Fractions.

143

319 4785

Mult. $\frac{319}{960}$ by $\frac{15}{32}$

80720 | 4785

20

3072 | 09570 | 03 Sh.

354

12

3072 | 4248 | 1 Penny.

1176

4

8072 | 4704 | $1 \frac{1632}{3072} = \frac{17}{32}$

1632

Decimally.

319 .33229166

960 | 319.00000000

640

15 .46875

32 | 15.00000

£.

Mult. .33229166
by .46875

.1557617156250

20

Sh. 3.1152343125000

12

Penny 1.3828117500000

4

Farth. 1.5312470000000

Thus you see, if a Shilling be } s. d. f.
Integer, the Product is - - - } 62 3 2 $\frac{1}{2}$
If a Pound be Integer - - - } 3 1 1 $\frac{17}{32}$

7. Multiply 12 l. 16 s. 4 d. by 22 l. 16 s. 8 $\frac{1}{2}$ d. Fractionally and Decimally.

16 s. 4 d. is 196 Pence; that is, $\frac{196}{249}$ or $\frac{49}{60}$ of a £.

16 s. 8 $\frac{1}{2}$ d. is 802 Farthings; that is, $\frac{802}{960}$ or $\frac{401}{480}$ of a £.

$$\begin{array}{r} 8429009 \\ 769 \times 10961 \end{array}$$

$$\text{Mult. } 12 \frac{49}{60} \text{ by } 22 \frac{401}{480}$$

$$28800.8429009 | 292 \text{ £.}$$

$$19409$$

$$20$$

$$2880 | 0 | 38818 | 0 | 13 \text{ Sh.}$$

$$1378$$

$$12$$

$$\frac{2784}{2880} = \frac{29}{30}$$

$$2880 | 16536 | 5 \text{ d.}$$

$$2136$$

$$4$$

$$2880 | 8544 | 2 \frac{29}{30}$$

$$2784$$

Decimally.

$$\begin{array}{r} 49 \quad .816666 \\ 60 \overline{) 49.000000} \\ \underline{40} \end{array}$$

$$\begin{array}{r} 401 \quad .8354166 \\ 480 \overline{) 401.0000000} \\ \underline{320} \end{array}$$

£.

$$\text{Mult. } 22.83541666$$

$$\text{by } - - 12.816666$$

$$\text{£. } 292.67390830205556$$

$$20$$

$$\text{Sh. } 13.47816604111120$$

$$12$$

$$\text{Pence } 5.73799249333440$$

$$4$$

$$\text{Farth. } 2.951969973333760$$

8. Mul-

$$\begin{array}{r}
 \text{£.} \\
 \text{Mult. } 3.63888888 \\
 \quad 2.5025 \\
 \hline
 \text{£. } 9.106319421200 \\
 \quad 20 \\
 \hline
 \text{Sh. } 2.126388424000 \\
 \quad 12 \\
 \hline
 \text{Penny } 1.516661088000 \\
 \quad 4 \\
 \hline
 \text{Farth. } 2.066644352000
 \end{array}$$

The ingenious Mr. *Hodgson*, in his *System of Mathematicks*, Vol. I. p. 21. says, That he cannot omit taking Notice of a very notorious Blunder, or rather Error in Judgment, that prevails amongst almost all the Teachers of Arithmetic, viz. that of multiplying *Money* by *Money*, the Result of which they call *Money*, which is in its own Nature absurd; and after his Way of reasoning, concludes, That if the Product arising from the Multiplication of 4 Pounds by 4 Pounds, be 16 Pounds, then it follows, that 4 p is equal to 4 pp (where p in this Case is put for a *Pound Sterling*) a Part to the Whole, which is impossible.

I must confess, that the Multiplication of *Money* by *Money* is not applicable to any practical Uses, but a mere Speculation, and serves only to exercise Youth in the Doctrine of *Fractions*. But if this ingenious Gentleman's Way of Reasoning be allowed, then *Feet*, multiplied by *Feet*, must not produce *Feet*; and so a very useful Branch of the Mathematicks, viz. *Mensuration*, is founded upon an Absurdity: For substituting f for a Foot, 4 f will be equal to 4 ff, after the same Manner as 4 p is equal to 4 pp. But to remove the Absurdity: p is equal to 1 *Pound Sterling*;

Sterling ; therefore 4 p is equal to 4 Times 1 *Pound*, that is, 4 *Pounds*.

Again, p is equal to 1 *Pound Sterling* ; therefore pp is equal to but 1 *Pound*, and consequently 4 pp is equal to 4 *Pound* ; so that 4 p is equal to 4 pp, without any Absurdity.



DIVISION of FRACTIONS.

R U L E.

Multiply the *Numerator* of the *Dividend*, by the *Denominator* of the *Divisor*, for the *Numerator* of the *Quotient* ; then the *Denominator* of the *Dividend*, by the *Numerator* of the *Divisor*, for the *Denominator* of the *Quotient*.

1. Divide $\frac{12}{13}$ by $\frac{7}{8}$ *Fractionally* and *Decimally*.

$$\begin{array}{r} 96 \\ \hline \frac{12}{13} \quad \frac{7}{8} \end{array}$$

91)96($1\frac{5}{91}$ Quotient.

5
Decimally.

$$\begin{array}{r} 12 \quad .923076923 \\ 13 \overline{)12.000000000} \\ \text{Rem. } 1 \\ 1.054945 \\ .875 \overline{)1.054945} \\ 18 \end{array}$$

$$\begin{array}{r} 5 \quad .054945 \\ 91 \overline{)5.000000} \\ \text{Rem. } 5 \end{array}$$

U 2

2. Di-

2. Divide $\frac{7}{8}$ of $\frac{8}{9}$ by $\frac{2}{3}$ of $\frac{3}{4}$ *Fractionally* and *Decimally*.

$$\begin{array}{r} 56 \\ \hline \frac{7}{8} \text{ of } \frac{8}{9} = \frac{7}{9} \\ \hline 72 \end{array}$$

$$\begin{array}{r} 6 \\ \hline \frac{2}{3} \text{ of } \frac{3}{4} = \frac{1}{2} \\ \hline 12 \end{array}$$

$$\begin{array}{r} 14 \\ \hline \text{Divide } \frac{7}{9} \text{ by } \frac{1}{2} \\ \hline \end{array}$$

9) 14 ($1\frac{5}{9}$ Quotient.

5

Decimally.

$$\begin{array}{r} 7.777777 \\ \hline 9 \overline{) 7.000000} \\ \hline 7 \end{array}$$

$$\begin{array}{r} 1.5 \\ \hline 2 \overline{) 1.0} \end{array}$$

$$\begin{array}{r} 1.55555 \\ \hline 5 \overline{) 7.77777} \\ \hline 2 \end{array}$$

$$\begin{array}{r} 5.55555 \\ \hline 9 \overline{) 5.00000} \\ \hline 5 \end{array}$$

3. Divide $7\frac{2}{8}$ by $3\frac{1}{4}$ *Fractionally* and *Decimally*.

$$\begin{array}{r} 252 \\ \hline 63 \quad 15 \\ \hline 7\frac{2}{8} \quad 3\frac{1}{4} \\ \hline 120 \overline{) 252} (2\frac{1}{10} \\ \hline 12 \end{array}$$

$$\frac{12}{120} = \frac{1}{10}$$

Decimally.

$$\begin{array}{r} 7.875 \\ \hline 8 \overline{) 7.000} \end{array}$$

$$\begin{array}{r} 3.75 \\ \hline 4 \overline{) 3.00} \end{array}$$

2.1

$$3.75 \overline{) 7.875}$$

Vulgar Fractions.

149

4. Divide $3\frac{1}{2}$ more $\frac{3}{80}$ by $2\frac{1}{2}$ more $\frac{1}{7}$ of $\frac{11}{21}$
Fractionally and Decimally.

$$\begin{array}{r} 252 \\ \hline 240 \quad 12 \\ \hline \frac{1}{4} \text{ add } \frac{3}{80} = \frac{63}{80} \end{array}$$

$$\begin{array}{r} 11 \\ \hline \frac{1}{7} \text{ of } \frac{11}{21} \\ \hline 147 \end{array}$$

$$\begin{array}{r} 320 \\ 169 \\ \hline 147 \quad 22 \end{array}$$

$$\begin{array}{r} 89082 \\ \hline 303 \quad 757 \end{array}$$

$$\frac{1}{2} \text{ add } \frac{11}{147} \text{ Divide } 3\frac{63}{80} \text{ by } 2\frac{169}{294}$$

$$294$$

$$60560 | 89082 | 1 \quad \frac{14268}{30280}$$

$$28522 = 14261$$

$$28522$$

$$60560 \quad 30280$$

Decimally.

$$3 \quad .75$$

$$3 \quad .0375$$

$$1 \quad .5$$

$$4 | 3.00$$

$$80 | 3.0000$$

$$2 | 1.0$$

$$11 \quad .0748299$$

$$.75$$

$$147 | 11.0000000$$

$$.0375$$

$$.0748299$$

$$.7875$$

$$.5$$

$$.5748299$$

$$1.4709$$

$$2.5748299 | 3.7875000000$$

$$19270109$$

5. Di-

5. Divide 19s. 7d. $\frac{1}{4}$ by 7s. 11d. $\frac{1}{4}$ Fractionally and Decimally.

Note: Before the Operation of this Example, it ought first to be determined whether a *Pound* or *Shilling* ought to be the *Integer*. But for the Learners Benefit I have inserted both Operations.

1. If a *Shilling* be *Integer*.

Then 7d. $\frac{1}{4}$ is 29 *Farthings*, which is $\frac{29}{48}$ of a *Shilling*.

And 11d. $\frac{1}{4}$ is 47 *Farth.* that is $\frac{47}{48}$ of a *Shil.*

45168

941 383

Divide $19\frac{29}{48}$ by $7\frac{47}{48}$

18384) 45168 (2s.

8400

12

18384) 100800 (5d.

8880

4

18384) 35520 (1 $\frac{232}{384}$ f.

17136

$$\frac{17136}{18384} = \frac{232}{384}$$

Decimally.

29 .604166

48 | 29.000000

32

47 .979166

48 | 47.000000

32

7.979166) 19.604166666666 (2.45691 *Skill.*

12

5.48292 d.

4

1.95168 f.

Vulgar Fractions.

151

2. If a Pound be Integer.

Then 19s. 7 $\frac{1}{4}$ d. is 941 Farthings, that is $\frac{941}{960}$ of a Pound.

And 7s. 11 $\frac{1}{4}$ d. is 383 Farthings, that is $\frac{383}{960}$ of a Pound.

Divide $\frac{941}{960}$ by $\frac{383}{960}$

367680)903360(2l.

168000

20

367680)3360000(9s.

50880

12

367680)610560(1d.

242880

$\frac{236160}{367680} = \frac{246}{383}$

4

367680)971520(2 $\frac{246}{383}$ f.

236160

Decimally.

$\frac{941}{960} \cdot 9802083$
960)941.0000000
320

$\frac{383}{960} \cdot 398958$
960)383.0000000
320

$\cdot 398958) \cdot 980208333333(2.456921$

20

Sb. 9.138420

12

P. 1.661040

4

F. 2.644160

Note :

Note : In the *Fractional* Operations of this Example, the *Denominators* being equal, might have been cancelled, and the *Numerator* of the *Dividend*, divided by the *Numerator* of the *Divisor*, would have answered the Question, as may be seen by the following Operations.

$$\begin{array}{r} \text{Divide } \frac{941}{48} \text{ by } \frac{383}{48} \\ \hline 383 \overline{) 941} (2s. \\ \underline{175} \\ 12 \\ 383 \overline{) 2100} (5d. \\ \underline{185} \\ 4 \\ 383 \overline{) 740} (1 \frac{357}{383}f. \\ \underline{357} \end{array}$$

$$\begin{array}{r} \text{Divide } \frac{941}{960} \text{ by } \frac{383}{960} \\ \hline 383 \overline{) 941} (2l. \\ \underline{175} \\ 20 \\ 383 \overline{) 3500} (9s. \\ \underline{53} \\ 12 \\ 383 \overline{) 636} (1d. \\ \underline{253} \\ 4 \\ 383 \overline{) 1012} (2 \frac{246}{383}f. \\ \underline{246} \end{array}$$

From hence you plainly see, that in Division of *Fractions*, if the *Denominators* be equal, they may be

be cancelled, and the Operation thereby more compendiously performed, because the *Numerator* of the *Dividend*, becomes the *Numerator* of the *Quotient*; and the *Numerator* of the *Divisor*, becomes the *Denominator* of the *Quotient*.

6. Divide $17l. \frac{7}{9}$, and $\frac{1}{4}$ of $\frac{1}{9}$ of a *Penny*, by $10l. \frac{2}{3}$, and $\frac{2}{48}$ of a *Shilling*, both by *Vulgar* and *Decimal Fractions*.

$$\frac{\frac{3}{4} \text{ of } \frac{5}{9} \text{ of } \frac{1}{240}}{8640} = \frac{1}{576} \quad \frac{4041}{4032 \quad 9} \quad \frac{7}{9} \text{ add } \frac{1}{576} = \frac{449}{576}$$

$$\frac{\frac{1}{48} \text{ of } \frac{1}{20}}{960} \quad \frac{1923}{1920 \quad 3} \quad \frac{2}{3} \text{ add } \frac{1}{960} = \frac{641}{960}$$

$$\begin{array}{r} 9831360 \\ \hline 10241 \quad 10241 \\ \hline \text{Divide } 17 \frac{449}{576} \text{ by } 10 \frac{641}{960} \\ \hline 5898816 \quad 9831360 (1l. \\ 3932544 \\ 20 \\ 5898816 \quad 78650880 (13s. \\ 1966272 \\ 12 \\ 5898816 \quad 23595264 (4d. \end{array}$$

Decimally.

$$\frac{7}{9} \cdot 777777 \quad \frac{1}{576} \cdot 00173611$$

X

$$\begin{array}{r} .00173611 \\ .77777777 \\ \hline .77951388 \end{array}$$

$$\begin{array}{r} 2 \quad .666666 \\ 3 \overline{) 2.000000} \end{array}$$

$$\begin{array}{r} 1 \quad .0010416 \\ 960 \overline{) 1.0000000} \end{array}$$

$$\begin{array}{r} .0010416 \\ .6666666 \\ \hline \end{array}$$

$$10.6677082 \quad) \quad 17.7795138888 \quad (\quad 1.666666$$

20

$$\begin{array}{r} Sb. \quad 13.33320 \\ 12 \end{array}$$

$$\begin{array}{r} D. \quad 3.99840 \\ 4 \end{array}$$

$$F. \quad 3.99360$$

Note : In the *Fractional* Operation of this Example, the *Numerators* being equal, might have been cancelled, and the Operation thereby more compendiously performed, because the *Denominator* of the *Divisor* becomes the *Numerator* of the *Quotient*, and the *Denominator* of the *Dividend* becomes the *Denominator* of the *Quotient*, as followeth.

$$\begin{array}{r} \text{Divide } \frac{10241}{576} \text{ by } \frac{10241}{960} \\ \hline 576 \overline{) 960} (11. \\ \quad 384 \\ \quad \quad 20 \\ 576 \overline{) 7680} (135. \\ \quad \quad 192 \\ \quad \quad \quad 12 \\ 576 \overline{) 2304} (4d. \end{array}$$

7. Divide 27 l. $\frac{5}{8}$ and $\frac{1}{2}$ of a Shilling; by 3 l. $\frac{2}{13}$ and $\frac{1}{4}$ of a Farthing, both by *Vulgar* and *Decimal Fractions*.

$$\begin{array}{r} \frac{1}{2} \text{ of } \frac{1}{20} \\ \hline 40 \end{array} \quad \begin{array}{r} 208 \\ \hline 200 \quad 8 \\ \hline \frac{5}{8} \text{ add } \frac{1}{40} = \frac{13}{20} \\ \hline 320 \end{array} \quad \begin{array}{r} 11533 \\ \hline 11520 \quad 13 \\ \hline \frac{9}{13} \text{ add } \frac{1}{1280} \\ \hline 16640 \end{array}$$

$$\begin{array}{r} 3 \\ \hline \frac{3}{4} \text{ of } \frac{1}{960} = \frac{1}{1280} \\ \hline 3840 \end{array} \quad \begin{array}{r} 553 \quad 61453 \\ \hline 9201920 \end{array} \quad \begin{array}{r} 11533 \\ \hline 16640 \end{array}$$

Divide 27 $\frac{13}{20}$ by 3 $\frac{11533}{16640}$

$$122906 \overline{) 920192} (7 \text{ l.}$$

$$\begin{array}{r} 59850 \\ 20 \end{array}$$

$$122906 \overline{) 1197000} (9 \text{ s.}$$

$$\begin{array}{r} 90846 \\ 12 \end{array}$$

$$122906 \overline{) 1090152} (8 \text{ d.}$$

$$\begin{array}{r} 106904 \\ 4 \end{array}$$

$$\frac{58898}{122906} = \frac{4207}{8779}$$

$$122906 \overline{) 427616} (3 \frac{4207}{8779} \text{ f.}$$

$$\begin{array}{r} 58898 \end{array}$$

X 2

Decimally.

	Decimally.	
$\frac{5}{8} \cdot 625$	$\frac{1}{40} \cdot 025$	$\cdot 625$
$\frac{9}{13} \cdot 692307$	$\frac{1}{1280} \cdot 000781$	$\cdot 025$
$\frac{1}{13} \cdot 9.000000$	$\frac{1}{1280} \cdot 1.000000$	$\cdot 650$
	£.	$\cdot 000781$
		$\cdot 692307$
		$\cdot 693088$

3.693088) 27.650000000000 (7.48695

20

Sb. 9.73900

12

d. 8.86800

4

f. 3.47200

8. Divide 113*l.* 14*s.* 10*d.* by 22*l.* 19*s.* 7½, *Fractionally and Decimally.*

14*s.* 10*d.* is 178 *d.* that is $\frac{178}{240}$ or $\frac{89}{120}$ of a *£.*

19*s.* 7½*d.* is 942 *Farth.* that is $\frac{942}{960}$ or $\frac{471}{480}$ of a *£.*

6551520

13649

11031

Divide $113 \frac{89}{120}$ by $22 \frac{471}{480}$

132372) 06551520 (4*l.*

125664

20

132372) 2513280 (18*s.*

130584

12

132372) 1567008 (11*d.*

$\frac{46548}{132372}$

$= \frac{1293}{3677}$

110916

4

132372) 443664 (3 $\frac{1293}{3677}$

46548

Decimally.

$$\begin{array}{r} 89 \quad .741666 \\ 120 \overline{) 89.000000} \end{array}$$

$$\begin{array}{r} 471 \quad .981250 \\ 480 \overline{) 471.000000} \end{array}$$

$$\begin{array}{r} \text{£.} \\ 22.98125 \text{) } 113.7416666666 \text{ (} 4.94932 \\ \underline{20} \\ \text{Sh. } 18.98640 \\ \underline{12} \\ \text{d. } 11.83680 \\ \underline{4} \\ \text{f. } 3.34720 \end{array}$$

Note : From what has been done you may observe, That when a *greater* Fraction is divided by a *less*, the *Quotient* will be greater than the *Dividend*, as you may see in the first and second Examples, and in the fifth, where a *Pound* is the *Integer*. Consequently, If *Integers* be divided by a *Fraction*, the *Quotient* will also be greater than the *Dividend*. So likewise, if *Integers* or *Fractions* be multiplied by another *Fraction*, the *Product* will be less than the *Multiplicand*.

Thus have I gone through the whole Work of *Fractions*, and illustrated the same with variety of Examples wrought at large, and such as have not been treated of in any Books of *Arithmetic* that I have had the Opportunity of perusing. I shall now proceed to the *Golden Rule*, and therein apply the Work of *Decimals*; it being my chief Design in this Treatise to recommend the Knowledge of so useful and excellent an Invention, as that of *Decimal Arithmetic*. Although the chief Use and Excellency thereof, consisteth more in *Geometrical Computations*, than in the common
and

and practical Parts of *Arithmetic* : Yet how very useful it is even in these, especially in the Computations of *Interest* and *Annuities* will appear as we go on.



The GOLDEN RULE.

SO called by reason of its Excellency and universal Use, hath always *Three* Numbers given to find a *Fourth*. The greatest Difficulty here, if any, will be the right stating the Question, or placing down the *three* given Numbers in their proper Order. To do which observe,

That always *two* of the *three* given Numbers are a *Supposition*, and limits the Proportion, and the other is a *Demand*, and moves the Question, and hath commonly before it some such like Words as these, *What will, How many, How long, How far, How much, &c.* and is the *third* Number in the Proportion. The *first* Number is that Number in the *Supposition*, which is of the same Kind or Species with the *Demand*. And the *other* Number in the *Supposition* is the second.

For Instance : If four *Pound* of Sugar cost 2*s.* 6*d.* what will seven *Hundred Weight* cost at the same Rate.

Here 4*lb.* and 2*s.* 6*d.* is the *Supposition*, and 7 *Hundred Weight*, is the *Demand*, as appears by the Words *What will* before it. Therefore 4*lb.* in the *Supposition* being of the same Kind and Species with 7 \oplus the *Demand*, is the *first* Number in the Proportion, and 2*s.* 6*d.* is the *second*, and 7 \oplus the *Demand* is the *third*, and must be placed thus :
If

lb. s. d. £
If 4 - - - - - 2 : 6 - - - - - 7

The next thing to be observed is,

That the *first* and *third* Numbers be reduced into the same Denomination, and the *second* Number into the *lowest* Name mentioned.

That is, 7 £ must be reduced into *Pounds*, to make it of the same Denomination with the *first* Number; and 2 s. 6 d. must be reduced into *Pence*, the lowest Name here mentioned; and then the Question will appear thus.

lb.	s.	d.	£
If 4 - - - - -	2 : 6 - - - - -		7
	12		4
	<hr/>		<hr/>
	30 Pence.		28 Qrs.
			28
			<hr/>
			224
			56
			<hr/>
			784 lb.

The next Thing to be observed is, That the *first* and *third* Numbers are called *Extremes*; and in order to find the *fourth* Number, which must bear the same Proportion to the *third* Number, as the *second* doth to the *first*. You are to consider, whether the *third* Number requires *more* or *less* than the *second* Number for the *fourth*.

If *more* be required, multiply the *second* Number and *greater* Extreme together, and divide the Product by the *lesser* Extreme.

But if *less* be required, multiply the *second* Number and *lesser* Extreme together, and divide the Product by the *greater* Extreme, the Quotient shews the *fourth* Number or *Answer* to the Question in the same Denomination with your *second* Number.

Note :

Note : When Numbers, be they *Money*, *Weight*, or *Measure*, are required to be reduced into lesser or the least Denomination, *Multiplication* performs the Work, by beginning at the greatest Denomination, and multiplying it with the Number of *Units* of the next lesser Denomination, that make one of that Denomination you are reducing, and so proceeding till you are arrived to that lesser or least Denomination required.

To reduce Numbers of lesser Denominations into greater, is only the *Converse* of the former, and is performed by *Division*. The Practice of which you will meet with in the following Questions, inserted at large for the Learner's Instruction.

1. If 4 *Pounds* of Sugar cost 2 s. 6 d. what will 7 *Hundred Weight* amount to?

lb	s.	d.	£
If 4 . . .	2	6	:: 7
	12		4
	30		28
			28
			784 lb
			30
			4 23520
			12 5880 Pence.
			2 0 49 0 Sh.
			£. 24 : 10

In this Question it is easy to conceive, that *more* is required ; therefore the *second* Number 30, and *third* Number 784, which is the *greater* Extreme, are multiplied together, and divided by the *first* Number 4, which is the *lesser* Extreme. The

Quotient

The Golden Rule.

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Quotient 5880 is the fourth Number, and being Pence, is reduced into Pounds, and gives 24 l. 10 s. for the Answer to the Question.

Decimally.

4 lb is $\frac{4}{112}$ or $\frac{1}{28}$ of an Hundred Weight.

2 s. 6 d. is 30 Pence; that is, $\frac{30}{240}$ or $\frac{1}{8}$ of a £.

$$\begin{array}{r} 1 \quad .035714 \\ \hline 28 \overline{) 1.000000} \end{array}$$

$$\begin{array}{r} 1 \quad .125 \\ \hline 8 \overline{) 1.000} \end{array}$$

If $\text{£} \quad \text{£} \quad \text{£}$
 $.035714 : .125 :: 7$

$$\begin{array}{r} .035714 \cdot 8750000 \cdot 24.5 \text{ £.} \\ \hline 20 \\ \hline \text{Sb. } 10.0 \end{array}$$

2. If when Wheat is 6 s. 6 d. per Bushel the Penny Loaf weigheth 8 Ounces, how much must the Penny Loaf weigh when Wheat is 9 s. per Bushel?

$$\begin{array}{r} \text{s. d. oz.} \\ \text{If } 6 : 6 : 8 :: 9 \\ \hline 12 \quad \quad \quad 12 \\ \hline 78 \quad \quad \quad 108 \\ \hline 8 \end{array}$$

$$108 \overline{) 624} 5 \text{ oz.}$$

$$\begin{array}{r} 84 \\ \hline 16 \end{array}$$

$$\begin{array}{r} 504 \\ \hline 84 \end{array}$$

$$\begin{array}{r} 108 \overline{) 1344} 12 \frac{4}{9} \\ \hline 48 \end{array}$$

Decimally.

lb. oz. lb.
 If $6.5 : 8 :: 9$

$$9 \overline{) 52.0000}$$

$$\begin{array}{r} \text{oz. } 5.7777 \\ \hline 16 \end{array}$$

$$46662$$

$$7777$$

$$\text{dr. } 12.4432$$

Y

Note:

Note : The dearer the Wheat, the less must be the Penny Loaf; therefore, the *second* Number and *lesser* Extreme are multiplied together, and the *Product* divided by the *greater* Extreme.

3. If 17 lb of Figs cost 5 s. 2 d. what will 15 Frails amount to, each Frail containing 47 lb?

15 Frails.	lb	s.	d.	lb	
47					If 17 : 5 : 3 :: 705
<hr/>					12
105					63
60					<hr/>
<hr/>					63 Pence: 2115
705 lb					4230

17)44415(2612 d.

12|2612

21|7 : 8

£. 10 : 17 : 8 ½

11

4

17)44(2 ½ f.

10

Decimally.

3 d. is $\frac{3}{12}$ or $\frac{1}{4}$ of a Shilling.

1 .25

4|1.00

lb.	Sh.	lb.
If 17	: 5.25	:: 705
	705	

17|3701.2500|217.7205 Sh.

12

d. 8.6460

4

f. 2.5840

It is plain, that 705 lb will require more than 5 s. 3 d. therefore the *second* Number and *greater* Extreme are multiplied together, and the *Product* divided by the *lesser* Extreme.

The Golden Rule.

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4. A yearly Pension being 37 l. 15 s. what is that a Week?

Days £. Sb. Days.
If 365 : 37 : 15 :: 7

20
755 Sb.
7
365)5285(14 Sb.

175
12
365)2100(5 d.

$$\frac{5}{365} = \frac{1}{73}$$

275
4
365)1100(3 $\frac{1}{73}$ f.
5

Decimally.

15 s. is $\frac{15}{20}$ or $\frac{3}{4}$ of a Pound.

$$\frac{3}{4} = .75$$

Days. £. Days.
If 365 : 37.75 :: 7

7 £.
365)264.250000(.723972

20
Sb. 14.479440

12
d. 5.753280

1
f. 3.013120

Less being required, the second Number and lesser Extreme are multiplied together, and the Product divided by the greater Extreme.

Y 2

5. A

5. A *Vintner* bought 1 Tun of White Wine for 50 *l.* and a Tun of Canary for 80 *l.* 7 *s.* 6 *d.* and mingleth them together; I demand what *one Gallon* of this Mixture cost him?

$$\begin{array}{r} 50 : 00 : 0 \\ 80 : 07 : 6 \\ \hline 130 : 07 : 6 \end{array}$$

$$\begin{array}{r} \text{Tun.} \quad \text{£.} \quad \text{s.} \quad \text{d.} \quad \text{Gall.} \\ \text{If } 2 : 130 : 7 : 6 :: 1 \end{array}$$

$$\begin{array}{r} 4 \quad 20 \\ \hline 8 \quad 2607 \\ 63 \quad 12 \\ \hline \end{array}$$

$$\text{Gall. } 504$$

$$504)31290(62\frac{1}{12} \text{ d.}$$

$$\frac{12}{504} = \frac{1}{12}$$

$$12 \quad \text{Sh. } 5 : 2\frac{1}{12}$$

Decimally.

7 *s.* 6 *d.* is 90 *d.* that is $\frac{90}{240}$ or $\frac{3}{8}$ of a Pound.

$$\frac{3 \cdot 375}{8 \cdot 3000}$$

$$\begin{array}{r} \text{Gall.} \quad \text{£.} \quad \text{Gall.} \\ \text{If } 504 : 130.375 :: 1 \end{array}$$

$$504)130.37500(.25868 \text{ £.}$$

20

$$5.17360$$

12

$$\text{d. } 2.08320$$

I need add no more, because the *Reader* cannot but see that *less* is required in this Question.

6. If for 3 *s.* 6 *d.* I have One Hundred Weight carried 60 Miles, how far shall 20 *lb.* be carried for the same Money, at the same Rate?

If

The Golden Rule.

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$\frac{112}{60} : 60 :: 29$
 $\frac{60}{29} 6720 (231 \text{ Miles.})$

$\frac{21}{8}$
 $29)168(5 \frac{23}{29} f.$
 $\frac{23}{23}$

Decimally.

$29 .258928$
 $\frac{112}{29} 29.000000$

$\frac{23}{231.724}$
 $\frac{1}{231.724}$

$.258928 | 60.000000000$
 $\text{Miles } 231.724$
 $\frac{8}{8}$

Furlongs 5.792

In this Question *more* is required, because a *less* Weight ought be carried *farther* for the same Money.

7. A *Mariner* enter'd on Board a Man of War, July 29, 1729, and was discharged from thence December 24, 1731; what came his Wages to, at 1 l. 3 s. 6 d. per Month?

Days.

From July 29, 1729, to July 29, 1730, is -- 365
 From July 29, 1730, to July 29, 1731, is -- 365
 Remaining Days in July is - - - - 3
 Days in August - - - - 31
 September - - - - 30
 October - - - - 31
 November - - - - 30
 To the Time of Discharge, December - - - 24
 Total Days of Service - - - - 879

a 6 + Days. 0-12-0

Days. £. s. d. Days.

If 28 : 1 : 3 : 6 :: 879

20 282

23

12

Pence 282

28)247878(8852 Pence.

22

4

28)88(3 $\frac{1}{7}$ f.

12)8852

20)73(7 : 8

36 : 17 : 8 $\frac{1}{4}$ the Mariner's Wages.

Decimally.

3 s. 6 d. is 42 d. that is $\frac{42}{240}$ or $\frac{7}{40}$ of a £.

7 .175

40)7.000

Days.

If 28 : 1.175 :: 879

879

28)1032.8250(36.8896 £.

20

Sh. 17.7320

12

d. 8.7840

4

f. 3.1360

Mon. Days. l. s. d.

28)879(31 : 11 at 1 : 3 : 6 per Month.

11

10

11 15 0

3

35 05 0

D. 1 3 6

7 - - - - - 5 10 $\frac{1}{2}$

4 - - - - - 3 4 $\frac{1}{4}$

Mariner's Wages - - 36 17 8 $\frac{1}{4}$

Note :

Note: The Pay of *Mariners* belonging to any of his Majesty's Ships or Vessels, is always accounted *per Month*, of 4 *Weeks*, or 28 *Days*.

But the Pay of *Mariners* in Merchant-Ships differ, being reckoned sometimes by the *Voyage*, and sometimes by the *Month*; and their *Month* is always a *Kalendar Month*, which is *one Month* and a *Day* in a *Year* *less* than the other.

8. *A* borrowed of *B* 137 *l.* 10 *s.* which at the Year's End he paid with Thanks, and promised to do *B* the like Courtesy: Some Time after, *B* borrowed of *A* 322 *l.* 15 *s.* How long must he keep it to requite his former Courtesy to *A*?

<i>l.</i>	<i>s.</i>	<i>Days.</i>	<i>l.</i>	<i>s.</i>	
If 137	: 10	: 365	::	322 : 15	
20				20	
<hr/>				<hr/>	
2750				6445	
365					
<hr/>					
6445)1003750(155					
3225					
				$\frac{3225}{6445} = \frac{645}{1289}$	

Decimally.

<i>£.</i>	<i>Days.</i>	<i>£.</i>
If 137.5	: 365	: 322.7
365		
<hr/>		
322.75)50187.500000(155.4996		

It is plain, that *more* Money being lent, *less* Time is required to keep it.

9. If

9. If $\frac{3}{4}$ of $\frac{5}{9}$ of a Yard of Cloth cost $\frac{2}{3}$ of $\frac{7}{9}$ of a Pound Sterling, what will 791 Yards amount to?

$$\frac{\frac{3}{4} \text{ of } \frac{5}{9}}{36} = \frac{5}{12}$$

$$\frac{\frac{2}{3} \text{ of } \frac{7}{9}}{54}$$

rd. $\frac{27685}{54}$

If $\frac{5}{12} : \frac{35}{54} :: \frac{791}{1}$

54

Divide $\frac{27685}{54}$ by $\frac{5}{12}$

$$27 \overline{) 0333220} (1230 \text{ £.}$$

$$12$$

$$20$$

$$27 \overline{) 240} (8 \text{ Sh.}$$

$$24$$

$$12$$

$$27 \overline{) 288} (10 \text{ d.}$$

$$18$$

$$4$$

$$27 \overline{) 72} (2 \frac{2}{3} \text{ f,}$$

$$18$$

$$\frac{18}{27} = \frac{2}{3}$$

Decimally.

Decimally.

$$\begin{array}{r} 5 \quad .4166 \\ 12 \overline{) 5.0000} \end{array}$$

$$\begin{array}{r} 35 \quad .6481481 \\ 54 \overline{) 35.0000000} \end{array}$$

Yds. *£.* *Yds.*
If .416666666 : .6481481481481 :: 791

$$\begin{array}{r} .416666666 \overline{) 512.6851851851471} \quad \begin{array}{r} 791 \quad \text{£.} \\ 193387767 \end{array} \quad \begin{array}{r} 1230.4444 \\ 20 \end{array} \end{array}$$

Sb. 8.8880

12

d. 10.6560

4

f. 2.6240

10. Bought 19 $\frac{5}{9}$ Tun of Wine, at 3 s. $\frac{1}{4}$ and $\frac{7}{8}$ of a Penny, per Quart. I demand how much I paid for the whole?

$$\begin{array}{r} 7 \quad \quad \quad 316 \\ \hline 28 \quad 288 \\ \hline \frac{7}{8} \text{ of } \frac{1}{12} \quad \frac{7}{96} \text{ add } \frac{3}{4} = \frac{79}{96} \\ \hline 96 \quad \quad 384. \end{array}$$

Hbds 4

63

Gallons 252

Quarts $\frac{4}{1008}$

Tun. *Sb.* *Tun.*
If $\frac{1}{1008}$: 3 $\frac{79}{96}$:: 19 $\frac{5}{9}$

$$\begin{array}{r} 64592 \\ \hline 367 \quad 176 \\ \hline \text{Mult. } 3 \frac{79}{96} \text{ by } 19 \frac{5}{9} \\ \hline 864 \end{array}$$

Z

65108736

The Golden Rule.

65108736

Divide $\frac{64592}{864}$ by $\frac{1}{1008}$ 864) 65108736 (75357 *Shill.*

288

12

864) 3456 (4 *Pence.*

Decimally.

$$\begin{array}{r} 1 \quad .000992063492 \\ 1008 \overline{) 1.000000000000} \end{array}$$

$$\begin{array}{r} 7 \quad .07291666 \\ 96 \overline{) 7.00000000} \end{array}$$

$$\begin{array}{r} .07291666 \\ 3 \quad .75 \\ 4 \overline{) 3.00} \end{array}$$

$$\begin{array}{r} .75 \\ .82291666 \end{array}$$

$$\begin{array}{r} 5 \quad .5555 \\ 9 \overline{) 5.0000} \end{array}$$
*Tun.**Shillings.**Tun.*

If .000992063492 : 3.8229166666 :: 19.555555

19.555555

Shillings.

.000992063462) 74.7592571341129630 (75357.3311

12

d. 3.9732*f.* 3.8928

In the following Examples, wherein Allowance is made for *Tare* and *Tret*, observe,

That *Gross* is the whole Weight of any Commodity, together with the *Hogshead*, *Chest*, or *Box*, or whatsoever else contains it.

Tare is the Weight of the *Hogshead*, *Chest*, or *Box*, &c. and is sometimes rated at a certain Number of *Pounds per Hundred Weight*.

Trett is an Allowance of 4 *lb.* in every 104 *lb.* for Waste and Dust on some Sort of Goods; such as *Tobacco*, *Indigo*, and all *Spices*.

Neat

The Golden Rule.

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Neat is the Weight of a Commodity, after the *Tare* and *Trest* is deducted.

11. What will 362 lb. 3 qrs. 16 *Gross* amount to,
at 3 l. 11 s. 4 d. per cwt. *Neat*. *Tare* 18 lb. per cwt.
 lb. lb. cwt. qrs. lb.
If 112 : 18 :: 362 : 3 : 16

$$\begin{array}{r} 4 \\ \hline 1451 \text{ qrs.} \\ 28 \end{array}$$

$$\begin{array}{r} 40644 \text{ lb.} \\ 18 \end{array}$$

$$112)731592(6532 \text{ Tare.}$$

$$\text{lb.} \quad 8$$

From 40644 *Gross*.

Take 6532 *Tare*.

Rem. 34112 *Neat*.

cwt. l. s. d. lb.

If 112 : 3 11 4 :: 34112

$$\begin{array}{r} 20 \\ \hline 856 \end{array} \text{ Pence.}$$

$$71 \quad 112)29199872(260713 \frac{1}{2}$$

$$\begin{array}{r} 12 \\ \hline 16 \end{array}$$

$$856$$

$$12)260713$$

$$2|0)2172|6 : 1$$

$$\text{£. } 1086 : 6 : 1 \frac{1}{7}$$

Decimally.

$$18 \text{ lb. is } \frac{18}{112} \text{ or } \frac{9}{56} \text{ of an } \text{cwt.} \quad \frac{9}{56} \frac{.160714}{9.000000}$$

$$\text{qrs. lb.} \quad 3 : 16 \text{ is } 100 \text{ that is } \frac{100}{112} \text{ or } \frac{25}{28} \text{ of an } \text{cwt.}$$

$$\begin{array}{r} 25 \quad .892857 \\ 28 \overline{)25.000000} \end{array}$$

The Golden Rule.

$$\begin{array}{r} \text{If } 1 : .160714 :: 362.892857 \\ \quad \quad \quad .160714 \end{array}$$

Tare 58.321962619898

From 362.892857 *Gross.*

Take 58.321962 *Tare.*

Rem. 304.570895 *Neat.*

11 s. 4 d. is 136 Pence, that is $\frac{136}{240}$ or $\frac{17}{30}$ of a £.

$$\begin{array}{r} 17 \quad .5666 \\ 30 \overline{) 17.0000} \end{array}$$

$$\begin{array}{r} \text{If } 1 : 3.566666 :: 304.570895 \\ \quad \quad \quad 3.566666 \end{array}$$

£. 1086.302655786070

Shil. 6.053115721400

Pence .637388656800

Far. 2.549554627200

Note : The *Decimal* Operation gathers up the *Fractions* in the *Tare*, which is omitted in the *Vulgar* Operation, and from hence it is, that the *Vulgar* exceeds the *Decimal* by 2 *Fartbings*.

The Golden Rule.

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12. At 7 d. $\frac{1}{2}$ per lb . what will 9 Hbds. of Sugar amount to, each Hbd. weighing 4 c . 2 q . 17 lb . Tare 47 lb . $\frac{1}{2}$ per Hbd.

c	q	lb	lb
4	: 2	: 17	47 $\frac{1}{2}$
4			4
<u>18</u>			<u>189</u>
28			9
<u>521</u>			1701 <i>Qrs. lb. Tare.</i>
9			
<u>4689</u>			
4			

From 18756 *Qrs. lb. Gross.*

Take 1701 *Qrs. lb. Tare.*

Rem. 17055 *Qrs. lb. Neat.*

Qrs. lb. d. *Qrs. lb.*

If 4 : 7 $\frac{1}{2}$:: 17055

4 30

30 4511650

Fartb. 127912 = $\frac{1}{2}$

4|127912

12|31978

2|0)266(4 : 10

£. 133 : 4 10 : 0 $\frac{1}{2}$

Decimally.

The Golden Rule.

Decimally.

521 lb the Weight of 1 Hbd.

$\begin{array}{r} 9 \\ \hline \text{From } 4689 \text{ Grofs.} \\ \text{Take } 425.25 \text{ Tare.} \\ \hline \text{Rem. } 4263.75 \text{ Neat.} \end{array}$	$\begin{array}{r} \text{lb} \\ 47.25 \text{ Tare of 1 Hbd.} \\ 9 \\ \hline 425.25 \end{array}$
$\begin{array}{r} \text{lb} \quad \text{Pence.} \\ \text{If } 1 : 7.5 : 4263.75 \\ \hline 7.5 \end{array}$	
$\begin{array}{r} 12 \overline{) 31978} \\ 20 \overline{) 266} (4 : 10 \\ \hline \text{£. } 133 : 4 : 10 \end{array}$	$\begin{array}{r} \text{Pence } 31978.125 \end{array}$

13. At 6 s. 10 d. per lb . what cost 6 Barrels of Nutmegs, each Barrel weighing 3 c . 2 q . 21 lb . Tare 37 $\frac{1}{2}$ lb . per Barrel, Trett 4 lb . per 104?

Where Allowance is made for Trett, when the Tare is subtracted from the Grofs, the Remainder is called *Subtle*, (not *Neat*, as aforegoing) which divided by 26, gives the whole Allowance for Trett, because 26 lb . is $\frac{1}{4}$ Part of 104; this subtracted from the *Subtle*, leaves the *Neat* Weight.

$\begin{array}{r} \text{c} \quad \text{q} \quad \text{lb} \\ 3 : 2 : 21 \\ \hline 4 \\ 14 \\ 28 \\ \hline 413 \\ 4 \\ \hline 1652 \text{ Qrs. lb. in 1 Barrel.} \\ 6 \\ \hline 9912 \text{ Qrs. lb. Grofs.} \end{array}$	$\begin{array}{r} \text{lb} \\ 37 \frac{1}{2} \\ \hline 4 \\ 149 \\ 6 \\ \hline 894 \text{ Qrs. lb. Tare.} \end{array}$
--	---

Qrs. lb.

Qrs. lb.

From 9912 *Gross*

Take 894 *Tare.*

Rem. 9018 *Subtle.*

Take 346 *Trett.*

8672 *Neat.*

$$\begin{array}{r} 26 \overline{) 9018 (346 \frac{22}{28}} \\ 22 \end{array}$$

Qrs. lb. s. d. Qrs. lb.

If 4 : 6 : 10 :: 8672

$$\begin{array}{r} 12 \\ 82 \end{array}$$

$$\begin{array}{r} 82 \\ 4 \overline{) 711104} \\ 12 \overline{) 177776} \text{ Pence.} \\ 2 \overline{) 01481 (4 : 8} \\ \text{£. 740 : 14 : 8} \end{array}$$

Decimally.

413 $\frac{11}{6}$ in a Barrel ; 37.25 *Tare* of a Barrel.

2478 *Gross.*

223.50 *Tare*

2254.50 *Subtle.*

86.711538 *Trett.*

2167.788462 *Neat.*

223.50 whole *Tare.*

86.711538

$$26 \overline{) 2254.500000}$$

6 s. 10 d. is 82 Pence ; that is, $\frac{82}{240}$ or $\frac{41}{120}$ of a £.

$$\begin{array}{r} 41 \quad .341666 \\ 120 \overline{) 41.000000} \end{array}$$

If $\frac{11}{6}$: £. : $\frac{11}{6}$
If 1 : .341666 :: 2167.788462

$$\begin{array}{r} .341666 \\ \text{£. 740.659612657692} \\ 120 \end{array}$$

$$\begin{array}{r} \text{Sh. 13.192253153840} \\ 12 \end{array}$$

$$\begin{array}{r} \text{d. 2.307037846080} \\ 4 \end{array}$$

$$\begin{array}{r} \text{f. 1.228151384320} \end{array}$$

15. Two Men, *A* and *B*, barter. *A* hath 357 Ream of Paper, worth 9 s. 7 d. per Ream; for which *B* giveth 74 l. 11 s. 6 d. ready Money, and the rest he must, by Agreement, give in Broad Cloth, at 21 s. 9 d. per Yard. I demand how much Broad Cloth will satisfy?

R.	s.	d.	R.	l.	s.	d.
If 1	:	9	:	7	:	357
		<u>12</u>				<u>115</u>
		115				12 41055
						2 0)342(1 : 3
						£. 171:1:3

From 171:1:3 Pr. of Pap.
Take 74:11:6 Paid.
Rem. 96:9:9 to be re-
turned in Broad Cloth.

l.	s.	d.	Yd.	l.	s.	d.
If 1	:	1	:	9	:	1
		<u>20</u>				<u>20</u>
		21				1929
		<u>12</u>				<u>12</u>
		261				261)23157(88 $\frac{21}{29}$

89 Yards of Broad Cloth that will satisfy.

Decimally.

9 s. 7 d. is 115 d. that is, $\frac{115}{240}$ or $\frac{23}{48}$ of a £.

23	.479166	R.	£.	R.
48	23.000 00	If 1	:	.479166 :: 357

357
£. 171.062262

11 s. 6 d. is 138 d. that is, $\frac{138}{240}$ or $\frac{23}{40}$ of a £.

23	.575	From 171.062262 £.	Price of Paper.
40	23.000	Take 74.575 - - -	Money paid.

£. 96.487262 to be return'd in Cl.

1 s. 9 d. is 21 d. that is, $\frac{21}{240}$ or $\frac{7}{80}$ of a £.

7	.0875	£.	Yd.	£.
80	7.0000	If 1.0875	:	1 :: 96.487262

1 Yds.
1.0875)96.487262(88.72
A a 16. Two

16. Two Men, *A* and *B*, barter. *A* hath Nutmegs, which cost him 7 s. 8 d. per lb; but in Barter will have 9 s. 6 d. per lb. *B* hath Cinnamon, which cost him 8 s. 8 d. per lb. I demand how he must rate his Cinnamon, to make his Gain in Barter equal to that of *A*?

$$\begin{array}{r}
 \begin{array}{ccc}
 s. & d. & s. \\
 \text{If } 7 & : 8 & : 9 \\
 \hline
 12 & & 12 \\
 \hline
 92 & & 114 \\
 & & \hline
 & & 104
 \end{array}
 \end{array}$$

$$\begin{array}{r}
 92 \overline{) 11856} (128 \\
 \underline{80} \\
 4 \\
 92 \overline{) 320} (3 \frac{11}{23} \\
 \underline{44}
 \end{array}$$

$$\begin{array}{r}
 s. & d. \\
 8 & : 8 \\
 \hline
 12 & \\
 \hline
 104
 \end{array}$$

$$\frac{44}{92} = \frac{11}{23}$$

Decimally.

8 d. is $\frac{8}{12}$ or $\frac{2}{3}$ of a Shilling. $\frac{2}{3} .666666$

6 d. is $\frac{6}{12}$ or $\frac{1}{2}$ of a Shilling. $\frac{1}{2} .5$

If 7.666 : 9.5 :: 8.666666

$$\begin{array}{r}
 9.5 \\
 \hline
 7.666 \overline{) 82.3333270} (10.74 \text{ Sh.} \\
 \underline{12} \\
 d. 8.88 \\
 \underline{4} \\
 f. 3.52
 \end{array}$$

17. Two Merchants, *A* and *B*, barter. *A* had 13 $\text{lb. } 3 \text{ Q. } 10 \text{ lb.}$ of Sugar, worth 6 d. $\frac{1}{2}$ per lb; for which

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which B gave him 27 p. 2 q. 20 lb. of Figs. I demand how B rated his Figs?

lb. d.	p. q. lb.	$\text{p. q. lb. Farth. lb.}$
If 1 : 6 $\frac{1}{2}$:: 13 : 3 10		If 27 : 2 : 20 : 40300 :: 1
$\frac{4}{26}$	$\frac{4}{55}$	$\frac{4}{110}$
	28	28
	1550	3100)40300(13 f.
	26	$d. 3 \frac{1}{4}$
Farth. 40300 whole 3100		Rate of B's Figs.
Worth of A's Sugar.		

Decimally.

lb. d.	lb.	lb. d.	lb.
If 1 : 6.5 :: 1550		If 3100 : 10075 :: 1	
$\frac{6.5}{10075.0}$		$\frac{1}{10075.00}$	
Pence 10075.0		3100)10075.00(3.25 d.	
		$\frac{4}{F. 1.00}$	

18. A Merchant bought 13 p. 3 q. 21 lb. of Sugar, at 5 $d. \frac{1}{4}$ per lb. and selleth it again for 6 $d. \frac{1}{2}$ per lb. I demand how much he gained by the Whole?

$d.$	lb. d.	p. q. lb.
From 6 $\frac{1}{2}$	If 1 : 1 $\frac{1}{4}$:: 13 : 3 ; 21	
Take 5 $\frac{1}{4}$	$\frac{4}{5}$	$\frac{4}{55}$
Rem. 1 $\frac{1}{4}$		28
		1561

Decimally.

$d.$	
From 6.5	4 7805
Take 5.25	12 1951 $\frac{1}{4}$
Rem. 1.25	2 0)16(2 : 7
	$\text{£. } 8 : 2 : 7 \frac{1}{4}$

A a 2

lb.	d.	lb.
If 1 :	1.25 ::	1561
		1.25
12 1951		d. 1951.25
2 0) 16 (2 : 7		4
£. 8 : 2 : 7 $\frac{1}{4}$		f. 1.00

19. A Draper bought 2569 Yards of Holland for 510 *l.* 17 *s.* 8 *d.* being damnified; he is willing, in the Sale thereof, to lose 8 per Cent. I demand how he must sell it per Yard.

l.	l.	l.	l.	s.	d.
From 100	If 100 :	92 ::	510 :	17 :	8
Take 8	20		20		
Rem. 92	2000		10217		
	12		12		
	24000		122612		
			92		

$$24000 \overline{) 11280304} (470 \text{ } \text{£}.$$

$$\begin{array}{r} 304 \\ 20 \\ \hline 6080 \\ 12 \end{array}$$

l.	s.	d.
Again : 510 :	17 :	8
	8	24000) 72960 (3 $\frac{1}{25}$ d.
£. 40 87 : 01 : 4		960

$$\begin{array}{r} 20 \\ \hline \text{Sh. } 17 | 41 \\ 12 \\ \hline \text{d. } 496 \\ \hline \text{f. } 3.84 \end{array}$$

l.	s.	d.
From 510 :	17 :	8
Take 40 :	17 :	4 $\frac{1}{4}$
£. 470 :	00 :	3 $\frac{1}{4}$

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Yds. *l.* *s.* *d.* *Yd.*
If 2569 : 470 : 00 : 3 :: 1

20

9400

12

d. *s.* *d.*
2569)112800(43 is 3 : 7 $\frac{1}{2}$ per Yard.

2336

4

2569)9344(3 $\frac{1637}{2569}$ f.

1637

Decimally.

17 s. 8 d. is 212 d. that is, $\frac{212}{240}$ or $\frac{53}{60}$ of a £.

53 .883333

60|53.000000

If £. £. £.
If 100 : 92 : 510.83333

92

100)47001.266636(470.01266 £.

20

Yds. *£.* *Yd.* .253320
If 2569 : 470.012666 :: 1

12

1

d. 3.039840

2569)470.012666(.182955 £.

20

Sh. 3.659100

12

d. 7.909200

4

f. 3.636800

20. A Merchant in *Flanders* delivers 500 *l. Flemish*, to receive the same again at *London*; the Exchange at 35 *s.* 6 *d. Flemish* per Pound Sterling: How much must he receive?

<i>s.</i>	<i>d. Flem.</i>	<i>l. Eng.</i>	<i>l. Flem.</i>
If 35	: 6	. . . 1	:: 500
12			20
426			10000
			12
			426)120000(281 <i>£.</i>
			Rem. 294
			20
			426)5880(13 <i>Sb.</i>
			342
			12
			426)4104(9 <i>d.</i>
			270
			4
			426)1080(2 <i>f.</i>
			228

Decimally.

15 *s.* 6 *d. Flemish* is $\frac{186}{240}$ or $\frac{31}{40}$ of a *£. Flemish*.

	31 .775
	40 31.000
If <i>£. Flem.</i>	<i>£. Ster.</i>
1.775 . . . 1	:: 500
	1.775)500.00000(281.69 <i>£. Sterl.</i>
	Rem. 25
	20
	Sb. 13.80
	12
	d. 9.60
	4
	f. 2.40

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21. A Merchant at *London* delivered 2220*l.* to receive the same at *Paris*, at $3\frac{1}{3}$ French Crowns per Pound Sterling, What must he receive?

£.	Fr. Crowns.	
If 1 - - - -	$3\frac{1}{3}$:: 2220	
	<u>3</u>	<u>10</u>
	10	3 22200
		7400 Crowns.

Decimally.

If 1 - - - -	3.3333 :: 2220
	<u>2220</u>
	7399.9260 French Crowns.

22. A Merchant at *Cadiz* receiveth 1500 *Ducats*, to pay the same again by his Correspondent at *London*; the Exchange $58\frac{1}{2}$ Pence per Ducat; How much doth it amount to?

Ducat.	Pence.	Ducats.
If 1 - - - -	$58\frac{1}{2}$:: 1500	
	<u>4</u>	
	234	
	<u>1500</u>	
	4 351000 Farth.	
	<u>12 87750</u>	
	20 731 2 : 6	
	£. 365 : 12 : 6	

Decimally.

$58\frac{1}{2}$ d. is 234 Farthings, that is $\frac{234}{960}$ or $\frac{39}{160}$ of a £.

39	.24375
160	39.00000

If

Ducat.	£.	Ducats.
If 1 - - -	.24375	:: 1500
	<u>1500</u>	

£. 365.62500

20

Sb. 12.50000

12

d. 6.00000

23. A Merchant at *London* receiveth from his Correspondent abroad, his Account Currant, the Ballance of which is 756 *Dollars*, the Exchange at $53\frac{7}{8}$ *Pence per Dollar* : What doth it amount to ?

Dollar.	Pence.	Dollars.
If 1 - - - -	$53\frac{7}{8}$:: 756
	<u>8</u>	

431

756

8|325836

12|40729 : $\frac{1}{2}$

20|339|4 : 1

£. 169 : 14 : $1\frac{1}{2}$

Decimally.

Dollar.	Pence.	Dollars.
If 1 - - - -	53.875	:: 756
	<u>756</u>	

12|40729.500 Pence.

20|339|4 : 1

£. 169 : 4 : $1\frac{1}{2}$

24. A Carrier received 50 *Shillings* for the Carriage of 3 £ . 3 *qrs.* 21 *lb.* 137 *Miles*; I demand how much he ought to receive at that Rate, for the carrying of 2 £ . 1 *qr.* 21 *lb.* 279 *Mile.*

First, I find what he ought to receive for the Carriage of 3 £ . 3 *qrs.* 21 *lb.* 279 *Mile.*

$$\begin{array}{rcl} \text{M.} & \text{Sh.} & \text{M.} \\ \text{If } 137 & - - - 50 & :: 279 \\ & & \underline{50} \\ & & 137 \end{array}$$

$$\begin{array}{r} 113 \\ 12 \\ \hline \end{array}$$

$$137 \overline{) 13950} (101 \text{ Shil.}$$

$$\begin{array}{r} 123 \\ 4 \\ \hline \end{array}$$

$$137 \overline{) 492} (3 \frac{81}{137} f.$$

£ .	<i>qrs.</i>	<i>lb.</i>		<i>s.</i>	<i>d.</i>		£ .	<i>qrs.</i>	<i>lb.</i>
If 3	3	21	- - -	101	9 $\frac{1}{4}$::	2	1	17
	4				12			4	
	15				1221			9	
	28				4			28	
	<u>441</u>	<i>lb.</i>			<u>4887</u>	<i>Far.</i>		<u>269</u>	<i>lb.</i>

$$\begin{array}{r} 269 \\ \hline 441 \overline{) 1314603} (2980 \frac{423}{441} = \frac{47}{49} \\ 423 \quad 12 \overline{) 745} \\ 20 \overline{) 612} : 1 \end{array}$$

Answer £ . 3 : 2 : 1

B b

Decimally.

Decimally.

If $\overset{M.}{137} - - - \overset{£.}{2.5} :: 279$

$$\begin{array}{r} 2.5 \\ 137 \overline{) 697.50000} \end{array} \overset{£.}{(5.09124}$$

12

$\overset{qrs. lb.}{3} \overset{lb.}{21}$ is 105, that is $\frac{105}{112}$ of an \oplus . $\frac{105}{112} | .9375$
 $\overset{£.}{105.0000}$

$\overset{qrs. lb.}{1} \overset{lb.}{17}$ is 45, that is $\frac{45}{112}$ of an \oplus . $\frac{45}{112} | .401785$
 $\overset{£.}{45.000000}$

If $\overset{qrs. lb.}{3.9375} \dots \overset{lb.}{5.09124} :: \overset{qrs. lb.}{2.401785}$

$$\begin{array}{r} 5.09124 \\ 3.9375 \overline{) 12.22806386340} \end{array} \overset{£.}{(3.10554}$$

113 20

Skill. 2.11080

12

d. 1.32960

25. If 40 Acres of Grass can be mowed by 9 Men in 7 Days, How many Acres may be mowed by 24 Men in 28 Days?

First, I find how many Acres can be mowed by 24 Men in 7 Days.

$\overset{Men.}{9} \dots \overset{Acres.}{40} :: \overset{Men.}{24}$

$$\begin{array}{r} 40 \\ 9 \overline{) 960} \end{array} (106 \text{ Acres.}$$

6

$$\begin{array}{r} 4 \\ 9 \overline{) 24} \end{array} (2 \text{ Rood.}$$

6

$$\begin{array}{r} 40 \text{ Pole.} \\ 9 \overline{) 240} \end{array} (26 \frac{2}{3} = \frac{80}{3}$$

6

If

The Golden Rule.

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Days. Acres. R. P. Days.
If 7 - - - 106 2 26 :: 28

4		40 68264
426		4 1706 : 24
40		
17066 Poles.	Acres 426 : 2 : 24	
28		

7) 477848 (68264 Poles.

Decimally.

Men. Acres. Men.
If 9 . . 40 :: 24

40	Acres.
9) 960.00000	(106.66666
6	

Days. Acres. Days.
If 7 . . 106.66666 :: 28

28	Acres.
7) 2986.66648	(426.66664

4
Rood 2.66656

40
Pole 26.66240

26. If 48 *Pioneers* in 12 *Days*, can cast a Trench 24 *Yards* long : In how many *Days* will 162 *Pioneers* cast a Trench 108 *Yards*.

First, I find in how many *Days* 162 *Pioneers* will cast a Trench 24 *Yards* long.

Pio. da. pio. Yds. da. ho. min. yds.

If 48 . 12 :: 162. If 24 . 3 : 13 : 20 :: 108.

12
162) 576 (3 days.
90

24
85
60

24
162) 2160 13 hours.

5120 min.
108

54
162) 2160 13 hours.

24) 552960 (23040 min.

60
162) 3240 (20 min.

24) 384
Answer 16 days.

*The Golden Rule.**Decimally.**Pio. Days. Pio.*

If 48 . . 12 :: 162

12

162)576.000000(3.555555 *Days.**Yds.**Days.**Yds.*

If 24 . . 3.555555 :: 108

108

24)383999940(15.999997 *Days.*

12

27. Four Merchants, *A*, *B*, *C*, and *D*, make a Stock. *A* put in 227 *l.* *B* 349 *l.* *C* 115 *l.* and *D* 439 *l.* In trading they gained 428 *l.* I demand each Merchant's Share of the Gain?

All Questions of this Kind are answered by so many several Operations in the *Rule of Three*, as there are *Partners* in the Stock.

R U L E.

As the total Stock, is to the Gain or Loss, so is each Man's Stock, to his Share of the Gain or Loss.

						£.
<i>A</i> 's Stock	-	-	-	-	-	227
<i>B</i> 's	-	-	-	-	-	349
<i>C</i> 's	-	-	-	-	-	115
<i>D</i> 's	-	-	-	-	-	439
						<hr/>
Total	-	-	-	-	-	1130

If

The Golden Rule.

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$$\begin{array}{r} \text{If } 1130 \dots 428 :: 349 \\ \hline 1130)149372(132 \text{ £.} \\ \hline 212 \\ \hline 20 \end{array}$$

$$113|0)424|0(3 \text{ Sh.}$$

85

12

$$113)1020(9 \text{ d.}$$

3

4

$$113)12(0 \frac{12}{113} \text{ f.}$$

$$\begin{array}{r} \text{If } 1130 \dots 428 :: 227 \\ \hline 1130)97156(85 \text{ £.} \\ \hline 1106 \\ \hline 20 \end{array}$$

$$113|0)2212|0(19 \text{ sh.}$$

65

12

$$113)780(6 \text{ d.}$$

102

4

$$113)408(3 \frac{69}{113} \text{ f.}$$

69

$$\begin{array}{r} \text{If } 1130 \dots 428 :: 115 \\ \hline 1130)4922|0(43 \text{ £.} \\ \hline 63 \\ \hline 20 \end{array}$$

$$113|0)4922|0(43 \text{ £.}$$

63

20

$$113)1260(11 \text{ Sh.}$$

17

12

$$113)204(1 \text{ d.}$$

91

4

$$113(364(3 \frac{25}{113} \text{ f.}$$

25

$$\begin{array}{r} \text{If } 1130 \dots 428 :: 439 \\ \hline 1130)187892(166 \text{ £.} \\ \hline 312 \\ \hline 20 \end{array}$$

$$113|0)187892(166 \text{ £.}$$

312

20

$$113|0)624|0(5 \text{ Sh.}$$

59

12

$$113)708(6 \text{ d.}$$

30

4

$$113)120(1 \frac{7}{113} \text{ f.}$$

7

	<i>l.</i>	<i>s.</i>	<i>d.</i>	
A's Part of the Gain	85	19	6 $\frac{1}{2}$	- - 69
B's - - - - -	132	3	9	- - 12
C's - - - - -	43	11	1 $\frac{1}{2}$	- - 25
D's - - - - -	166	5	6 $\frac{1}{2}$	- - 7
Proof - -	£. 427	19	11 $\frac{1}{2}$	113

Decimally.

If the *Gain* be divided by the Sum of all the Stocks, the *Quotient* becomes a *Multiplier*, or *Multiplicand*, to each Man's Stock, and will determine the Gain.

$$1130 \overline{) 428.000000} (.378761$$

70

$$\begin{array}{r} .378761 \\ \text{C's Stock } 115 \\ \hline \end{array}$$

$$\begin{array}{r} \text{£. } 43.557515 \\ 20 \\ \hline \end{array}$$

$$\begin{array}{r} \text{Sh. } 11.150300 \\ 12 \\ \hline \end{array}$$

$$\begin{array}{r} \text{d. } 1.803600 \\ 4 \\ \hline \end{array}$$

$$\text{f. } 3.214400$$

$$\begin{array}{r} .378761 \\ \text{B's Stock } 349 \\ \hline \end{array}$$

$$\begin{array}{r} \text{£. } 132.187589 \\ 20 \\ \hline \end{array}$$

$$\begin{array}{r} \text{Sh. } 3.751780 \\ 12 \\ \hline \end{array}$$

$$\text{d. } 9.021360$$

$$\begin{array}{r} .378761 \\ \text{A's Stock } 227 \\ \hline \end{array}$$

$$\begin{array}{r} \text{£. } 85.978747 \\ 20 \\ \hline \end{array}$$

$$\begin{array}{r} \text{Sh. } 19.574940 \\ 12 \\ \hline \end{array}$$

$$\begin{array}{r} \text{d. } 6.899280 \\ 4 \\ \hline \end{array}$$

$$\text{f. } 3.597120$$

$$\begin{array}{r} .378761 \\ \text{D's Stock } 439 \\ \hline \end{array}$$

$$\begin{array}{r} \text{£. } 166.276079 \\ 20 \\ \hline \end{array}$$

$$\begin{array}{r} \text{Sh. } 5.521580 \\ 12 \\ \hline \end{array}$$

$$\begin{array}{r} \text{d. } 6.258960 \\ 4 \\ \hline \end{array}$$

$$\text{f. } 1.035840$$

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28. Two Men, *A* and *B*, join their Stocks.
A had 344 *l.* 15 *s.* 9 *d.* *B* 213 *l.* 5 *s.* They
gained in trading 578 *l.* 14 *s.* 9 *d.* I demand each
Man's Share of the Gain?

	<i>l.</i>	<i>s.</i>	<i>d.</i>
The Stock of <i>A</i> - - -	344	15	9
The Stock of <i>B</i> - - -	213	5	0
Total - - -	558	0	9

<i>l.</i>	<i>s.</i>	<i>d.</i>	<i>l.</i>	<i>s.</i>	<i>d.</i>	<i>l.</i>	<i>s.</i>	<i>d.</i>
If 558 : 0 : 9	..	578 : 14 : 9	::	344 : 15 : 9				
20		20		20				
11160		11574		6895				
12		12		12				
133929 <i>d.</i>		138897 <i>d.</i>		82749 <i>d.</i>				
		82749						

133929)11493587853(85818 *d.*

68931

4

12|85818

133929)275724(2 *f.*

2|0)715(1 : 6

7866

£. 357 : 11 : 6½

Pence.

Pence.

l.

s.

If 133929

.. 138897

::

213

:: 05

51180

20

133929)7108748460(53078

4265

64998

12

4

51180 *d.*

133929)259992(1

12|53078

126063

2|0)442(3 : 2

221 : 3 : 2¼

Decimally.

Decimally.

s. d.

15 : 9 is 189 d. that is, $\frac{189}{240}$ or $\frac{63}{80}$ of a £.5 : 0 is $\frac{5}{20}$ or $\frac{1}{4}$ of a £.14 : 9 is 177 d. that is, $\frac{177}{240}$ or $\frac{59}{80}$ of a £.

$$\begin{array}{r} 63 \quad .7875 \\ \hline 80 \overline{) 63.0000} \end{array} \quad \begin{array}{r} 1 \quad .25 \\ \hline 4 \overline{) 1.00} \end{array} \quad \begin{array}{r} 59 \quad .7375 \\ \hline 80 \overline{) 59.0000} \end{array}$$

A's Stock 344.7875

B's - - - 213.25

1.037094

Total 558.0375 578.7375000000

1569750

344.7875	1.037094
1.037094	213.25
<hr/>	<hr/>
£. 357.5770475250	£. 221.16029550
20	20
<hr/>	<hr/>
Sb. 11.5409505000	Sb. 3.20591000
12	12
<hr/>	<hr/>
d. 6.4914060000	d. 2.47092000
4	4
<hr/>	<hr/>
f. 1.9656240000	f. 1.88368000

29. Three Merchants accompany. A had in Stock 230 l. 7 Months; B 290 l. 10 Months; and C 326 l. 4 Months. They gained together 219 l. 10 s. What was each Man's Share of the Gain?

Note: When *Time* is annexed to the *Stock*, then each Man's *Stock* must be multiplied by its *Time*, and proceed as afore.

A's

The Golden Rule.

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A's Stock - - 230 £. 7 Months.

7

1610

B's Stock - - 290 £. 10 Months.

1610

2900

10

1304

2900

Total 5814

C's Stock - - 326 £. 4 Months.

4

1304

If 5814 . . l. 219 : 10 :: 1610

20

Sb. 4390

1610

5814)7067900(121|5 Sb.

3890 £. 60 : 15 : 8 A's Share.

12

5814)46680(8 d.

168

4

672

If 5814 . . s. 4390 :: 2900

2900

5814)12731000(218|9 Sb.

4154 £. 109 : 9 8 ½ B's Share.

12

5814)49848(8 d.

3336

4

5814)13344(2 f.

1716

C q

If

Sb.

If 5814 . . 4390 :: 1304

1304

5814)5724560(9814 Sb.

3584 l. 49 : 4 : 7 $\frac{1}{4}$ C's Share.

12

5814)43008(7 d.

2310

4

5814)9240(1 f.

3426

	<i>l.</i>	<i>s.</i>	<i>d.</i>
A's Share of the Gain - - -	60	15	8
B's - - - - -	109	9	8 $\frac{1}{2}$
C's - - - - -	49	4	7 $\frac{1}{4}$
Proof - - -	£. 219	9	11 $\frac{1}{2}$

Decimally.

5814)219.500000(.037753

4058

.037753 1610	.037753 2900	.037753 1304
£. 60.782330 20	£. 109.483700 20	£. 49.229912 20
Sb. 15.646600 12	Sb. 9.674000 12	Sb. 4.598240 12
d. 7.759200 4	d. 8.088000	d. 7.178880
f. 3.036800		

30. Three

30. Three Merchants accompany. *A* put in *January* 1, 137 *l.* 10 *s.* and *April* 9, he put in more 36 *l.* 14 *s.* 6 *d.* *B* put in *January* 1, 257 *l.* and takes out *May* 15, 119 *l.* 14 *s.* *C* put in *January* 1, 119 *l.* 18 *s.* 9 *d.* *March* 19, he put in more 227 *l.* and takes out *September* 27, 195 *l.* 18 *s.* 3 *d.* At the Year's End they cast up their Stock, and found they had lost by trading 359 *l.* 18 *s.* 10 *d.* I demand each Man's Share of the Loss.

	<i>l.</i>	<i>s.</i>	<i>days.</i>		<i>l.</i>	<i>s.</i>	<i>d.</i>	<i>days</i>
<i>A</i> had in Stock	137	10	99	and	174	4	6	266
137	10		20			20		
36	14	6						<i>Jan.</i> 31
			2750		3484			<i>Feb.</i> 28
174	4	6	12			12		<i>Mar.</i> 31
			33000		41814			<i>April</i> 9
			99		266			99
			3267000		11122524			365
			11122524					266

14389524 *A's* Stock and Time.

	<i>l.</i>	<i>s.</i>	<i>days.</i>		<i>l.</i>	<i>s.</i>	<i>days.</i>
<i>B</i> had in Stock	257	135		and	137	6	230
257			20			20	
119	14						<i>Jan.</i> 31
			5140		2746		<i>Feb.</i> 28
137	6		12			12	<i>Mar.</i> 31
			61680		32952		<i>Apr.</i> 9
			135		230		<i>May</i> 15
			8326800		7578960		135
			7578960				365

15905760 *B's* Stock and Time. 230

198

The Golden Rule.

	<i>l.</i>	<i>s.</i>	<i>d.</i>	<i>days.</i>	<i>l.</i>	<i>s.</i>	<i>d.</i>	<i>days.</i>
C had in Stock	119	18	9	78	and	346	18	9, 192
119 18 9			20				20	
227 00 0				2398		6938		Jan. 31
346 18 9			12			12		Feb. 28
195 18 3				28785		83265		Mar. 19
151 00 6			78			192		78
				2245230		15986880		

	<i>l.</i>	<i>s.</i>	<i>d.</i>	<i>days.</i>		<i>l.</i>	<i>s.</i>	<i>d.</i>
Likewise C had in Stock	151	00	6	95				
Mar. 12			20					
April 30				3020				
May 31			12		Loss	359	18	10
June 30						20		
July 31				36246		7198		
Sept. 27			95			12		
192				3443370				
78				15986880		86386	d.	
270				2245230				
365				21675480	C's Stock and Time			
95				15905760	B's Stock and Time			
				14389524	A's Stock and Time			

Total - - 51970764

Pence.

If 51970764 ... 86386 :: 14389524
86386

51970764)1243053420264(23918 d.

16686912 199|3 : 2

4

l. 99:13:2½

51970764)66747648(1 f.

14776884

If

The Golden Rule.

199

Pence.

If 51970764... 86386 :: 15905760
86386

51970764)1374034983360(26438 d.

31924728 220|3 : 2

4 110 : 3 : 2 1/2

51970764)127698912(2 f.

If 51970764... 86386 : 21675480
86386

51970764)1872458015280(36029 d.

3359124 300|2 : 5

4 l. 150 : 2 : 5
13436496

	<i>l.</i>	<i>s.</i>	<i>d.</i>
A's Share of the Loss - -	99	13	2 1/4
B's - - - - -	110	3	2 1/2
C's - - - - -	150	2	5
Proof - - -	359	18	9 1/4

s. d. Decimally.

10 : 0 is $\frac{10}{20}$ or $\frac{1}{2}$ of a £.

14 : 6 is 174 d. that is, $\frac{174}{240}$ or $\frac{29}{40}$ of a £.

14 : 0 is $\frac{14}{20}$ or $\frac{7}{10}$ of a £.

18 : 9 is 225 d. that is, $\frac{225}{240}$ or $\frac{75}{80}$ of a £.

18 : 3 is 219 d. that is, $\frac{219}{240}$ or $\frac{73}{80}$ of a £.

18 : 10 is 226 d. that is, $\frac{226}{240}$ or $\frac{113}{120}$ of a £.

$\frac{1}{2}$.5 $\frac{29}{40}$.725 $\frac{75}{80}$.9375
21.0 40|29.000 80|75.0000

$\frac{73}{80}$.9125 $\frac{113}{120}$.941666666
80|73.0000 120|113.00000000

80

The Golden Rule.

	£.	Days.	£.	Days.
A had in Stock	137.5	: 99	and 174.225	: 266
		<u>99</u>		<u>266</u>
137.5		13612.5		46343.850
<u>36.725</u>		<u>46343.85</u>		
174.225		59956.35	A's Stock and Time.	

	£.	Days.	£.	Days.
B had in Stock	257	: 135	and 137.3	: 230
		<u>135</u>		<u>230</u>
257		34695		31579.0
<u>119.7</u>		<u>31579</u>		
137.3		66274	B's Stock and Time.	

	£.	Days.	£.	Days.
C had in Stock	119.9375	: 78	and 346.2375	: 192
		<u>78</u>		<u>192</u>
	9355.1250		66612.0000	

	£.	Days.
Likewise C had in Stock	151.025	: 95
		<u>95</u>
119.9375		14347.375
<u>227</u>		<u>9355.125</u>
346.9375		66612.
<u>195.9125</u>		<u>90314.500</u>
151.0250		

59956.35 A's Stock and Time.

66274. B's Stock and Time.

90314.5 C's Stock and Time.

216544.85) 359.941666666 (.0016622

The Golden Rule.

201

<u>.0016622</u>	<u>.0016622</u>
59956.35	66274
<u>£. 99.659444970</u>	<u>£. 110.1606428</u>
20	20
<u>Sb. 13.188899400</u>	<u>Sb. 3.2128560</u>
12	12
<u>d. 2.266792800</u>	<u>d. 2.5542720</u>
4	4
<u>f. 1.067171200</u>	<u>f. 2.2170880</u>

<u>.0016622</u>
90314.5
<u>£. 150.12076190</u>
20
<u>Sb. 2.41523800</u>
12
<u>d. 4.98285600</u>
4
<u>f. 3.93142400</u>

31. At 5 per Cent. per Ann. What is the Interest of 3752 l. forborn 9 Years?

First, I find what is the Interest of it for One Year.

If $\frac{£.}{100} = \frac{£.}{5} :: \frac{£.}{3752}$

	<u>5</u>	
100)	18760	(187 £.
	<u>60</u>	
	20	
	<u>100)</u>	1200 (12 Sb.

11

The Golden Rule

year.	l.	s.	years.
If 1 - - -	187	: 12	: 9

20

3752

9

2|0)3376|8 *Shillings.*

£. 1688 : 8 The Interest of 9 Years.

Decimally.

If the Sum of Money be multiplied by the *Rate of Interest*, made so many *Decimal* Parts of 100, the *Product* will be the *Interest* of *One Year*, which multiplied by the *Time*, gives the *Interest* for the *Time* required.

£.

3752

.05

£. 187.60 The Interest of 1 Year.

9

£. 1688.40 The Interest of 9 Years.

20

Sh. 8.00

32. At 6 per Cent. per Ann. What is the Interest of 354*l.* 14*s.* 7*d.* forborn 7 Years, 25 Days.

l. *l.* *l.* *s.* *d.*
If 100 - - - 6 :: 354 : 14 : 7

20	20
2000	7094
12	12
24000	85135
	6

2400|0) 51081|0 (21*l.*

681

20

240|0) 1362|0 (5*s.*

162

12

d.

240) 1944 (8 $\frac{24}{240} = \frac{1}{10}$

24

days. *l.* *s.* *d.* *years.* *days.*
If 365 - - 21 : 5 : 8 :: 7 : 25

20	365
425	2580
12	5108

5108 365) 13178640 (36105 Pence,

315 300|8 : 9

4

150 : 8 : 9

365) 1260 (3 $\frac{165}{365} = \frac{33}{73}$

165

D d

Decimally.

Decimally.

14s. 7d. is 175 Pence, that is, $\frac{275}{240}$ or $\frac{25}{24}$ of a £.

25 Days is $\frac{25}{365}$ or $\frac{5}{73}$ of a Year.

$$\begin{array}{r} 35 \quad .729166 \\ 48 \overline{) 35.000000} \end{array}$$

$$\begin{array}{r} 5 \quad .068493 \\ 73 \overline{) 5.000000} \end{array}$$

$$\begin{array}{r} \text{£.} \\ 354.729166 \\ .06 \end{array}$$

$$\begin{array}{r} \text{Interest of 1 Year :} \quad 21.28374996 \\ \quad \quad \quad 7.068493 \end{array}$$

$$\begin{array}{r} \text{£. 150.44403760601028} \\ 20 \end{array}$$

$$\begin{array}{r} \text{Sh. 8.880752} \\ 12 \end{array}$$

$$\begin{array}{r} d. 10.569024 \\ 4 \end{array}$$

$$\begin{array}{r} f. 2.276096 \end{array}$$

The Excess in this Operation proceeds from the $\frac{1}{10}$ of a Penny, which the former takes not in.

If $\frac{1}{4}$ $\frac{1}{2}$ or $\frac{3}{4}$ be annexed to the Rate of Interest, then 25, 5, or 75, must also be annexed in the *Decimal* Operation. That is, If the Rate of Interest be $5 \frac{1}{4}$, then the Money must be multiplied by .0525 for the Interest of 1 Year. If it should be $5 \frac{1}{2}$, or $5 \frac{3}{4}$, then by .055, or .0575; understand the like for any other Rate of Interest.

Some Bankers compute the Interest of Money by the following Rule, viz. Multiply the Sum of Money by the Number of Days it is out at Interest, for a Dividend; and then divide 36500 (the Days in a Year multiplied by 100) by the Rate of Interest for a Divisor.

Quest.

The Golden Rule.

203

	<i>Days.</i>		
<i>Quest. 31.</i>	365		3752 Principal.
	9		3285 Days at Int.
5 36500	3285	730 0)1232532 0	(16887.
<u>7300</u>			
		292	
		20	
		73 0)584 0	(8s.

		<i>l.</i>	<i>s.</i>	<i>d.</i>
<i>Quest. 32.</i>		354	14	7 Principal.
		20		
<i>yea. days.</i>				
7 : 25		7094		
365	6 36500	12		
<u>2580</u>	6083	85135		<i>Pence.</i>
		2580		
	6083	219648300		(36108 <i>Pence.</i>
		3336	300 9	
				£. 150 : 09

The Excess in this Operation proceeds from the *Divisor*, which, if it had been carry'd on with some *Decimal* Parts annexed thereunto, would have been 6083.3333.

33. At 6 *per Cent. per Annum*, What comes 282*l.* 15*s.* 6*d.* to, forborn *Five Years*, allowing *Interest* upon *Interest* ?

R U L E.

First find the *Interest* of the Money for the *first* Year, and add it to the *Principal* ; and find the *Interest* thereof for the next Year ; and so proceed to the End of the Time required,

Do 2

If

The Golden Rule.

l. l. l. s. d.
If 100 . . 6 :: 282 : 15 : 6

20 20 20

2000 120 5655

12 12 12

24000 1440 67866

4 4 4

96000 5760 271464
Mult. 5760

96000)1563632640(16287 f.

80640

To 271464

Add 16287

287751

If 96000 . . 5760 :: 287751
5760

96000)1657445760(17265 f.

5760

To 287751

Add 17265

305016

If 96000 . . 5760 :: 305016

5760

96000)1756892160(18300 f.

92160

To 305016

Add 18300

323316

The Golden Rule.

207

If 96000 . . 5760 : : 323316
5760

96000)1862300160(19398 f.
92160

To 323316
 Add 19398
342714

If 96000 . . 5760 : : 342714
5760

96000)1974032640(20562
80640

To 342714
 Add 20562

41363276

12190819

210756(8 : 3

Answer £. 378 : 8 : 3

This Method of working *Interest* upon *Interest* will be found very troublesome and tedious, especially when required for many Years; I will therefore shew you a Method much more compendious and easy, that will save all the Operations in the *Golden Rule*.

R U L E.

Reduce the *Principal* Money into *Farthings*, and multiply it by the *Rate* of Interest; cut off two Figures in the *Product*; the rest are the *Interest* of the *first* Year in *Farthings*; and so proceed for every Year required,

The Golden Rule.

	<i>l.</i>	<i>s.</i>	<i>d.</i>
	282	: 15	: 6
	<u>20</u>		
	5655		
	<u>12</u>		
	67866		
	<u>4</u>		
1st Year's Interest	271464	16287	84
2d Year's Interest	287751	17265	06
3d Year's Interest	305016	18300	96
4th Year's Interest	323316	19398	96
5th Year's Interest	342714	20562	84
	<u>4</u>	363276	
		<u>12</u>	90819
		210	756(8 : 3
			£. 378 : 8 : 3 as afore.

Decimally.

Decimally.

15 s. 6 d. is 186 d. that is, $\frac{186}{240}$ or $\frac{31}{40}$ of a £.

$$\begin{array}{r} 31 \quad .775 \\ 40 \overline{) 31.000} \end{array} \quad \begin{array}{r} \text{£.} \\ 282.775 \\ .06 \end{array}$$

Interest of the 1st Year 16.96650
Add 282.775

$$\begin{array}{r} 299.74150 \\ .06 \end{array}$$

Interest of the 2d Year 17.9844900
Add 299.74150

$$\begin{array}{r} 317.7259900 \\ .06 \end{array}$$

Interest of the 3d Year 19.063559400
Add 317.72599

$$\begin{array}{r} 336.789549400 \\ .06 \end{array}$$

Interest of the 4th Year 20.20737296400
Add 336.7895494

$$\begin{array}{r} 356.99692236400 \\ .06 \end{array}$$

Interest of the 5th Year 21.4198153418400
Add 356.99692236400

$$\begin{array}{r} \text{£. } 378.4167377058400 \\ 20 \end{array}$$

$$\begin{array}{r} \text{Sh. } 8.3347541168000 \\ 12 \end{array}$$

$$\begin{array}{r} \text{d. } 4.0170494016000 \end{array}$$

This

This Operation also may be very much contracted, if in multiplying by the *Rate of Interest* made so many *Decimal Parts* of 100, you cut off two Figures in the *Product*.

Note : Three Cyphers are annexed to the *Decimal Parts* of the *Principal*, that so six *Decimal Places* may be retained throughout the Operation.

£.		
282.775000		
16.966500	00	
299.741500		
17.984490	00	
317.725990		
19.063559	40	
336.789549		
20.207372	94	
356.996921		
21.419815	26	
£. 378.416736		
	20	
Sb. 8.334720		
	12	
d. 4.016640		

These *Decimal Operations* takes in the several *Remainders*, which the *Vulgar* does not ; and from hence comes the Difference between them.

34. At 5 per Cent. per Annum, what will 278 l. 13 s. 4 d. amount to, forborn 11 Years, allowing Interest upon Interest ?

l. s. d.

Decimally.

278:13:4

13 s. 4 d. is $\frac{160}{140}$ or $\frac{2}{5}$ of a £.

20

2 .666666

5573

3|2.000000

12

2

66880

£. 278.666666

4

13.933333 3-

267520

1st Year 292.599999

13376 00

14.629999 95

1st Year 280896

2d — 307.229998

14044 80

15.361499 90

2d — 294940

3d — 322.591497

14747 00

16.129574 85

3d — 309687

4th — 338.721071

15484 35

16.936053 55

4th — 325171

5th — 355.657124

16258 55

17.782856 20

5th — 341429

6th — 373.439980

17071 45

18.671999 00

6th — 358500

7th — 392.111979

17925 00

19.605598 95

7th — 376425

8th — 411.717577

18821 25

20.585878 85

8th — 395246

9th — 432.303455

19762 30

21.615172 75

9th — 415008

10th — 453.918627

20750 40

22.695931 35

10th — 435758

11th £. 476.614558

21787 90

20

11th — 457545

Sh. 12.291160

12

12|114386

d. 3.493920

2|0)953(2 : 2

4

£. 476:12:2

f. 1.975680

E e

Tho' this Kind of *Interest* is counted very unlawful by many, yet it is easy to make it appear, that it is far more reasonable, than that which is called *Simple Interest*; for what is more unjust, than after the Rate of 6 per Cent. to take 3 *l.* for half a Year, or 30 *s.* for a *Quarter*, when 3 *l.* will give in half a Year 1 *s.* 9 *d.* $\frac{1}{2}$?

But in *Leases*, *Reversions*, *Annuities*, &c. it is far more unreasonable, as divers can by Experience witness, who compounding according to the Rules of *Simple Interest*, have paid more for their *Tenements*, *Annuities*, *Leases in Reversion*, &c. than they have been really worth, as will appear in the three following Questions.

35. What is 289 *l.* 15 *s.* 6 *d.* payable at the End of 7 Years, worth in ready Money, discounting after the Rate of 6 per Cent.

First: Find the *Interest* of 100 *l.* for the Time mentioned.

Then, as 100 *l.* with the *Interest*, is to 100 *l.* so is the *Debt* to be paid, to its Worth in ready Money.

6 *l.* the *Interest* of 100 *l.* for a Year.

$\frac{7}{42}$ *l.* the *Interest* of 100 *l.* for 7 Years.

l. *l.* *l.* *s.* *d.*

If 142 . . 100 :: 289 : 15 : 6

$$\begin{array}{r} 20 \\ \hline 2840 \\ 12 \\ \hline 34080 \end{array}$$

$$\begin{array}{r} 20 \\ \hline 5785 \\ 12 \\ \hline 69426 \end{array}$$

$$\begin{array}{r} 100 \\ \hline 3408 \overline{) 694260} 10 (204 \text{ £} \\ 228 \\ \hline \end{array}$$

$$\begin{array}{r} 20 \\ \hline 3408 \overline{) 4560} (1 \text{ Sh.} \\ 1152 \\ \hline \end{array}$$

$$\begin{array}{r} 12 \\ \hline 3408 \overline{) 13824} (4 \text{ d.} \end{array}$$

Decimally.

15 s. 6 d. is 186 d. that is, $\frac{186}{240}$ or $\frac{31}{40}$ of a £.

$$\begin{array}{r} 31 \quad .775 \\ 40 \overline{) 31.000} \end{array}$$

If £. - - £. : : £.
If 142 - - 100 : : 289.775

$$\begin{array}{r} 100 \\ 142 \overline{) 28977.500} (204.0669 \text{ £.} \\ \underline{2840} \\ 577 \\ \underline{568} \\ 97 \\ \underline{96} \\ 100 \\ \underline{100} \\ 0 \end{array}$$

20
Sb. 1.3380
12
d. 4.0560

To find the Worth of the said Sum of Money, allowing *Interest* upon *Interest*.

100 £.	Decimally.
20	100.000000
2000	1 Yea. Int. 6.000000 00
12	106.000000
24000	2d - - - 6.360000 00
4	112.360000
96000	3d - - - 6.741600 00
1 Yea. Int. 5760 00	119.101600
101760	4th - - - 7.146096 00
2d - - - 6105 00	126.247696
107865	5th - - - 7.574861 76
3d - - - 6471 90	133.822557
114336	6th - - - 8.029353 42
4th - - - 6860 16	141.851910
121196	7th - - - 8.511114 60
5th - - - 7271 76	£. 150.363024
128467	
6th - - - 7708 02	
136175	
7th - - - 8170 50	
144345 f.	

The Golden Rule.

Farth. Farth. l. s. d.
 If 144345 . . 96000 :: 289 : 15 : 6

4 185012	20
12 46253	5795
3854 : 5	12
£. 192 : 14 : 5	69546
	4
	278184
	96000

144345)26705664000(185012 f.
Decimally.

£. £. £.
 If 150.363 - 100 :: 289.775
100

150.363)28977.50000000(192.71695 £.
20

8b. 14.33900
12

d. 4.06800
 l. s. d.

The Purchase by Simple Interest 204 01 4
 By Compound Interest - - 192 14 4

The Difference - - - 11 07 0

Thus you see, how much those wrong themselves, who purchase, and compound by the Rules of *Simple Interest*.

36. What is 728 l. 16 s. 8 d. payable at the End of $5\frac{1}{2}$ Years, worth in *ready Money*, compounding after the Rate of 5 per Cent.

5 l. the Interest of 100 l. for 1 Year.

5

25 l. the Interest of 100 l. for 5 Years.

2 l. --- 10 s. the Interest of half a Year.

27 l. 10 s. the Interest of $5\frac{1}{2}$ Years.

The Golden Rule.

215

l. s. l. l. s. d.
If 127 : 10 . . 100 :: 728 ; 16 : 8

20
2550
12
30600

20
14576
12
174920
100
17492000

306|00)174920|00(571 £.

194
20

306)3880(12 s.

208

12

306)2496(8 d.

48

Years.

Decimally.

5.5

.5

27.5 the Interest of 100 l, for 5 $\frac{1}{2}$ Years.

16 s. 8 d. is 200 d. that is, $\frac{200}{240}$ or $\frac{5}{6}$ of a £.

5
6|5.0000000
.8333333

£. £. £.
If 127.5 - - 100 - - 728.8333333

100
72883.3333300

127.5)72883.3333300(571.633986

1150

20

Sb. 12.679720

12

d. 8.156640

*The Golden Rule.
Interest upon Interest.*

<i>£.</i>	<i>Decimally.</i>
100	<i>£.</i> 100.000000
20	5.000000 00
<u>200120</u>	<u>105.000000</u>
24000	5.250000 00
<u>4</u>	<u>110.250000</u>
96000	5.512500 00
1st Year's Int. 4800 00	<u>115.762500</u>
<u>100800</u>	5.788125 00
2d - - - - - 5040 00	<u>121.550625</u>
<u>105840</u>	6.077531 25
3d - - - - - 5292 00	<u>127.628156</u>
<u>111132</u>	2)6.381407 80
4th - - - - - 5556 60	<u>3.190703</u>
<u>116688</u>	<u>127.628156</u>
5th - - - - - 5834 40	<u>130.818859</u>
<u>122522</u>	
6th - - - - - 2)6126 10	
<u>3063</u>	
<u>122522</u>	
125585	

Far. Far. l. s. d.
If 125585 . . 96000 :: 728 : 16 : 8

20
14576
12
174920
4
699680
96000

125585)67169280000(534851 f.

Rem. 17165 133712 : 3

1114|2 : 8

£. 557 : 2 : 8 $\frac{1}{2}$

The Golden Rule.

217

If $\overset{\text{£.}}{130.818859} - - \overset{\text{£.}}{100} : : \overset{\text{£.}}{728.833333333333}$
100

$\overset{\text{£.}}{130.818859} 72883.333333333300 \overset{\text{£.}}{(557.131708}$

Rem. $\overset{\text{£.}}{94315544}$ 20

Sb. $\overset{\text{£.}}{2.634160}$
12

d. $\overset{\text{£.}}{7.609920}$
4

f. $\overset{\text{£.}}{2.439680}$

	<i>l.</i>	<i>s.</i>	<i>d.</i>
Thus you see the Purchase by			
Simple Interest is - - - - }	571	12	8
By Compound Interest - - -	557	2	8
The Difference - - -	$\overset{\text{£.}}{14}$	10	0

Note: To answer this Question exactly, a mean Proportion, should have been found between the *Interest* and *Principal* of the 5th Year, and the *Interest* and *Principal* of the 6th Year, for the odd half Year; but because that is not to be performed, without the *Square Root*, therefore I take half the *Interest* of the 6th Year, and add it to the *Principal* of the 5th Year; which tho' it be not exact, according to the Rules of *Compound Interest*, yet its Difference is less than a *Shilling*, therefore inconsiderable, and may in Practice be used.

37. How much ready Money will purchase an Annuity of 59 *l.* to continue 5 Years, after the Rate of 6 per Cent. Discount?

If

If £. 106 . . 100 :: 59 If £. 112 . . 100 :: 59

$$\begin{array}{r} 59 \\ \hline 106 \overline{) 5900} (55 l. \end{array}$$

$$\begin{array}{r} 70 \\ 20 \\ \hline \end{array}$$

$$106 \overline{) 1400} (13 s.$$

$$\begin{array}{r} 22 \\ 12 \\ \hline \end{array}$$

$$106 \overline{) 264} (2 d.$$

$$\begin{array}{r} 52 \\ 4 \\ \hline \end{array}$$

$$106 \overline{) 208} (1 f.$$

$$\begin{array}{r} 102 \\ \hline \end{array}$$

If £. 118 . . 100 :: 59 If £. 124 . . 100 :: 59

$$\begin{array}{r} 59 \\ \hline 118 \overline{) 5900} (50 l. \\ 0 \end{array}$$

If £. 130 . . 100 :: 59

$$\begin{array}{r} 59 \\ \hline 13 \overline{) 590} (45 l. \end{array}$$

$$\begin{array}{r} 5 \\ 20 \\ \hline \end{array}$$

$$13 \overline{) 100} (7 s.$$

$$\begin{array}{r} 9 \\ 12 \\ \hline \end{array}$$

$$13 \overline{) 108} (8 d.$$

$$\begin{array}{r} 4 \\ 4 \\ \hline \end{array}$$

$$13 \overline{) 16} (1 f.$$

$$\begin{array}{r} 3 \\ \hline \end{array}$$

$$\begin{array}{r} 59 \\ \hline 112 \overline{) 5900} (52 l. \end{array}$$

$$\begin{array}{r} 76 \\ 20 \\ \hline \end{array}$$

$$112 \overline{) 1520} (13 s.$$

$$\begin{array}{r} 64 \\ 12 \\ \hline \end{array}$$

$$112 \overline{) 768} (6 d.$$

$$\begin{array}{r} 96 \\ 4 \\ \hline \end{array}$$

$$112 \overline{) 384} (3 f.$$

$$\begin{array}{r} 48 \\ \hline \end{array}$$

If £. 124 . . 100 :: 59

$$\begin{array}{r} 59 \\ \hline 124 \overline{) 5900} (47 l. \end{array}$$

$$\begin{array}{r} 72 \\ 20 \\ \hline \end{array}$$

$$124 \overline{) 1440} (11 s.$$

$$\begin{array}{r} 76 \\ 12 \\ \hline \end{array}$$

$$124 \overline{) 912} (7 d.$$

$$\begin{array}{r} 44 \\ 4 \\ \hline \end{array}$$

$$124 \overline{) 176} (1 f.$$

$$\begin{array}{r} 52 \\ \hline \end{array}$$

The Golden Rule.

219

The first Year's Purchase is	- - -	55	13	2	1
The second Year's	- - -	52	13	6	3
The third	- - -	50	-	-	-
The fourth	- - -	47	11	7	1
The fifth	- - -	45	7	8	1
<hr/>					
The Purchase of the whole	- -	£. 251	06	0	2

Decimally.

$$\begin{array}{r} \text{If } \overset{\text{£.}}{106} \dots \overset{\text{£.}}{100} :: \overset{\text{£.}}{59} \\ \hline \text{106) } 5900.000000 \text{ (} \overset{\text{£.}}{55.660377} \end{array}$$

$$\begin{array}{r} \text{If } \overset{\text{£.}}{112} \dots \overset{\text{£.}}{100} :: \overset{\text{£.}}{59} \\ \hline \text{112) } 5900.000000 \text{ (} \overset{\text{£.}}{52.678571} \end{array}$$

$$\begin{array}{r} \text{If } \overset{\text{£.}}{118} \dots \overset{\text{£.}}{100} :: \overset{\text{£.}}{59} \\ \hline \text{118) } 5900.000000 \text{ (} \overset{\text{£.}}{50} \end{array}$$

$$\begin{array}{r} \text{If } \overset{\text{£.}}{124} \dots \overset{\text{£.}}{100} :: \overset{\text{£.}}{59} \\ \hline \text{124) } 5900.000000 \text{ (} \overset{\text{£.}}{47.580645} \end{array}$$

$$\begin{array}{r} \text{If } \overset{\text{£.}}{130} \dots \overset{\text{£.}}{100} :: \overset{\text{£.}}{59} \\ \hline \text{130) } 5900.000000 \text{ (} \overset{\text{£.}}{45.384615} \end{array}$$

F f

First

The Golden Rule.

First Year's Purchase	-	55.660377
Second	-	52.678571
Third	-	50.
Fourth	-	47.580645
Fifth	-	45.384615

£. 251.304208

20

Sh. 6.084160

12

1.009920

Interest upon Interest.

59 £.
20

100 £.
20

1180

2000

12

12

14160

24000

4

4

Farthings 56640

96000

Farthings.

5760 00

Int. & Princ. of the 1st Ye. 101760

6105 60

of the 2d . . 107865

6471 90

of the 3d . . 114336

6860 16

of the 4th . . 121196

7271 36

of the 5th . . 128467

$$\begin{array}{r} \text{F.} \quad \text{F.} \quad \text{F.} \\ \text{If } 101760 \dots 96000 :: 56640 \\ \hline 96000 \\ 101760 \overline{) 5437440000} (53433 \\ \hline 97920 \end{array}$$

$$\begin{array}{r} \text{F.} \quad \text{F.} \quad \text{F.} \\ \text{If } 107865 \dots 96000 :: 56640 \\ \hline 96000 \\ 107865 \overline{) 5437440000} (50409 \\ \hline 73215 \end{array}$$

$$\begin{array}{r} \text{F.} \quad \text{F.} \quad \text{F.} \\ \text{If } 114336 \dots 96000 :: 56640 \\ \hline 96000 \\ 114336 \overline{) 5437440000} (47556 \\ \hline 77184 \end{array}$$

$$\begin{array}{r} \text{F.} \quad \text{F.} \quad \text{F.} \\ \text{If } 121196 \dots 96000 :: 56640 \\ \hline 96000 \\ 121196 \overline{) 5437440000} (44864 \\ \hline 102656 \end{array}$$

$$\begin{array}{r} \text{F.} \quad \text{F.} \quad \text{F.} \\ \text{If } 128467 \dots 96000 :: 56640 \\ \hline 96000 \\ 128467 \overline{) 5437440000} (42325 \\ \hline 74225 \end{array}$$

Farthings.

The First Year's Purchase	- - - - -	53433
The Second Year	- - - - -	50409
The Third Year	- - - - -	47556
The Fourth Year	- - - - -	44864
The Fifth Year	- - - - -	42325

$$4 \overline{) 238587}$$

$$12 \overline{) 59646} \frac{1}{2}$$

$$497 \overline{) 10} : 6$$

$$£. 248 : 10 : 6 \frac{1}{2}$$

F f 2

Decimally.

The Golden Rule.

Decimally.

	£.	
	100.000000	
	6.000000	00
Int. and Prin. of the first Year	106.000000	
	6.360000	00
Of the second - -	112.360000	
	6.741600	00
Of the third - -	119.101600	
	7.146096	00
Of the fourth - -	126.247696	
	7.574861	76
Of the fifth - -	£. 133.822557	

$$\begin{array}{r} \text{£.} \quad \text{£.} \quad \text{£.} \\ \text{If } 106 \dots 100 :: 59 \\ \hline 59 \end{array}$$

$$\begin{array}{r} 106)5900.000000(55.660377 \\ \hline 38 \end{array}$$

$$\begin{array}{r} \text{£.} \quad \text{£.} \quad \text{£.} \\ \text{If } 112.36 \dots 100 :: 59 \\ \hline 59 \end{array}$$

$$\begin{array}{r} 112.36)5900.00000000(52.509789 \\ \hline 10796 \end{array}$$

$$\begin{array}{r} \text{£.} \quad \text{£.} \quad \text{£.} \\ \text{If } 119.1016 \dots 100 :: 59 \\ \hline 59 \end{array}$$

$$\begin{array}{r} 119.1016)5900.0000000000(49.537537 \\ \hline 832408 \end{array}$$

If

If 126.247696 ... 100 :: 59

59

126.247696)5900.000000000000(46.733526

If 133.822557 ... 100 :: 59

59

133.822557)5900.000000000000(44.088232

The first Year's Purchase	- - -	£. 55.660377
The second	- - -	52.509789
The third	- - -	49.537537
The fourth	- - -	46.733526
The fifth	- - -	44.088232

£. 248.529461

20

Sb. 10.589220

12

d. 7.070640

The Purchase by Simple Interest - £. 251 6 1

By Compound Interest - £. 248 10 7

Difference - - - - £. 2 15 6

38. One Man owes another the Sum of 350 l. to be paid in Manner following, viz. the Sum of 70 l. in 6 Months, and so every 6 Months following the Sum of 70 l. until the Whole be paid. Now it is agreed between both to pay the Whole at one Payment; I demand the Time of Payment.

Note: When the Sums of Money and Times of Payment are equal, as in this Question, by adding one Term of Time more, and taking half thereof, you have the equated Time of Payment.

70 £.

The Golden Rule.

70 £. in 6 Months.

70 £. in 6 Months following:

70 £. in 6 Months after the preceeding.

70 L. in 6 Months after that.

70 £. in 6 Months following.

6 Months more added.

350 £. —

2136

18 Months the Time of Payment.

39. One Man is indebted to another 20 *Shillings*, to be paid at *one Shilling per Week*. What is the Time of Payment to pay the Whole at once?

20

1 Week more aided.

221

10 $\frac{1}{2}$ Weeks the Time of Payment.

From hence may be observed, the great *Extor-*
tion of those *Ufurers*, who lend 18 *Shillings*, to
receive again 20, at one *Shilling* per *Week*.

W. D.

W.

A Year is 52 : 1

Decimally, 52.142857

W.

Sb.

W.

If $10.5 \cdot 2 :: 52.142857$

2

10.5) 104.285714 (9.93197 Sb.

18 Shillings lent, is $\frac{9}{10}$ of a £.

If

£. Sh. £.
If .9 . . 9.93197 :: 100

100
9|993.19700

Sh. 1103.5522
12

d. 6.6264

f. 2.5056

210|110|3

£. 55 : 3

Thus it appears, that those who practise this unreasonable Way of letting out Money to Use, gain 55 *l.* 3 *s.* 6 *d.* $\frac{1}{4}$ for every 100 *l.* they so let out. No Wonder then, that such heap up great Possessions; but let them remember, it is heaped together for the last Days.

40. One Merchant owes to another 1500 *l.* to be paid in Manner following, viz. 300 *l.* at 6 Months; 300 *l.* more at 10 Months; 300 *l.* more at 12 Months; 300 *l.* more at 18 Months; and 300 *l.* more at 24 Months. I demand the Time of paying it altogether?

Note: If the Sums of Money or Times of Payment be different, then each Sum must be multiplied by its Time of Payment; and the Total divided by the Debt, the Quotient gives the equated Time of Payment.

300 *l.* at 6 Months.

6
1800

300 *l.* at 10 Months.

10
3000

300 *l.* at 12 Months.

12
3600

300 *l.* at 18 Months.

18
5400

300 *l.*

300 l. at 24 Months.

$$\begin{array}{r} 24 \\ \hline 7200 \end{array}$$

1800

3000

3600

5400

7200

15|00)210(00|14 Mon. the Time of Payment.

41. Suppose 477 l. to be paid in Manner following, *viz.* 47 l. ready Money; 20 l. at 3 Months; 60 l. at 6 Months; 100 l. at 9 Months; 100 l. at 12 Months; 150 l. at 15 Months; when would be the Time of paying it at one Payment?

20 l. at 3 Months.

$$\begin{array}{r} 3 \\ \hline 60 \end{array}$$

60 l. at 6 Months.

$$\begin{array}{r} 6 \\ \hline 360 \end{array}$$

100 l. at 9 Months.

$$\begin{array}{r} 9 \\ \hline 900 \end{array}$$

100 l. at 12 Months.

$$\begin{array}{r} 12 \\ \hline 1200 \end{array}$$

150 l. at 15 Months.

$$\begin{array}{r} 15 \\ \hline 2250 \end{array}$$

60

360

900

1200

2250

477)4770(10 Mon.
the Time of Payment.

These Rules on *Equation of Payments*, resolve the Question exactly, according to the Rules of Simple Interest; for in Question 38,

The Interest of 70 l. for 6 Months is

12 Months - - 4 4 0

18 Months - - 6 6 0

24 Months - - 8 8 0

30 Months - 10 10 0

Total £. 31 10 0

So likewise the Interest of 350 *l.* for } *l. s. d.*
 18 Months is - - - - - } 31 10 0

Again, in Question 40, *l. s. d.*
 The Interest of 300 *l.* for 6 Months is - 9 0 0
 10 Months - - 15 0 0
 12 Months - - 18 0 0
 18 Months - - 27 0 0
 24 Months - - 36 0 0
 Total - - £. 105 0 0

Which agrees with the Interest of }
 1500 *l.* for 4 Months, that comes to } 105 0 0

But to answer these Questions exactly, is by the Rebate, and not by the Interest of the Money.

For in Question 38, *l. s. d.*
 The Rebate of 70 *l.* for 6 Months is - 2 0 9 $\frac{1}{2}$
 12 Months - - 3 19 3
 18 Months - - 5 15 8 $\frac{1}{2}$
 24 Months - - 7 10 0
 30 Months - - 9 2 7 $\frac{1}{2}$
 Total - - £. 28 8 4 $\frac{1}{2}$

The Rebate of 350 *l.* for 18 Months is 28 18 0
 Difference - - 0 9 7 $\frac{1}{2}$

So that the Time of Payment must not be full 18 Months, but must want 8.346 Days, in which Time the Interest of 350 *l.* comes to the Difference, viz. 9 *s.* 7 *d.* $\frac{1}{2}$

		<i>l.</i>	<i>s.</i>	<i>d.</i>
Again, in Question 40,				
The Rebate of 300 <i>l.</i> for 6 Months is	-	8	14	9 $\frac{1}{2}$
10 Months	-	14	5	8 $\frac{1}{2}$
12 Months	-	16	19	7 $\frac{1}{2}$
18 Months	-	24	15	5
24 Months	-	32	2	10 $\frac{1}{2}$
Total		-	£. 96	18 5
The Rebate of 2500 <i>l.</i> for 14 Months is		98	2	7 $\frac{1}{2}$
Difference		-	1	4 2 $\frac{1}{2}$

So that the Time of Payment must not be 14 Months, by 4.908 Days, in which Time the Interest of 1500 *l.* will amount to the Difference, viz. 1 *l.* 4 *s.* 2 *d.* $\frac{1}{2}$

Thus, tho' the aforesaid Rules for finding the equated Time of Payment be not exact, according to the Rules of *Rebate*; yet they come very near it, and may be used as a good Help to find the exact Time of Payment, which otherwise will be very difficult to resolve.

The End of the First Part.



T H E
Arithmetician's Guide.

CONTAINING,
The Principles of ALGEBRA, in a
more plain and intelligible a Manner, than
any heretofore extant.

ILLUSTRATED
In the Numerical and Literal Solution, of many
Examples, both in Simple and Quadratic Equa-
tions.

WHEREIN
Every Process, both by Figures, and Letters,
are from Rules so plain, easy, and clear, that
Persons of a mean Understanding may compre-
hend them.

By *THO. CROSBY.*

P A R T II.

L O N D O N :

Printed for the AUTHOR ; and sold by JOHN CLARK,
Bookfeller, under the *Royal Exchange* ; and JOHN
WILCOX, Bookfeller, in the *Strand*.

M.DCC.XLVI.

THE MONUMENTAL CHURCH

OF THE
CITY OF ALBANY, N. Y.
AND THE
COUNTY OF ALBANY, N. Y.

THE MONUMENTAL CHURCH
OF THE CITY OF ALBANY, N. Y.
AND THE COUNTY OF ALBANY, N. Y.
WAS
ERECTED
IN
THE
YEAR
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THE
Arithmetician's Guide.

PART II.

ALGEBRA,

 ALLED by some the *Analytic Art* and universal Mathematicks, was first invented by *Francis Vieta*, but since, by the Labours of several eminent Persons, it has not only found very great Improvements, but has in a Manner been brought to the very Heighth of Perfection.

It is divided into Two Parts, *viz. Numerical* and *Literal*: The former of which is more ancient, and the latter more modern; and the Difference between the two consists chiefly in this, That in every Problem, the *Literal* does not only find out the very Thing sought for, as the *Numerical* does, but at the same Time discovers a general Rule, and Mode of Construction; which the *Numerical* cannot do, because all the Footsteps of the Process are confounded and lost in it, which the *Literal* always keeps entire, that they may be readily made use of back again, in order to *Composition*.

The following Signs, or Characters, are used in this Art.

- + More, the Sign of *Addition*.
- Less, the Sign of *Subtraction*.

× Mul-

ALGEBRA

- × Multiply, the Sign of *Multiplication*.
- = Equal, the Sign of *Equality*.
- ✓ Root, the Sign of the *Root* of any Quantity.
- ∴ Therefore.
- ⊃ Greater.
- ⊂ Lesser.

Note: When two or more Letters are joined together, without any Sign betwixt them, they are supposed to be *multiplied* together.

Division is expressed by a Line, like a *Fraction*, the *Numerator* shewing the *Dividend*, and the *Denominator* the *Divisor*.

And this is carefully to be observed, that the *Quantity*, that hath no Sign before it, if it be the *leading Quantity*, it is to be understood, to have the Sign +.

Axioms.

1. If *equal Quantities* be added to *equal Quantities*, the Sum shall be *equal*.
2. If *equal Quantities* be taken from *equal Quantities*, the Remainders shall be *equal*.
3. If *equal Quantities* be multiplied with *equal Quantities*, the Products will be *equal*.
4. If *equal Quantities* be divided by *equal Quantities*, the Quotients will be *equal*.
5. Those *Quantities* that are *equal* to one and the same Thing, are also *equal* to one another.

ADDITION

JOINS together all proposed *Quantities*, keeping the same Signs + and -.

If the Signs are different, *Addition* is performed by *Subtraction*; for *Quantities* of the same Name, being

being affected by contrary Signs, in *Addition*, destroy one another.

In *Simple Quantities* of the same Kind.

To $2a$	a	$3d$	$-5c$
Add a	$-a$	$5d$	$-2c$
Sum $3a$	0	$8d$	$-7c$

To $5b$	$7m$	$4b$	$11e$
Add $-3b$	$-9m$	$2b$	$-2e$
Sum $2b$	$-2m$	$6b$	$9e$

In *Simple Quantities* of a different Kind.

To a	$4c$	$3c$
Add b	$2d$	$2e$
Sum $a + b$	$4c + 2d$	$3c + 2e$

To b	$4a$	$2d$
Add $-a$	1	-2
Sum $b - a$	$4a + 1$	$2d - 2$

In *Compound Quantities* of the same Kind.

To $2a + 4d$	$3c + 2a$	$5b - 3c$
Add $a + 5d$	$8c + 4a$	$7c - 2b$
Sum $3a + 9d$	$11c + 6a$	$3b + 4c$

To $a + b - 2c$	$bb + 2abc$	$ab + ac$
Add $a - b + 3c$	$bb - abc$	$ab - 3ac$
Sum $2a + c$	$2bb + abc$	$-2ac$

In

In *Compound Quantities* of a different Kind.

$$\begin{array}{r} \text{To } 3a + c \\ \text{Add } 2b + d \\ \hline \text{Sum } 3a + c + 2b + d \end{array} \quad \begin{array}{r} 5d - 4c + 2a \\ 2b - e - 4n \\ \hline 5d - 4c + 2a + 2b - e - 4n \end{array}$$

$$\begin{array}{r} \text{To } 5a - 3b + 4c - d \\ \text{Add } 5e + 3i - 2m + 4n \\ \hline \text{Sum } 5a - 3b + 4c - d + 5e + 3i - 2m + 4n \end{array}$$

In *Compound Quantities* of a mix'd Kind.

$$\begin{array}{r} \text{To } 3a + 4c - d + 2b \\ \text{Add } 7d + m - 2b + 2a \\ \hline \text{Sum } 5a + 4c + 6d + m \end{array} \quad \begin{array}{r} 2aa + 3b \\ 7c - 7aa \\ \hline 3b + 7c - 5aa \end{array}$$

$$\begin{array}{r} \text{To } 7aa + 3ab + 2cc - 3d \\ \text{Add } 7a - ab + aa + 7d \\ \hline \text{Sum } 8aa + 2ab + 2cc + 4d + 7a \end{array}$$

$$\begin{array}{r} \text{To } 5cca + 2aab + 357c - 18aa - 27bc \\ \text{Add } 2ca - 7aa + 3aab + 29c + abc - 18bc \\ \hline \text{Sum } 5cca + 5aab + 386c - 25aa - 45bc + 2ca + abc \end{array}$$



SUBTRACTION

JOINS together the two proposed *Quantities*, changing all the Signs of the *Subducend*.

Hence, if the Signs be different, *Subtraction* will be performed by *Addition*, keeping the Sign of the *upper Quantity*.

In

In Simple Quantities of the same Kind.

From	$5a$	$2bc$	$4acd$
Take	$2a$	bc	$4acd$
Rem.	$3a$	bc	0

In Simple Quantities of a different Kind.

From	a	$3d$
Take	b	$2c$
Rem.	$a - b$	$3d - 2c$

From	dm	$4bc$
Take	$2bd$	4
Rem.	$dm - 2bd$	$4bc - 4$

In Compound Quantities of the same Kind.

From	$4a + 3b$	$6c - 5b$
Take	$2a + 2b$	$4c - 3b$
Rem.	$2a + b$	$2c - 2b$

From	$3c + 2a - b$	$ab - 4cd$
Take	$c + 3a - 2b$	$2ab - 5cd$
Rem.	$2c - a + b$	$cd - ab$

From	$3c + 2a - d$	$4a - 2c$
Take	$2c + 3a + d$	$2a - 6c$
Rem.	$c - a - 2d$	$2a + 4c$

From	$6a + 9b - 3c + 4d$	$4a + 3b$
Take	$7a + 2b - 3c + 8d$	$2a - 2b$
Rem.	$7b - a - 4d$	$2a + 5b$

H h

From

$$\begin{array}{r}
 \text{From } 2b - 2c + d \\
 \text{Take } b + 4c - d \\
 \hline
 \text{Rem. } b - 6c + 2d
 \end{array}
 \qquad
 \begin{array}{r}
 5bc - bb \\
 3bb - 3bc \\
 \hline
 8bc - 4bb
 \end{array}$$

$$\begin{array}{r}
 \text{From } 6bb + 3aa - 3c + 9b + 8a \\
 \text{Take } 4bb - 4aa - 6c - 5b + 3a \\
 \hline
 \text{Rem. } 2bb + 7aa + 3c + 14b + 5a
 \end{array}$$

In Compound Quantities of a different Kind.

$$\begin{array}{r}
 \text{From } aa - bb \\
 \text{Take } ac - ab \\
 \hline
 \text{Rem. } aa - bb - ac + ab
 \end{array}
 \qquad
 \begin{array}{r}
 5b - 2a \\
 3c + d \\
 \hline
 5b - 2a - 3c - d
 \end{array}$$

$$\begin{array}{r}
 \text{From } bc - dd \\
 \text{Take } ad + bd - 2cd \\
 \hline
 \text{Rem. } bc - dd - ad - bd + 2cd
 \end{array}$$

In Compound Quantities of a mix'd Kind.

$$\begin{array}{r}
 \text{From } ecc + bbd - aac - eee \\
 \text{Take } 2bbd - 2ccc + eee \\
 \hline
 \text{Rem. } 3ecc - bbd - aac - 2eee
 \end{array}$$

$$\begin{array}{r}
 \text{From } 7abc + 2aa - 3aac \\
 \text{Take } 4ab - 7abc + 2aac \\
 \hline
 \text{Rem. } 14abc + 2aa - 5aac - 4ab
 \end{array}$$

$$\begin{array}{r}
 \text{From } 24aaa + 2bcd - 33ac + 49 \\
 \text{Take } 13bcd + 27ac - 72 + abcd \\
 \hline
 \text{Rem. } 24aaa - 11bcd - 60ac + 121 - abcd
 \end{array}$$

MULTIPLICATION

JOINS together the Letters of the *Multipli-*
cand, and *Multiplier*, without any Sign be-
tween them.

Like Signs, whether $+$ or $-$, make in the *Pro-*
duct $+$, and different Signs $-$.

Multiply a	a	aa	d
by b	a	a	b
Product \overline{ab}	\overline{aa}	\overline{aaa}	\overline{db}

Multiply $4a$	$3bd$	ccc
by $2b$	$2ab$	$5dd$
Product $\overline{8ab}$	$\overline{6abbd}$	$\overline{5cccd}$

Multiply $5cd$	$2ac$	c
by $3ab$	10	12
Product $\overline{15abcd}$	$\overline{20ac}$	$\overline{12c}$

Multiply $6bbbb$	$4abb$
by $3bb$	$4ca$
Product $\overline{18bbbbbb}$	$\overline{16aabbcc}$

Multiply $a + b$	$b - a$
by c	$2d$
Product $\overline{ca + cb}$	$\overline{2db - 2da}$

Multiply $a + 2b - 3c + d$
by $3e$
Product $\overline{3ea + 6eb - 3ce + 3ed}$

Multiply $6b - 2a - c - 4b$
by $2ab$
Product $\overline{12abb - 4aab - 2abc - 8abb}$

H h 2

Multiply

$$\begin{array}{r} \text{Multiply } c + d \\ \text{by } a + b \\ \hline \text{Product } ac + ad + bc + db \end{array}$$

$$\begin{array}{r} \text{Multiply } b - a \\ \text{by } c - d \\ \hline \text{Product } bc - ca - db + ad \end{array}$$

$$\begin{array}{r} \text{Multiply } a + b \\ \text{by } a + b \\ \hline aa + ab \\ ab + bb \\ \hline \text{Product } aa + 2ab + bb \end{array}$$

$$\begin{array}{r} \text{Multiply } a - b \\ \text{by } a - b \\ \hline aa - ab \\ - ab + bb \\ \hline \text{Product } aa - 2ab + bb \end{array}$$

$$\begin{array}{r} \text{Multiply } b + c \\ \text{by } b - c \\ \hline bb + bc \\ - bc - cc \\ \hline \text{Product } bb - cc \end{array}$$

$$\begin{array}{r} \text{Multiply } b + c - 2d \\ \text{by } b + c - 2d \\ \hline bb + bc - 2bd \\ bc + cc - 2cd \\ - 2bd - 2dc + 4dd \\ \hline \text{Product } bb + 2bc - 4bd + cc - 2cd + 4dd \end{array}$$

$$\begin{array}{r} \text{Multiply } 6a - d - 2 \\ \text{by } a + 2d \\ \hline 6aa - ad - 2a \\ 12ad - 2dd - 4d \\ \hline \text{Product } 6aa + 11ad - 2a - 2dd - 4d \end{array}$$

$$\begin{array}{r} \text{Multiply } 3bb - 2aa + 5 \\ \text{by } 9a + 4b - 1 \end{array}$$

$$\begin{array}{r} 27abb - 18aaa + 45a \\ 12bbb - 8aab + 20b \\ - 3bb + 2aa - 5 \end{array}$$

$$\text{Product } 27abb - 18aaa - 45a + 12bbb - 8aab + 20b - 3bb + 2aa - 5.$$

$$\begin{array}{r} \text{Multiply } a + b - c \\ \text{by } d + e \end{array}$$

$$\text{Product } ad + bd - cd + ae + eb - ec$$

D I V I S I O N

Resolves the *Dividend* into the *Divisor* and *Quotient*; or, as it were, extracts the *Divisor* from the *Dividend*, and takes the *Residue* for the *Quotient*.

Like Signs make in the *Quotient* +, and different Signs —.

$$\begin{array}{r} a)aa(a \\ aa \\ \hline 0 \end{array}$$

$$\begin{array}{r} c)bc(b \\ bc \\ \hline 0 \end{array}$$

$$\begin{array}{r} 2b)2abc(ac \\ 2abc \\ \hline 0 \end{array}$$

$$\begin{array}{r} 3c)9cd(3d \\ 9cd \\ \hline 0 \end{array}$$

$$\begin{array}{r} b)3b(3 \\ 3b \\ \hline 0 \end{array}$$

$$\begin{array}{r} 2bb)4aabb(2aa \\ 4aabb \\ \hline 0 \end{array}$$

$$\begin{array}{r} 2ccbbb)6cccbbbb(3cb \\ 6cccbbbb \\ \hline 0 \end{array}$$

$$\begin{array}{r} 3abb)3aabb(ab \\ 3aabb \\ \hline 0 \end{array}$$

$$\begin{array}{r} 9)18cd(2cd \\ 18cd \\ \hline 0 \end{array}$$

$$\begin{array}{r} 5aab)40aabb(8b \\ 40aabb \\ \hline 0 \end{array}$$

$$\begin{array}{r} 4bd) 12bbccddddd(3bccddd \\ \underline{12bbccddddd} \\ 0 \end{array}$$

$$\begin{array}{r} 4aac) 12aaaaacccddddd(3aacccddd \\ \underline{12aaaaacccddddd} \\ 0 \end{array}$$

$$\begin{array}{r} b) ab + bc(a + c \\ \underline{ab + bc} \\ 0 \end{array} \quad \begin{array}{r} c) bc - cd(b - d \\ \underline{bc - cd} \\ 0 \end{array}$$

$$\begin{array}{r} b + c) ab + ac + db + dc(a + d \\ \underline{ab + ac + db + dc} \\ 0 \end{array} \quad \begin{array}{r} d + 1) dd + d(d \\ \underline{dd + d} \\ 0 \end{array}$$

$$\begin{array}{r} d - c) bd - bc - ad + ac(b - a \\ \underline{bd - bc - ad + ac} \\ 0 \end{array} \quad \begin{array}{r} 2a - 1) 2aaa - aa(aa \\ \underline{2aaa - aa} \\ 0 \end{array}$$

$$\begin{array}{r} 2aa + 4ac) 6aab + 12abc - 10aacccc - 20acccccc(3b - 5cccc \\ \underline{6aab + 12abc - 10aacccc - 20acccccc} \\ 0 \end{array}$$

$$\begin{array}{r} a + b) aa - bb(a - b \\ \underline{aa + ab} \\ -ab - bb \\ \underline{-ab - bb} \\ 0 \end{array} \quad \begin{array}{r} a - b) aa - bb(a + b \\ \underline{aa - ab} \\ ab - bb \\ \underline{ab - bb} \\ 0 \end{array}$$

$$\begin{array}{r} b - a) bbb - 3bba + 3baa - aaa(bb - 2ab + aa \\ \underline{bbb - bba} \\ -2bba + 3baa \\ \underline{-2bba + 2baa} \\ baa - aaa \\ \underline{baa - aaa} \\ 0 \end{array} \quad \begin{array}{r} \times b \leftarrow a \\ bbb - 2abb + aab \\ \underline{-abb + 2aab - aaa} \\ bbb - 3abb + 3aab - aaa \\ \underline{\hspace{1cm}} \\ 0 \end{array}$$

c + d

$$\begin{array}{r}
 c+d)ccc+ddd(cc-cd+dd \\
 \underline{ccc+ecd} \quad \times c+d \\
 -ccc+ddd \quad \underline{ccc-ccd+ddd} \\
 -ccd-cdd \quad \underline{ccd-cdd+ddd} \\
 \underline{ddd+cdd} \quad \underline{ccc+ddd} \\
 \underline{ddd+cdd} \quad \underline{\hspace{1cm}} \\
 \hspace{1.5cm} 0
 \end{array}$$

Note : If *Division* cannot be made after the preceeding Manner, then by placing the *Divisor* under the *Dividend* there will be a *Fraction*, so as to include the *Quotient*.

$$b)ac(\frac{ac}{b}$$

$$d)c(\frac{c}{d}$$

$$d)a+b(\frac{a+b}{d}$$

PROPORTION,

FROM two or three given Terms rightly disposed, by the Help of *Multiplication* and *Division*, produces a *third* or *fourth* Proportional required. Thus,

Multiply the *second* Number by itself, and divide the *Product* by the *first*, it gives the *third* Proportional.

Multiply the *second* Number by the *third*, and divide the *Product* by the *first* Number, it gives the *fourth* Proportional.

As $a : b :: b : \frac{bb}{a}$ the third Proportional.

As $a : 2a :: 2a : \frac{4aa}{a}$ that is, $4a$, the third Proportional.

As

As $b : ab :: ab : \frac{aabb}{b}$ that is, aab , the third Proportional

As $2c : d :: d : \frac{dd}{2c}$ the third Proportional.

As $a : a + b :: a + b$

$a + b$

$aa + ab$

$ab + bb$

$aa + 2ab + bb$ the third Proportional.

a

As $a : b :: c : \frac{bc}{a}$ the fourth Proportional.

As $3a : b :: 3b : \frac{3bb}{3a}$ that is, $\frac{bb}{a}$ the fourth Proportional.

As $a : c :: 2ab : \frac{2abc}{a}$ that is, $2bc$, the fourth Proportional.

As $2b : 3a :: 4bd : \frac{12abd}{2b}$ that is, $6ad$, the fourth Proportional.

As $d : 5b :: a : \frac{5ba}{d}$ the fourth Proportional.

As $10aa : 7c :: 10dd : \frac{70ddc}{10aa}$ that is, $\frac{7ddc}{aa}$ the fourth Proportional.

As $a : a + b :: a - b$

$a - b$

$aa + ab$

$- ab - bb$

$aa - bb$

a

the fourth Proportional.

.As

As $b : b + c :: 2b - c$

$$\frac{2b - c}{2bb + 2bc}$$

$$\frac{-bc - cc}{2bb + bc - cc}$$

$$\frac{2bb + bc - cc}{b}$$

the fourth Proportional.

As $a + 2b : a :: 2a - b : \frac{2aa - ab}{a + 2b}$ the 4th

Proportional.

As $3b - 3a : 2a + c :: 3d : \frac{6ad + 3cd}{3b - a}$ the

fourth Proportional.

As $2d + c : 2d + b :: 2d - c$

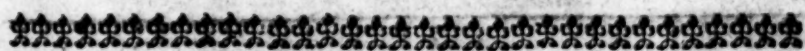
$$\frac{2d - c}{4dd + 2db}$$

$$\frac{-2dc - bc}{4dd + 2db - 2dc - bc}$$

$$\frac{4dd + 2db - 2dc - bc}{2d + c}$$

the fourth Pro-

portional.



REDUCTION of EQUATIONS,

IS the ordering of the Terms of an *Equation*, so that the given Quantities may possess one Side, and the required, the other.

Note, When *Quantities* are carried from one Side of an *Equation* to another, it is done by *Addition*, giving them the contrary Signs.

To find the *Value* of x in the following *Equations*.

Examples.

1. Let $a - b + x = b - x$.

Add $+x$ $+x$.

Then $a - b + 2x = b$. By adding $+x$ to each

Add $-a + b$ $-a + b$. Side of the Equat.

Then $2x = 2b - a$.

By adding $-a + b$
to both Sides. $\therefore x = \frac{2b - a}{2}$ By dividing each Side of the
Equation by 2 ; for if $2x$ be
equal to $2b - a$, then x must be equal to $\frac{1}{2} 2b - a$.

2. Let $x - a = b + c$. This Example is plain

Add $+a$ $+a$. from the foregoing.

Then $x = b + c + a$.

3. Let $x + a = b + c$. This Example needs

Add $-a$ $-a$. no Explanation.

Then $x = b + c - a$.

4. Let $x + a - b = 2x + b$. This Example

Add $-x$ $-x$ also needs no
Explanation.

Then $a - b = x + b$

Add $-b$ $-b$

Then $a - 2b = x$

5. Let $\frac{x}{a} = b$. For if $\frac{x}{a} = b$, then conse-

Then $x = ab$. quently b multiplied by a
must be equal to x .

6. Let

6. Let $\frac{x}{3b} = a - c.$

Then $x = 3ba - 3bc$; as is plain from the Argument in the preceeding.

7. Let $\frac{5x + b - da}{2} = 156.$

Then $5x + b - da = 312$, as in former Example.
Add $-b$ $-b$.

Then $5x - da = 312 - b.$
Add $+da$ $+da$.

Then $5x = 312 - b + da.$

$\therefore x = \frac{312 - b + da}{5}$ on the same Argument
used in Example 1.

8. Let $xx = 6x$

Each Side of the Equation
divided by x .

Then $x = 6.$

9. Let $3xx = 9x.$

Then $xx = 3x$, as aforegoing.

$\therefore x = 3$, as is plain.

10. Let $xx + ax = bx + cx$

Then $x + a = b + c$, rejecting x from each

Add $-a$ $-a$ Quantity, is dividing the whole by x .

Then $x = b + c - a$

11. Let $xx = abc.$ For if xx be the Square of

Then $x = \sqrt{abc}$ x , then the Square Root
of abc , must be equal to x .

12. Let $xxx = 7abcd$. If xxx be the Cube of x ,
 Then $x = \sqrt[3]{7abcd}$ then the Cube Root
 of $7abcd$, must be
 equal to x .

13. Let $ax + ab + bc = cx + 2d - a$

Add $-cx$ $-cx$

Then $ax - cx + ab + bc = 2d - a$

Add $-ab$ $-ab$

Then $ax - cx + bc = 2d - a - ab$

Add $-bc$ $-bc$

Then $ax - cx = 2d - a - ab - bc$

$\therefore x = \frac{2d - a - ab - bc}{a - c}$

The Steps in the Process of this *Example* might have been in a less Number, by carrying more Quantities than one at a time. But as I have done here, I shall follow the like Method through the following *Examples*, and do hope the Reader will not deem it superfluous; my End being their Instruction, and an Endeavour to render this *Treatise* as sufficiently explicit as I could.

Note: I shall omit the *Additions* of the *Quantities* to each Side of the *Equation*, as not necessary, hoping the *Learner* will plainly see, by what has already been done, the *Process* in every Step, till it comes to a *Conclusion*, in finding the Value of the *unknown Quantity*, as may be required.

SIMPLE

SIMPLE EQUATIONS.

1. **T**O find a *Number*, which being multiplied by 3, and from the *Product* 5 subtracted; then the *Remainder* divided by 2, and to the *Quotient* if the *Number* sought be added, the *Sum* shall be 40.

Numerically.

x , the *Number* sought.

$$\frac{3x - 5}{2} + x = 40$$

$$\text{Then } \frac{3x - 5 + 2x}{2} = 40$$

$$\therefore 3x - 5 + 2x = 80$$

$$\therefore 5x - 5 = 80$$

$$\therefore 5x = 85$$

$$\therefore x = \frac{85}{5} = 17$$

17

3

51

5

2) 46(

23

17

40

Literally.

$$\text{Let } 3 = a \quad x$$

$$5 = b \quad a$$

$$2 = c \quad \frac{ax - b}{c} + x = d$$

$$40 = d$$

$$\text{Then } \frac{ax - b + cx}{c} = d$$

$$3 = a$$

$$d = 40$$

$$\therefore ax - b + cx = cd$$

$$2 = c$$

$$c = 2$$

$$\therefore ax + cx = cd + b$$

$$5$$

$$cd = 80$$

$$\therefore x = \frac{cd + b}{a + c} = 17$$

$$b = 5$$

$$a + c = 5$$

$$17$$

2. To find a Number, which being multiplied by 12, to the Product add 48, it shall produce the same, as if the Number sought, was multiplied by 48.

Numerically.

x , the Number sought.

$$\begin{array}{rcl} 12x + 48 & = & 48x \\ \therefore 48 & = & 6x \\ \therefore x & = & \frac{48}{6} = 8 \end{array}$$

Literally.

Let $12 = a$ x , the Number sought.

$$48 = b$$

$$48 = c$$

$$ax + b = cx$$

$$\therefore b = cx - ax$$

$$\therefore x = \frac{b}{c - a} = 8$$

$$c - a = 48 - 12 = 36$$

3. To find a Number, to which if 11 be added, and from the same Number 7 be subtracted; the Sum of the Addition shall be double to the Remainder of the Subtraction.

Numerically.

x , the Number sought.

$$\begin{array}{rcl} 11 & + & x \\ \hline 25 & & 25 \\ 11 & + & 7 \\ \hline 18 & & 18 \end{array}$$

Then $x + 11 = 2x - 14$
 $11 = x - 14$
 $x = 25$

Literally.

Literally.

$$\begin{array}{rcl} \text{Let } 11 = a & x & x \\ 7 = b & a & b \\ & \hline & x + a & x - b \\ & & 2 \end{array} \quad \begin{array}{rcl} 11 = a \\ 14 = 2b \\ \hline 25 \end{array}$$

$$\text{Then } x + a = 2x - 2b$$

$$\therefore a = x - 2b$$

$$\therefore x = a + 2b = 25$$

4. To find a Number, to which, if the double, treble, or Quadruple of it be added, it shall produce its Square.

x , the Number sought.

$$\begin{array}{rcl} 2x & x & x \\ \hline 3x = xx & 3x & 4x \\ \therefore x = 3 & 4x = xx & 5x = xx \\ 6 & \therefore x = 4 & \therefore x = 5 \\ \hline 9 & 12 & 20 \\ & \hline & 16 & 25 \end{array}$$

5. To find a Number, to which if you add it-self, and multiply the Sum by the same, then if from the Product you subtract the same Number, and divide the Remainder by the same, the Quotient shall be 13.

Numerically.

x , the Number required.

$$\begin{array}{rcl} x & & 7 \\ \hline 2x & \text{Then } \frac{2xx - x}{x} = 13 & 7 \\ x & \therefore 2xx - x = 13x & 14 \\ \hline 2xx - x & \therefore 2x - 1 = 13 & 7 \\ a & \therefore 2x = 14 & 98 \\ & \therefore x = \frac{14}{2} = 7 & 7 \\ & & 7)91 \\ & & 13 \end{array}$$

Literally.

Literally.

Let $13 = a$

$$\begin{array}{r} x \\ x \\ \hline 2x \\ x \\ \hline \end{array}$$

Then $\frac{2xx - x}{x} = a$

$\therefore 2xx - x = ax$

$\therefore 2x - 1 = a$

$\therefore 2x = a + 1$

$\therefore x = \frac{a + 1}{2} = 7$

6. To divide the Number 16, into 2 Parts; so that the Square of the greater, may exceed the Square of the lesser by 32.

Numerically.

$$\begin{array}{r} \boxed{x} \quad 16 - x \quad \text{Then } 256 - 32x + xx + 32 = xx \\ x \quad 16 - x \quad \therefore 256 - 32x + 32 = 0 \end{array}$$

$$\begin{array}{r} xx \quad 256 - 16x \\ 16x + xx \end{array} \quad \therefore 288 = 32x = 0$$

$$\begin{array}{r} 256 - 32x + xx \\ \hline \end{array} \quad \therefore 288 = 32x$$

$$\boxed{9} \quad \boxed{7} \quad \therefore x = \frac{288}{32} = 9$$

$$\begin{array}{r} 9 \quad 7 \\ \hline 81 \quad 49 \\ \hline 32 \\ \hline 81 \end{array}$$

Literally.

Literally.

$$\begin{array}{rcl} \text{Let } 16 = a & \boxed{x} & a - x \\ 32 = b & x & a - x \\ & \hline & xx & aa - ax \\ & & \hline & & - ax + xx \\ & & \hline & & aa - 2ax + xx \end{array}$$

$$\text{Then } aa - 2ax + xx + b = xx$$

$$\therefore aa - 2ax + b = 0$$

$$\therefore aa + b = 2ax$$

$$\therefore x = \frac{aa + b}{2}$$

$$2x =$$

$$\frac{aa - 2ax + xx}{aa - 2ax + xx}$$

$$a = 16$$

$$a = 16$$

$$96$$

$$16$$

$$aa = 256$$

$$b = 32$$

$$2x = 32 \quad 288 \quad (9$$

$$288$$

7. To divide the Number 36 into Two Parts; so that if 12 be added to the greater, and 6 to the lesser, the Sum of the former may be double to the Sum of the latter.

Numerically.

$$\boxed{x} \quad 36 - x$$

$$12$$

$$6$$

$$x + 12 \quad 42 - x$$

$$2$$

$$84 - 2x$$

$$\boxed{24} \quad 12$$

$$12$$

$$6$$

$$36$$

$$18$$

$$\text{Then } x + 12 = 84 - 2x$$

$$\therefore 3x + 12 = 84$$

$$\therefore 3x = 72$$

$$\therefore x = 24$$

$$\frac{72}{3} = 24$$

$$3$$

K k

Literally.

Literally.

$$\begin{array}{rcl} \text{Let } 36 = a & \boxed{x} & a - x \\ 12 = b & b & c \\ 6 = c & \frac{x+b}{2} & \frac{a-x+c}{2} \end{array}$$

$$\begin{array}{rcl} \text{Then } x+b=2a-2x+2c & 2a-2x+2c & \\ \therefore 3x+b=2a+2c & 2a=72 & \\ \therefore 3x=2a+2c-b & 2c=12 & \\ \therefore x=2a+2c-b & & \\ \hline & 3 & 2a+2c=84 \\ & & b=12 \\ & & \hline & & 3)72 \\ & & \hline & & 24 \end{array}$$

8. A General, disposing his Army into a Square Battle, finds he has 284 Soldiers, over and above; but increasing each Side by one Soldier, he wants 25 Soldiers to fill up the Square. How many Soldiers had he?

Numerically.

x , the Number of Soldiers in a Side.

$$\begin{array}{rcl} x & x+1 & \\ \hline xx+284 & \text{the whole Army.} & x+1 \\ & & \hline & & xx+x \end{array}$$

$$\begin{array}{rcl} \text{Then } xx+2x+1-25=xx+284 & x+1 & \\ \therefore 2x+1-25=284 & & \hline \therefore 2x-24=284 & 154 & xx+2x+1 \\ \therefore 2x=308 & 154 & 155 \\ \therefore x=308 & 616 & 155 \\ \hline & 2 & 770 \\ & & 154 \\ & & \hline & & 23716 \\ & & 284 \\ & & \hline & & \text{Whole Army } 24000 \end{array}$$

Literally.

$$\begin{array}{rcl} \text{Let } 284 = a & x & x + b \\ 1 = b & x & x + b \\ 25 = c & \frac{x}{xx + a} & \frac{xx + bx}{bx + bb} \end{array}$$

$$\begin{array}{rcl} \text{Then } xx + a = xx + 2bx + bb - c & & xx + 2bx + bb \\ \therefore a = 2bx + bb - c & & 25 = c \\ \therefore 2bx - c = a - bb & & 284 = a \\ \therefore 2bx = c + a - bb & & 309 = c + a \\ \therefore x = c + a - bb & & 1 = b \\ \hline & & 2) 308 = c + a - 2b \\ & & 154 \end{array}$$

9. A Captain sends out $\frac{1}{3}$ of his Soldiers + 10, there remains with him $\frac{1}{2}$ + 15. How many Soldiers had he?

Numerically.

x , the Number of Soldiers he had.

$\frac{x}{3} + 10$, that is, $\frac{x + 30}{3}$ Soldiers sent out.

$\frac{x}{2} + 15$, that is, $\frac{x + 30}{2}$ Soldiers remaining.

$$\text{Then } \frac{x + 30}{3} + \frac{x + 30}{2} = x \quad \begin{array}{r} 3) 150 \\ 50 \end{array} \quad \begin{array}{r} 2) 150 \\ 75 \end{array}$$

$$\therefore \frac{2x + 60 + 3x + 90}{6} = x \quad \begin{array}{r} 10 \\ 60 \\ 90 \end{array} \quad \begin{array}{r} 15 \\ 90 \end{array}$$

$$\therefore 2x + 60 + 3x + 90 = 6x \quad \begin{array}{r} 90 \\ 150 \end{array}$$

$$\therefore 5x + 150 = 6x$$

$$\therefore x = 150$$

K k 2

Literally.

Literally.

Let $10 = a$ $\frac{x}{3} + a$ is $\frac{x+3a}{3}$ sent out.
 $15 = b$

$\frac{x}{2} + b$ is $\frac{x+2b}{2}$ remained.

$$\text{Then } \frac{x+3a}{3} + \frac{x+2b}{2} = x$$

$$\therefore \frac{2x+6a+3x+6b}{6} = x \quad \begin{array}{r} 6a = 60 \\ 6b = 90 \\ \hline 150 \end{array}$$

$$\therefore 2x+6a+3x+6b = 6x$$

$$\therefore 5x+6a+6b = 6x$$

$$\therefore x = 6a+6b = 150$$

10. There is an *Army*, to which if you add $\frac{1}{2} \frac{2}{3} \frac{3}{4}$ of itself, and from the *Total* take away 5000, the *Remainder* shall be 100000. What was the *Number* of the *Army*?

Numerically.

x , the *Army*.

$$\begin{array}{r} 46x \\ 12x+16x+18x \\ \hline \frac{x}{2} + \frac{2x}{3} + \frac{3x}{4} \\ \hline 24 \end{array}$$

$$\text{Then } x + \frac{46x}{24} - 5000 = 100000$$

$$\therefore 24x + 46x - 120000 = 2400000 \quad 36000$$

$$\therefore 70x - 120000 = 2400000 \quad 18000$$

$$\therefore 70x = 2520000 \quad 24000$$

$$\therefore x = \frac{2520000}{70} = 36000 \quad 27000$$

$$105000$$

$$5000$$

$$100000$$

Literally.

$$\begin{array}{rcl}
 \text{Let } 5000 = a & \text{Then } x + \frac{46x}{24} - a = b \\
 100000 = b & \therefore 24x + 46x - 24a = 24b \\
 24 & \therefore 70x - 24a = 24b \\
 \frac{5000}{24} = a & \therefore 70x = 24a + 24b \\
 120000 = 24a & \therefore x = \frac{24a + 24b}{70} = 36000 \\
 100000 = b & \\
 \frac{24}{24} & \\
 2400000 = 24b & \\
 120000 & \\
 710)252000(0 = 24a + 24b & \\
 \underline{36000} &
 \end{array}$$

11. To find *two* Numbers in the *Proportion* of 2 to 3, whose *Product*, when *multiplied*, shall be 54.

Numerically.

x , one Number. As $2 : 3 :: x : \frac{3x}{2}$ the other Number.

$$\begin{array}{rcl}
 \frac{3xx}{2} \times \frac{x}{1} & \text{Then } \frac{3xx}{2} = 54 \\
 \frac{3xx}{2} & \therefore 3xx = 108 \\
 & \therefore xx = \frac{108}{3} = 36 \\
 \text{As } 2 : 3 :: 6 : 9 & \therefore x = \sqrt{36} = 6 \\
 \frac{9}{54} &
 \end{array}$$

Literally.

$$\begin{array}{l}
 \text{Let } 2 = a \\
 3 = b \\
 54 = c
 \end{array}$$

$$\text{As } a : b :: x : \frac{bx}{a} \times \frac{x}{1}$$

Then

$$\text{Then } \frac{bxx}{a} = c$$

$$\therefore bxx = ac$$

$$\therefore xx = \frac{ac}{b}$$

$$\therefore x = \sqrt{\frac{ac}{b}} = 6$$

$$54 = c$$

$$2 = a$$

$$b = 3 \quad 108 = 2c$$

$$\sqrt[3]{36(6)}$$

$$36$$

$$0$$

12. To find *two* Numbers, in the *Ratio* of 4 to 5, and the Sum of their *Squares* to be 2624.

Numerically.

x , one Number. As $4 : 5 :: x : \frac{5x}{4}$ the other Number.

$$\begin{array}{r} 25xx \\ 5x \times 5x \\ 4 \quad 4 \\ \hline 16 \end{array}$$

$$41)41984(1024$$

$$98$$

$$164$$

$$0$$

$$\sqrt[3]{1024(32)}$$

$$62)124$$

$$0$$

$$\text{Then } \frac{25xx}{16} + xx = 2624$$

$$\therefore 25xx + 16xx = 41984$$

$$\therefore 41xx = 41984$$

$$\therefore xx = \frac{41984}{41} = 1024$$

$$\sqrt[3]{1024} = 32$$

$$\text{As } 4 : 5 :: 32 : 40$$

$$\begin{array}{r} 5 \\ 4)160 \\ \hline 40 \end{array}$$

$$40$$

$$40$$

$$1600$$

$$1024$$

$$2624$$

Literally.

Literally.

Let $4 = a$

$5 = b$

$2624 = c$

As $a : b : x ::$

$$\frac{bbxx}{aa} = \frac{bx}{a} \times \frac{bx}{a}$$

$2624 = c$
 $16 = aa$

$$\begin{array}{r} \sqrt{1024} (32 \\ 62 \overline{) 124} \\ \underline{} \\ 0 \end{array}$$

Then $\frac{bbxx}{aa} + xx = c$

$41 \overline{) 41984} \overline{) 1024}$

$\therefore bbxx + aaxx = aac$

$\therefore xx = aac$

$\frac{bb}{bb+aa} = \frac{25}{16}$

$\therefore x = \sqrt{\frac{aac}{bb+aa}} = 32$

15744
 2624

98
 82

164

9

13. To find the Side of a Square, whose Area is to the Sum of the Sides, as 45 is to 12.

Numerically.

x , the Side of the Square.

xx , the Area.

$4x$, the Sum of the Sides.

As $xx : 4x :: 45 : 12$

12

$4x$

15

4

15

15

75

15

Then $12xx = 180x$

60

225

$\therefore 12x = 180$

As $225 : 60 :: 45 : 12$

$\therefore x = 180$

12

60

$\frac{180}{12} = 15$

2700

2700

Literally.

Literally.

$$\text{Let } 45 = a \\ 12 = b$$

$$\text{As } xx : 4x :: a : b$$

$$\text{Then } \frac{bxx}{b} = \frac{4ax}{4x}$$

$$45 = a$$

$$\therefore bx = 4a$$

$$\therefore x = \frac{4a}{b} = 15$$

$$b = 12 \overline{) 180} = 4a \\ 15$$

14. To find the *Side* of a *Cube*, whose *Superficies* is to the *Solidity*, as 6 to 11.

Numerically.

x , the Side of the Cube.

11

11

xxx , the Solidity.

11

11

$6xx$, the Superficies.

121

121

As 6 : 11 :: $6xx$: xxx

6

11

 xxx

11

726

1331

$$\text{Then } 6xxx = 66xx$$

$$\text{As } 6 : 11 :: 726 : 1331$$

$$\therefore 6x = 66$$

$$11 \quad 6$$

$$\therefore x = 66$$

$$7986 = 7986$$

$$\frac{66}{6} = 11$$

Literally.

$$\text{Let } 6 = a$$

$$\text{As } a : b :: 6xx : xxx$$

$$11 = b$$

b

a

$$11 = b$$

$$\text{Then } 6bxx = axxx$$

$$6$$

$$\therefore 6b = ax$$

$$a = 6 \overline{) 66} = 6b$$

$$\therefore x = \frac{6b}{a}$$

$$= 11$$

$$11$$

15. A certain Man hires a *Labourer* on this Condition, that for every Day he worked, he should have 3 *Shillings*; but for every Day he worked not, he should pay 2 *Shillings*: When 390 Days were past, neither of them were indebted to each other. I demand how many Days he worked, and how many Days he worked not.

Numerically.

x , the Days that he worked.

$390 - x$, the Days that he worked not.

$$\begin{array}{r} 2 \\ \hline 780 - 2x = 3x \\ \therefore 780 = 5x \\ \therefore x = \frac{780}{5} = 156 \end{array} \quad \begin{array}{r} 156 \\ 3 \\ \hline 468 \end{array} \quad \begin{array}{r} 390 \\ 156 \\ \hline 234 \\ 2 \\ \hline 468 \end{array}$$

Literally.

Let $2 = a$

$b - x$

$b = 390$

$3 = c$

a

$a = 2$

$390 = b$

Then $ab - ax = cx$

$5)780 = ab$

$\therefore cx + ax = ab$

156

$\therefore x = \frac{ab}{c + a}$

$= 156$

16. A certain Gentleman hires a *Servant*, and promises him 24 *l.* yearly Wages, and a *Cloak*. At 8 Months End, the *Servant* obtains Leave to go away, and receives for his Wages the *Cloak*, with 13 *l.* I demand the Value of the *Cloak*.

Numerically.

x , the Value of the *Cloak*.

As $12 : x + 24 :: 8 : x + 13$

8

12

Then $8x + 192 = 12x + 156$

L 1

$\therefore 192$

$$\therefore 192 = 4x + 156$$

$$\therefore 4x = 36$$

$$\therefore x = \frac{36}{4} = 9$$

$$\text{As } 12 : 33 :: 8 : 22$$

$$\frac{8}{264} = \frac{12}{264}$$

Literally.

$$\text{Let } 24 = a$$

$$8 = b$$

$$12 = c$$

$$13 = d$$

$$a = 24$$

$$b = 8$$

$$13 = d$$

$$12 = c$$

$$\text{As } c : a + x :: b : d + x$$

$$\text{Then } ba + bx = cd + cx$$

$$\therefore ba = cx - bx + cd$$

$$\therefore cx - bx = ba - cd$$

$$\therefore x = \frac{ba - cd}{c - b} = 9$$

$$ba = 192 \quad 156 = cd$$

$$cd = 156$$

$$c - b = 4 \quad 36 = ba - cd$$

$$9$$

17. A Person being asked, How old he was? answered, If I *quadruple* $\frac{2}{3}$ of my Years, and add $\frac{2}{3}$ of them + 50, to the *Product*, the *Sum* will be so much *above* 100, as the *Number* of my Years is now *under* 100.

Numerically.

x , the Age of the Person.

$$\frac{8x}{3} + \frac{x}{2} + 50, \text{ is } \frac{16x + 3x + 300}{6}$$

$$\frac{8x}{3} \times \frac{4}{1} = \frac{32x}{3}$$

$$\therefore \text{Then } \frac{16x + 3x + 300}{6} - 100 = 100 - x$$

$$\therefore 16x$$

Simple Equations.

26

$$\begin{aligned} \therefore 16x + 3x + 300 - 600 &= 600 - 6x & 26 \\ \therefore 19x - 300 &= 600 - 6x & 2 \\ \therefore 25x - 300 &= 600 & 3)72 \\ \therefore 25x &= 900 & 24 \\ \therefore x &= \frac{900}{25} = 36 & 100 \\ & & 36 \\ & & 64 \end{aligned}$$

$$\begin{aligned} & 26 \\ & 2 \\ & 3)72 \\ & 24 \\ & 4 \\ & 96 \\ & 18 \\ & 50 \\ & 164 \\ & 100 \\ & 64 \end{aligned}$$

Literally.

Let $50 = a$ $\frac{8x}{3} + \frac{x}{2} + a$ is $\frac{16x + 3x + 6a}{6}$
 $100 = b$

Then $\frac{16x + 3x + 6a}{6} - b = b - x$ $\frac{50 = a}{6}$ $\frac{100 = b}{12}$
 $\therefore 19x + 6a - 6b = 6b - 6x$ $\frac{300 = 6a}{1200 = 12b}$
 $\therefore 25x + 6a - 6b = 6b$ $\frac{300 = 6a}{25)900(36}$
 $\therefore 25x = 12b - 6a$ $\frac{150}{0}$
 $\therefore x = \frac{12b - 6a}{25} = 36$

18, One being asked, What a Clock it was? answered, The Day at this Time is 16 Hours long. If to $\frac{1}{2}$ the Hours past, be added $\frac{2}{3}$ of the Hours to come, you will have the Hour, from Sun-rising.

Numerically.

x , the Hours past.
 $16 - x$, the Hours to come.

$$\frac{\frac{32 - 2x}{3}}{2} + \frac{3x - 2x}{3} \text{ is } \frac{3x + 64 - 4x}{6}$$

Then $\frac{3x + 64 - 4x}{6} = x$

L 1 2

64

$$\therefore 64 - x = 6x$$

$$\therefore 64 = 7x$$

$$\therefore x = \frac{64}{7} = 9 \frac{1}{7}$$

$$16$$

$$\frac{9 \frac{1}{7}}{6 \frac{1}{7}}$$

$$6 \frac{1}{7} \text{ is } 2 \frac{1}{7}$$

$$\frac{1}{2} \text{ of } \frac{64}{7} \text{ is } \frac{64}{14} = \frac{32}{7}$$

$$\frac{2}{3} \text{ of } \frac{48}{7} \text{ is } \frac{96}{21} = \frac{32}{7}$$

$$\frac{32}{7} + \frac{32}{7} \text{ is } \frac{64}{7} = 9 \frac{1}{7}$$

Literally.

$$\text{Let } 16 = a$$

$$a - x$$

$$\frac{2}{2a - 2x}$$

$$\frac{3x + 4a - 4x}{2} + \frac{2a - 2x}{3} = \frac{3}{6}$$

$$\text{Then } \frac{3x + 4a = 6x}{6} = x$$

$$16 = a$$

$$\frac{4}{7}$$

$$7)64 = 4a$$

$$9 \frac{1}{7}$$

$$\therefore 4a - x = 6x$$

$$\therefore 4a = 7x$$

$$\therefore x = \frac{4a}{7} = 9 \frac{1}{7}$$

19. From *Norimberg* to *Rome* is 140 Miles. A Traveller sets out at the same Time from each of the said Cities; the *one* goes 8 Miles a Day, and the *other* 6. In *how many* Days will they meet, and *how many* Miles will each of them have gone?

Numerically.

x , the Number of Days in which they meet.

$8x$, the Number of Miles one had gone.

$6x$, the Number of Miles the other had gone.

$$\text{Then } 8x - 6x = 140$$

$$10$$

$$10$$

$$\therefore 14x = 140$$

$$8$$

$$6$$

$$\therefore x = \frac{140}{14} = 10$$

$$\frac{80}{80}$$

$$\frac{60}{80}$$

$$140$$

Literally.

$$\begin{array}{lcl} \text{Let } 140 = a & \text{Then } bx + cx = a & \\ 8 = b & \therefore x = \frac{a}{b+c} & 14)140 \\ 6 = c & & \underline{10} \end{array}$$

20. A certain *Messenger* goes 6 *Miles* every Day; 8 Days after, another follows him, and goes 10 *Miles* every Day. In *how many Days* will the *latter* overtake the *former*?

Numerically.

x , the Days in which the *latter* will overtake the *former*.

$$\begin{array}{lcl} \text{Then } 6x + 48 = 10x & 10 & 12 \\ \therefore 48 = 4x & 12 & 6 \\ \therefore x = \frac{48}{4} = 12 & \underline{120} & \underline{72} \\ & & 48 \\ & & \underline{120} \end{array}$$

Literally.

$$\begin{array}{lcl} \text{Let } 6 = a & \text{Then } ax + ab = cx & 6 \\ 8 = b & \therefore ab = cx - ax & 8 \\ 10 = c & \therefore x = \frac{ab}{c-a} = 12 & \underline{4)48(12} \end{array}$$

21. One bought 3 Books, whose Prices were in *Proportion*, as 12, 5, and 1. If the Price of the *first* be *doubled*, of the *second* *trebled*, and of the *third* *quadrupled*, the *Sum* of those *Products* will as much *exceed* 10 *Crowns*, as the *Sum* of the Price of the *greatest* and *middlemost* is *under* 5 *Crowns*. How much did the said Books cost?

Numerically.

12 x , the Price of the first Book.

5 x , the Price of the second.

x , the Price of the third.

The first doubled is - - - $24x$

The second trebled is - - $15x$

The third quadrupled is - $4x$

$$12x + 5x \text{ is } 17x \quad \underline{\hspace{1cm}} \\ 43x$$

Then $43x - 10 = 5 - 17x$

$\therefore 60x - 10 = 5$

$\therefore 60x = 15$

$\therefore x = \frac{15}{60} = \frac{1}{4} = 1:3$

	s.	d.	℥.	s.	d.
1.	15	0	-	1	10:0
2.	6	3	-	0	18:9
3.	1	3	-	0	5:0
	0	15	0	2	13:9
	0	6	3	2	10:0
	1	1	3	0	3:9
	1	5	0		
	0	3	9		

Literally.

Let $12 = a$ Then $2ax + 3bx + 4cx - d = e - ax - bx$

$5 = b \quad \therefore 3ax + 3bx + 4cx - d = e - bx$

$1 = c \quad \therefore 3ax + 4bx + 4cx - d = e$

$10 = d \quad \therefore 3ax + 4bx + 4cx = e + d$

$5 = e \quad \therefore x = e + d$

$$\frac{3a + 4b + 4c}{60} = \frac{1}{4}$$

$d=10 \quad 36=3a$

$e=5 \quad 20=4b$

$\frac{15}{60} = \frac{1}{4}$

$\frac{15}{60} = \frac{1}{4}$

22. To divide the Number 50 into 2 Parts; so that if the greater be divided by 7, and the lesser multiplied by 3, the Sum of the Quotient and Product may produce just 50.

Numerically.

$[x] \quad] 50 - x$

$$\begin{array}{r} 3 \\ \hline 150 - 3x \end{array}$$

Then $\frac{x}{7} + 150 - 3x = 50$

$$\begin{array}{rcl} \therefore x + 1050 - 21x & = & 350 \quad [35] \quad 15 \\ \therefore 1050 - 20x & = & 350 \quad 3 \\ \therefore 1050 & = & 20x + 350 \quad \hline \therefore 20x & = & 700 \quad 45 \\ \therefore x & = & \frac{700}{20} = 35 \quad 5 \\ & & 50 \end{array}$$

Literally.

Let $50 = a$ $[x] \quad a - x$
 $7 = b$ c
 $3 = c$ $\hline ca - cx$

$bc = 21$ $a = 50$
 $a = 50$ $b = 7$

Then $\frac{x}{b} + ca - cx = a$

$bcx = 1050$ $350 = ba$
 $ba = 350$

$$\begin{array}{l} \therefore x + bca - bxc = ba \\ \therefore x + bca = ba + bcx \\ \therefore x + bc - aba = bcx \\ \therefore bca - ba = bcx - x \\ \therefore x = \frac{bca - ba}{bc - 1} = 35 \end{array}$$

$bc - 1 = 210 \overline{) 7010}$
 35

23. To divide the Number 20 into 2 Parts; so that the Square of the lesser, subtracted from the Square of the greater, may leave just 20.

Numerically.

$$\begin{array}{rcl} [x] & 20 - x \\ x & 20 - x \\ \hline xx & 400 - 20x \\ & - 20x + xx \\ \hline & 400 - 40x + xx \end{array}$$

Then

$$\text{Then } xx - 400 + 40x - xx = 20$$

$$\therefore 40x - 400 = 20$$

$$\therefore 40x = 420$$

$$\therefore x = \frac{420}{40} = 10.5$$

$$\begin{array}{r} \boxed{10.5} \end{array}$$

$$\begin{array}{r} \boxed{9.5} \end{array}$$

$$10.5$$

$$9.5$$

$$525$$

$$475$$

$$1050$$

$$855$$

$$110.25$$

$$90.25$$

$$90.25$$

$$20.00$$

Literally.

$$\text{Let } 20 = a$$

$$\begin{array}{r} \boxed{x} \end{array} \begin{array}{r} \boxed{a - x} \end{array}$$

$$\text{Then } xx - aa + 2ax - xx = a$$

$$a - x$$

$$\therefore 2ax - aa = a$$

$$aa - ax$$

$$\therefore 2ax = a + aa$$

$$-ax + xx$$

$$\therefore x = \frac{a + aa}{2a}$$

$$= 10.5$$

$$aa - 2ax + xx$$

$$20 = a$$

$$20$$

$$400 = aa$$

$$20$$

$$2a = 40 \quad 420 \quad (10.5)$$

24. If a Man gains 20 Crowns a Week, how much must he spend a Week, to have 500 Crowns, together with the Expence of 4 Weeks, at the Year's End.

Numerically.

x , Crowns spent in a Week.

$52x$, the Expence of a Year.

1560, the Gain of a Year.

$$56)1060(18 \frac{11}{14}$$

$$500$$

$$52$$

Then

Simple Equations.

267

$$\begin{array}{rcl} \text{Then } 52x + 4x + 500 & = & 1560 \\ \therefore 56x + 500 & = & 1560 \\ \therefore 56x & = & 1060 \\ \therefore x & = & \frac{1060}{56} = 18 \frac{13}{14} \end{array}$$

$$\begin{array}{r} 265 \\ \times 52 \\ \hline 530 \\ 1325 \\ \hline 13780 \end{array}$$

$$\frac{1060}{56} = \frac{265}{14} \times 52, \text{ is } \frac{13780}{14}$$

$$\frac{265}{14} \times 4, \text{ is } \frac{1060}{14}$$

$$\frac{13780}{14} + \frac{1060}{14} \text{ is } \frac{14840}{14} = \frac{7420}{7} = 1060$$

$$\begin{array}{r} 1060 \\ - 500 \\ \hline 1560 \end{array}$$

Literally.

$$\text{Let } 30 = a \quad \text{Then } dx + cx + b = ad$$

$$500 = b \quad \therefore dx + cx = ad - b$$

$$4 = c \quad \therefore x = \frac{ad - b}{d + c} = 18 \frac{13}{14}$$

$$52 = d$$

$$30 = a$$

$$1560 = ad$$

$$500 = b$$

$$d + c = 56$$

$$1060(18 \frac{13}{14})$$

25. A Labourer, after 40 Weeks in which he had been employed, laid up 28 Crowns, less the Pay of 3 Weeks, and finds he had expended 36 Crowns, more the Pay of 11 Weeks. What Pay did he receive per Week?

Numerically.

x , the Pay of a Week.

$40x$, the whole Pay.

M m

Then

$$\begin{array}{rcl}
 \text{Then } 28 - 3x + 36 + 11x & = & 40x \\
 64 + 8x & = & 40x \\
 64 & = & 32x \\
 x & = & \frac{64}{32} = 2 \\
 & & \frac{40}{80} = \frac{2}{2}
 \end{array}$$

Literally.

$$\begin{array}{rcl}
 \text{Let } 40 & = & a \\
 28 & = & b \\
 3 & = & c \\
 36 & = & d \\
 11 & = & e
 \end{array}
 \quad
 \begin{array}{rcl}
 \text{Then } b - cx + d + ex & = & ax \\
 b + d & = & ax + cx - ex \\
 x & = & \frac{b + d}{a + c - e} = 2
 \end{array}$$

$$\begin{array}{rcl}
 40 & = & a \\
 3 & = & c \\
 43 & = & a + c \\
 11 & = & e \\
 32 & = & a + c - e
 \end{array}
 \quad
 \begin{array}{rcl}
 28 & = & b \\
 36 & = & d \\
 64 & = & 2 \times 32 \\
 0 & = & 0
 \end{array}$$

26. Two Companions having got a Parcel of *Guineas*; says *A* to *B*, if you will give me one of your *Guineas*, I shall have as many as you. Nay, replies *B*, if you will give me one of yours, I shall have twice as many as you. How many *Guineas* had each of them?

A had x *Guineas*, and *B* y *Guineas*.

$$\begin{array}{rcl}
 \text{Then } x + 1 & = & y - 1 \\
 x & = & y - 2
 \end{array}
 \quad
 \begin{array}{rcl}
 \text{Again } y + 1 & = & 2x - 2 \\
 2x & = & y + 3 \\
 x & = & \frac{y + 3}{2}
 \end{array}$$

$$\text{Now } \frac{y + 3}{2} = y - 2$$

$$\begin{array}{rcl}
 \therefore y + 3 & = & 2y - 4 \\
 \therefore 2y & = & y + 7 \\
 \therefore y & = & 7, \text{ and } x = y - 2 = 5
 \end{array}$$

27. A

27. A Man bought two Horses with the Trappings, which together cost 100 Crowns. If the Trappings be laid on the first Horse, both the Horses will be of equal Value; but if the Trappings be laid on the other Horse, he will be double to the Value of the first. What did each of the Horses cost.

Numerically.

x , the Price of the first Horse.

y , the Price of the second.

$100 - y - x$, the Price of the Trappings.

Then $x + 100 - y - x = y$. And $y + 100 - y - x = 2x$

$$\therefore 100 - y = 2y$$

$$\therefore 100 = 3y$$

$$\therefore y = \frac{100}{3} = 33\frac{1}{3}$$

$$\begin{array}{r} 100 \\ - 183\frac{1}{3} \\ \hline 16\frac{2}{3} \end{array}$$

$$33\frac{1}{3} + 16\frac{2}{3} = 50$$

$$50 + 16\frac{2}{3} = 66\frac{2}{3}$$

Literally.

Let $100 = a$

Then $x + a - y - x = y$. And $y + a - y - x = 2x$

$$\therefore a - y = y$$

$$\therefore a = 2y$$

$$\therefore a - x = 2x$$

$$\therefore a = 3x$$

$$\therefore y = \frac{a}{2} = 50$$

$$\therefore x = \frac{a}{3} = 33\frac{1}{3}$$

28. A Vintner has two Sorts of Wine, viz. White Lisbon and Canary; which, if mixed in equal Parts, a Gallon of the mixed will cost 14 Pence; but if 2 Gallons of White Lisbon, be mixed with 3 of Canary, a Gallon will cost 15 Pence. What was the Price of a Gallon of each?

Numerically.

x , the Price of a Gallon of White Lisbon.

y , the Price of a Gallon of Canary.

M m 2

Then

Then $x + y = 28$. And $2x + 3y = 75$

$$\therefore x = 28 - y \quad \therefore 2x = 75 - 3y$$

$$\therefore 2(28 - y) = 75 - 3y$$

$$\text{Now } \frac{75 - 3y}{2} = 28 - y \quad 9 + 19 = \frac{28}{2} = 14$$

$$\therefore 75 - 3y = 56 - 2y \quad 18 + 57 = \frac{75}{5} = 15$$

$$\therefore 75 = 56 + y$$

$$\therefore y = 19, \text{ and } x = 18 - y = 9$$

Literally.

Let $14 = a$ Then $x + y = 2a$ And $cx + dy = 5b$

$$15 = b \quad \therefore x = 2a - y \quad \therefore cx = 5b - dy$$

$$2 = c \quad \therefore x = 5b - dy$$

$$3 = d$$

$$\text{Now } \frac{5b - dy}{c} = 2a - y \quad 15 = b \quad 14 = a$$

$$\therefore 5b - dy = 2ca - cy \quad 75 = 5b \quad 56 = 2ca$$

$$\therefore 5b = 2ca + dy - cy$$

$$\therefore dy - cy = 5b - 2ca \quad 56 = 2ca \quad 1) 19$$

$$\therefore y = \frac{5b - 2ca}{a - c} = 19, \text{ and } x = 2a - y = 9$$

29. A Son asked his Father, How old he was? His Father answered him thus: If you take away 5 from my Years, and divide the Remainder by 8, the Quotient will be $\frac{1}{3}$ of your Years. But if you add 2 to your Age, and multiply the Whole by 3, and subtract 7 from the Product, you will have the Number of the Years of my Age. What was the Age of the Father and Son?

Numerically.

x , the Age of the Father,

y , the Age of the Son.

Then

$$\text{Then } \frac{x-5}{8} = \frac{y}{3}$$

$$\text{And } y+2$$

$$\therefore 3x-15=8y$$

$$\therefore 3x=8y+15$$

$$\therefore x=8y+15$$

$$\frac{3}{3}$$

$$3y+6-7=x$$

$$\therefore 3y-1=x$$

$$18$$

$$2$$

$$20$$

$$3$$

$$60$$

$$7$$

$$53$$

$$\text{Now } \frac{8y+15}{3} = 3y-1$$

$$\therefore 8y+15=9y-3$$

$$\therefore 15=y-3$$

$$\therefore y=18, \text{ and } x=3y-1=53$$

Literally.

$$\text{Let } 5=a$$

$$8=b$$

$$2=c$$

$$3=d$$

$$7=e$$

$$\text{Then } \frac{x-a}{b} = \frac{y}{d}$$

$$\text{And } y+c$$

$$\therefore dx-da=by$$

$$\therefore dx=by+da$$

$$\therefore x=by+da$$

$$\frac{dy+dc-e}{d}$$

$$\text{Now } \frac{by+da}{d} = dy+dc-e$$

$$15=da$$

$$21=de$$

$$54=dy$$

$$6=de$$

$$\therefore by+da=ddy+ddc-de$$

$$\therefore da=ddy-by+ddc-de$$

$$\therefore ddy-by=da+de-ddc$$

$$\therefore y=\frac{da+de-ddc}{dd-b}=18$$

$$1)18$$

$$7=e$$

$$53$$

$$\text{And } x=dy+dc-e=53$$

30. To find 2 Numbers, to the Sum whereof, if you add 6, the Whole shall be double to the greater; and if you subtract 2 from their Difference, the Remainder will be $\frac{1}{2}$ of the lesser.

Num. — 0.16.3

Simple Equations.

[x] y Then $x + y + 6 = 2x$. And $x - y - 2 = \frac{y}{2}$
 $\therefore y + 6 = x$

Now $\frac{3y+4}{2} = y + 6$
 $\therefore 3y + 4 = 2y + 12$
 $\therefore y + 4 = 12$
 $\therefore y = 8$. And $x = y + 6 = 14$
 $14 + 8 + 6 = 28$. $6 - 2 = 4$

Literally.

Let $6 = a$ Then $x + y + a = 2x$. And $x - y - b = \frac{y}{b}$
 $2 = b$ $\therefore bx - x = y + a$ $\therefore bx - by - bb = y$
 $\therefore x = y + a$ $\therefore x = y + by + bb$
 $b - 1$

Now $\frac{y+a}{b-1} = \frac{y+by+bb}{b}$
 $\therefore by + ba = by + bby + bbb - y - by - bb$
 $\therefore ba = bby + bbb - y + by - bb$
 $\therefore bby - by - y = ba - bbb + bb$
 $\therefore y = \frac{ba - bbb + bb}{bb - b - 1} = 8$, and $x = \frac{y+a}{b-1} = 14$

31. To find two Numbers, the Product whereof is 240, and the Freble of the Greater divided by the Lesser is 5.

Numerically.

[x] y Then $xy = 240$, and $\frac{3x}{y} = 5$
 $\therefore x = 240$ $\therefore 3x = 5y$
 $\therefore x = 5y$
 3

Now

$$\text{Now } \frac{5x}{3} = \frac{240}{y}$$

$$\therefore 5xy = 720$$

$$\therefore xy = \frac{720}{5} = 144$$

$$\text{And } y = \sqrt{144} = 12$$

$$\text{And } x = \frac{240}{y} = 20$$

$$\begin{array}{r} 20 \times 12 = 240 \\ 12 \times 20 = 240 \\ \hline 240 \end{array}$$

Literally.

$$\text{Let } 240 = a \quad \text{Then } xy = a. \quad \text{And } \frac{3x}{y} = b$$

$$5 = b$$

$$\therefore x = \frac{a}{y}$$

$$\therefore 3x = by$$

$$\therefore x = \frac{by}{3}$$

$$\text{Now } \frac{a}{y} = \frac{by}{3}$$

$$\therefore 3a = byy$$

$$\therefore yy = \frac{3a}{b}$$

$$y = \sqrt{\frac{3a}{b}}$$

$$240 = a$$

$$3$$

$$b = 5 \quad 720 = 3a$$

$$\sqrt{144} = 12$$

$$\frac{3}{12} = \frac{by}{5}$$

$$5 = b$$

$$3 \times 60 = by$$

$$20$$

$$\therefore y = \sqrt{\frac{3a}{b}} = 12, \text{ and } x = \frac{by}{3} = 20$$

32. Two Men being minded to purchase a House; says John to Thomas, if you will give me $\frac{2}{3}$ of your Money, I can purchase the House myself: But, says Thomas to John, if you will give me $\frac{1}{4}$ of yours, I shall be able to buy the House. How much Money had each of them?

Numerically.

x , the Money John had.

y , the Money Thomas had.

Then

Then $\frac{2y}{3} + x = 1200$. And $\frac{3x}{4} + y = 1200$

$$\begin{aligned} \therefore 2y + 3x &= 3600 & \therefore 3x + 4y &= 4800 \\ \therefore 3x &= 3600 - 2y & \therefore 3x &= 4800 - 4y \\ \therefore x &= \frac{3600 - 2y}{3} & \therefore x &= \frac{4800 - 4y}{3} \end{aligned}$$

$$\begin{aligned} \text{Now } 3600 - 2y &= 4800 - 4y & 600 &= 1200 \\ \therefore 3600 + 2y &= 4800 & 2 &= 1200 \\ \therefore 2y &= 1200 & 3)1200 &= 400 \\ \therefore y &= \frac{1200}{2} = 600 & 3)2400 &= 800 \\ \text{And } x &= \frac{3600 - 2y}{3} = 800 & 400 &= 800 \\ & & 800 &= 3 \\ & & 1200 &= 3 \\ & & 4)2400 &= 600 \\ & & &= 600 \\ & & &= 1200 \end{aligned}$$

Literally.

Let $1200 = a$

Then $x + \frac{2y}{3} = a$. And $y + \frac{3x}{4} = a$

$$\begin{aligned} \therefore 3x + 2y &= 3a & \therefore 4y + 3x &= 4a \\ \therefore 3x &= 3a - 2y & \therefore 3x &= 4a - 4y \\ \therefore x &= \frac{3a - 2y}{3} & \therefore x &= \frac{4a - 4y}{3} \end{aligned}$$

$$\begin{aligned} \text{Now } 3a - 2y &= 4a - 4y & 3600 &= 2a \\ \therefore 3a + 2y &= 4a & 1200 &= 2y \\ \therefore 2y &= a & 3)2400 &= a - y \\ & & 800 &= 800 \end{aligned}$$

$\therefore y = \frac{a}{2} = 600$. And $x = \frac{3a - 2y}{3} = 800$

33. Some

Simple Equations.

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33. Some young Men and Maids had a Reckoning of 37 Shillings to pay for a Treat; and this was their Agreement, that every Man should pay 3 Shillings, and every Maid 2. Now, if there had been as many Men, as there were Maids, the Reckoning would have been 4 Shillings less than it was. How many young Men and Maids were there?

Numerically.

x , the Number of young Men.

y , the Number of Maids.

Then $3x + 2y = 37$. And $3y + 2x = 33$

$\therefore 3x = 37 - 2y$ $\therefore 2x = 33 - 3y$

$\therefore x = \frac{37 - 2y}{3}$ $\therefore x = \frac{33 - 3y}{2}$

Now $\frac{37 - 2y}{3} = \frac{33 - 3y}{2}$ $27 - 10 = 37$
 $15 - 18 = 33$

$\therefore 74 - 4y = 99 - 9y$

$\therefore 74 + 5y = 99$

$\therefore 5y = 25$

$\therefore y = \frac{25}{5} = 5$, and $x = \frac{37 - 2y}{3} = 9$

Literally.

Let $37 = a$ Then $bx + cy = a$. And $by + cx = a - d$

$3 = b$ $\therefore bx = a - cy$ $\therefore cx = a - d - by$

$2 = c$ $\therefore x = \frac{a - cy}{b}$ $\therefore x = \frac{a - d - by}{c}$

$4 = d$ $\frac{a - d - by}{c} = \frac{a - cy}{b}$

Now $\frac{a - d - by}{c} = \frac{a - cy}{b}$

$\therefore ba - bd - bby = ca - ccy$

$\therefore ba - bd = bby + ca - ccy$

$\therefore bby - ccy = ba - bd - ca$

$\therefore y = \frac{ba - bd - ca}{bb - cc} = 5$

And $x = \frac{a - cy}{b} = 9$

$37 = a$	$37 = cy$
$3 = b$	$10 = cy$
$111 = ba$	$3)27$
$12 = bd$	9
99	
$74 = ca$	
$5)25 = bb - cc$	
5	

N n

34. A

34. A *General*, who had fought a *Battle*, upon reviewing his *Army*, whose *Foot* was 3 *Times* the Number of his *Horse*, finds, that before the *Battle* $\frac{1}{12}$ — 120 of his *Foot* had deserted; and of his *Horse* $\frac{1}{20}$ + 120; besides $\frac{1}{4}$ of his whole *Army* was sent into *Garrison*. Reckoning the *Sick* and the *Wounded*, $\frac{1}{8}$ of his *Army* remained. The rest that were wanting, were either *Slain* or taken *Prisoners*. Now if you add 3000, to the Number of the *Slain* or *Taken*, the *Sum* will be equal to $\frac{1}{5}$ of the *Foot* he had at first; I demand the Number of *Horse* and *Foot*.

Numerically.

x , the Number of *Horse*.

$3x$, the Number of *Foot*.

$4x$, the whole *Army*.

$\frac{3x}{12}$ — 120, is $\frac{3x-1440}{12} = \frac{x-480}{4}$ *Foot* Deserters.

$\frac{x}{20}$ + 120, is $\frac{x+2400}{20}$ *Horse* Deserters.

x , sent into *Garrison*.

$\frac{12x}{8}$ is $\frac{3x}{2}$ Remained.

$448x$	$40x$
<hr/>	$8x$
$40x - 19200 + 8x - 19200 + 160x + 240x$	$160x$
<hr/>	$240x$
$\frac{x-480}{4} + \frac{x+2400}{20} + \frac{x}{1} + \frac{3x}{2}$	<hr/>
<hr/>	$448x$
160	

$$\text{Then } 4x - \frac{448x}{160} \text{ is } \frac{640x - 448x}{160} = \frac{192x}{160} = \frac{6x}{5}$$

$$\text{Now } \frac{6x}{5} + 3000, \text{ is } \frac{6x + 15000}{5}$$

$$\begin{aligned} \therefore \frac{6x+15000}{5} &= \frac{3x}{2} \\ \therefore 12x+30000 &= 15x \\ \therefore 30000 &= 3x \\ \therefore x &= \frac{30000}{3} = 10000 \end{aligned}$$

$$\begin{array}{r} 12 \overline{) 30000} \\ \underline{2500} \\ 120 \\ \underline{120} \\ 0 \end{array}$$

$$\begin{array}{r} \text{Horse } 10000 \quad 40000 \\ \text{Foot } 30000 \quad \underline{3} \\ \text{Army } 40000 \quad 8 \overline{) 120000} \\ \underline{15000} \end{array}$$

$$\begin{array}{r} 2 \overline{) 10000} \\ \underline{500} \\ 120 \\ \underline{120} \\ 0 \end{array}$$

Foot Deserters - - 2380

Horse Deserters - - 620

In Garrison - - - 10000

Remained - - - - 15000

28000

40000

Slain or Taken 12000

12000

3000

15000 = to $\frac{1}{2}$ the
Foot.*Literally.*Let $120 = a$ $3000 = b$ $\frac{3x}{12} - a$, is $\frac{3x-12a}{12} = \frac{x-4a}{4}$ Foot Deserters. $\frac{x}{20} + a$, is $\frac{x+20a}{20}$ Horse Deserters. $\frac{4x}{4}$ is x , in Garrison. $\frac{12x}{8}$ is $\frac{3x}{2}$ Remained.

$$\begin{array}{r} 448x \\ 40x - 160a + 8x + 160a + 160x - 240x \\ \hline \frac{x-4a}{4} + \frac{x+20a}{20} + \frac{x}{1} + \frac{3x}{2} \\ \hline 160 \end{array}$$

$$\begin{array}{r} 40x \\ 8x \\ 160x \\ 240x \\ \hline 448x \end{array}$$

Then $4x - \frac{448x}{160}$ is $\frac{6x}{5}$ as afore.

And $\frac{6x}{5} - b$ is $\frac{6x - 5b}{5} = \frac{3x}{2}$

$$\therefore 12x - 10b = 15x$$

$$\therefore 10b = 3x$$

$$\therefore x = \frac{10b}{3} = 10000$$

35. To divide 100, twice into 2 Parts, so that the *major* Part of the *first Division* may be *treble* to the *minor* Part of the *second*, and the *major* Part of the *second* may be *double* to the *minor* Part of the *first*.

Numerically.

$$\boxed{x} \quad \boxed{100 - x} \quad \boxed{y} \quad \boxed{100 - y}$$

Then $x = 300 - 3y$. And $y = 200 - 2x$

$$\text{Now } \frac{200 - y}{2} = 300 - 3y \quad \therefore 2x + y = 200$$

$$\therefore 2x = 200 - y$$

$$\therefore 200 - y = 600 - 6y \quad \therefore x = \frac{200 - y}{2}$$

$$\therefore 200 + 5y = 600$$

$$\therefore 5y = 400$$

$$\therefore y = \frac{400}{5} = 80$$

$$\boxed{60} \quad \boxed{40} \quad \boxed{80} \quad \boxed{20}$$

$$\text{And } x = 300 - 3y = 60$$

Literally.

$$\text{Let } 100 = a$$

Then $x = 3a - 3y$. And $y = 2a - 2x$

$$\text{Now } \frac{2a - y}{2} = 3a - 3y \quad \therefore y + 2x = 2a$$

$$\therefore 2x = 2a - y$$

$$\therefore x = \frac{2a - y}{2}$$

$$\therefore 2a - y = 6a - 6y$$

$$\therefore 2a + 5y = 6a$$

$$5)400 \quad 2$$

$$\therefore 5y = 4a$$

$$80$$

$$\therefore y = \frac{4a}{5}$$

$$= 80, \text{ and } x = 3a - 3y = 60$$

36. To divide 30, twice into 2 Parts, so that the *major* Part of the *first Division*, with the *minor* Part of the *second*, may be 33; and the Sum of the *minor* Parts, subtracted from the Sum of the *major*, may leave 14.

Numerically.

$$\begin{array}{l} \boxed{x} \quad \boxed{30-x} \quad \boxed{y} \quad \boxed{30-y} \\ \text{Then } x+30-y=33, \text{ and } x+y-60+x+y=14 \\ \therefore x=3+y \qquad \therefore 2x+2y-60=14 \\ \qquad \qquad \qquad \therefore 2x+2y=74 \qquad 30-x \\ \qquad \qquad \qquad \therefore 2x=74-2y \qquad \underline{30-y} \\ \qquad \qquad \qquad \therefore x=74-2y \qquad \underline{60-x-y} \end{array}$$

$$\begin{array}{l} \text{Now } \frac{74-2y}{2}=3+y \quad \boxed{20} \quad \boxed{10} \quad \boxed{17} \quad \boxed{13} \\ \qquad \qquad \qquad \frac{13}{33} \qquad \frac{20}{37} \qquad \frac{10}{23} \\ \therefore 74-2y=6+2y \\ \therefore 74=6+4y \\ \therefore 68=4y \\ \therefore y=17 \\ \qquad \qquad \qquad \frac{17}{4}=17, \text{ and } x=3+y=20 \end{array}$$

Literally.

$$\begin{array}{l} \text{Let } 30=a \quad \boxed{x} \quad \boxed{a-x} \quad \boxed{y} \quad \boxed{a-y} \\ 33=b \qquad \qquad \qquad a-x \\ 14=c \quad \text{Then } x+a-y=b \quad \underline{a-y} \\ \qquad \qquad \qquad \therefore x=b-a+y \quad \underline{2a-x-y} \end{array}$$

$$\begin{array}{l} \text{Again, } x+y-2a+x+y=c \\ \therefore 2x+2y-2a=c \qquad c=14 \\ \therefore 2x+2y=c+2a \qquad 4a=120 \\ \therefore 2x=c+2a-2y \qquad \underline{134} \\ \therefore x=\frac{c+2a-2y}{2} \qquad \underline{2b-66} \\ \qquad \qquad \qquad \qquad \qquad \underline{4)68} \\ \qquad \qquad \qquad \qquad \qquad \qquad 17 \end{array}$$

$$\text{Now } \frac{c+2a-2y}{2}=b-a+y$$

$$\begin{array}{rcl}
 \therefore c + 2a - 2y = 2b - 2a + 2y & b = 33 \\
 \therefore c + 2a = 2b - 2a + 4y & a = 30 \\
 \therefore 4y + 2b = c + 4a & 3 \\
 \therefore 4y = c + 4a - 2b & y = \frac{17}{20} \\
 \therefore y = c + 4a - 2b & 20 \\
 \hline
 4 & = 17 \text{ \& } x = b - a + y = 20
 \end{array}$$

37. A Man, his Wife, and his Son's Age together, make 96 Years; the Husband's, with the Son's, makes the Wife's + 15; and the Wife's, with the Son's, makes the Husband's + 2. What was the Age of each?

Numerically.

x . The Father's Age.

y , the Mother's Age.

z , the Son's Age.

Then $x + y + z = 96$, and $x + z = y + 15$, & $y + z = x + 2$

$\therefore x = 96 - y - z$. $\therefore x = y + 15 - z$. $\therefore x = y + z - 2$

Now $96 - y - z = y + 15 - z$, & $96 - y - z = y + z - 2$

$\therefore 96 - y = y + 15$ $\therefore 96 - z = 2y + z - 2$

$\therefore 96 = 2y + 15$ 98 $\therefore 96 = 2y + 2z - 2$

$\therefore 2y = 81$ 81 $\therefore 98 = 2y + 2z$

$\therefore y = 40\frac{1}{2}$ $2 \overline{) 17}$ $\therefore 2z = 98 - 2y$

$\frac{2}{2} = 40\frac{1}{2}$ $\frac{81}{81}$ $\therefore z = \frac{98 - 2y}{2} = 8\frac{1}{2}$

and $x = 96 - y - z = 47$

$47 + 40\frac{1}{2} + 8\frac{1}{2} = 96$

$47 + 8\frac{1}{2} = 55\frac{1}{2} = 40\frac{1}{2} + 15 = 55\frac{1}{2}$

$40\frac{1}{2} + 8\frac{1}{2} = 49 = 47 + 2 = 49$.

Literally.

Let $96 = a$ Then $x + y + z = a$

$15 = b$ $\therefore x = a - y - z$

$2 = c$

Again,

Again $x + z = y + b$. And $y + z = x + c$

$$\therefore x = y + b - z \quad \therefore x = y + z - c$$

Now $a - y - z = y + b - z$. And $a - y - z = y + z - c$

$$\begin{array}{rcl} \therefore a - y = y + b & 96 & \therefore a - z = 2y + z - c \\ \therefore a = 2y + b & 15 & \therefore a = 2y + 2z - c \\ \therefore 2y = a - b & 2 \overline{) 81} & \therefore 2z = a + c - 2y \\ \therefore y = a - b & 40\frac{1}{2} & \therefore z = \frac{a + c - 2y}{2} = 8\frac{1}{2} \\ \hline & 2 & \end{array}$$

$$x = y + b - z = 47$$

$$40\frac{1}{2} = y$$

$$15 = b$$

$$55\frac{1}{2}$$

$$8\frac{1}{2} = z$$

$$47$$

$$96 = a$$

$$2 = c$$

$$98$$

$$81 = 2y$$

$$2 \overline{) 17}$$

$$8\frac{1}{2}$$

38. Three *Merchants*, from two different Fairs, met together at an Inn, where they reckon up their Gains, and find them to be the Sum of 780 *Crowns*. If you add the Gain of the *first* and *second* together, and from the Sum, subtract the Gain of the *third*, the Remainder will be the Gain of the *first* + 82. But if you add the Gain of the *second* and *third* together, and from that Sum, subtract the Gain of the *first*, the Remainder will be the Gain of the *third* - 43. What was the Gain of each?

Numerically.

x , the Gain of the first Merchant.

y , the Gain of the second.

z , the Gain of the third.

Then $x + y + z = 780$. And $x + y - z = x + 82$

$$\therefore x = 780 - y - z \quad \therefore y - z = 82$$

$$\therefore y = z + 82$$

Again

Again $y + z - x = z - 43$

$$\therefore y = x - 43 \quad \text{Now } 780 - y - z = y + 43$$

$$\therefore x = y + 43 \quad \therefore 780 - z = 2y + 43$$

$$\therefore 2y = 737 - z$$

$$\text{Then } \frac{737 - z}{2} = z + 82 \quad \therefore y = \frac{737 - z}{2}$$

$$\therefore 737 - z = 2z + 164 \quad y = z + 82 = 273$$

$$\therefore 737 = 3z + 164 \quad x = y + 43 = 316$$

$$\therefore 3z = 573$$

$$\therefore z = \frac{573}{3} = 191$$

$$\begin{array}{r} 316 \\ 273 \\ 191 \\ \hline 780 \end{array}$$

$$\begin{array}{r} 316 \\ 273 \\ \hline 589 \\ 191 \\ \hline 398 \end{array}$$

$$\begin{array}{r} 316 \\ 82 \\ \hline 398 \end{array}$$

$$\begin{array}{r} 273 \\ 191 \\ \hline 464 \\ 316 \\ \hline 148 \end{array}$$

$$\begin{array}{r} 191 \\ 43 \\ \hline 148 \end{array}$$

Literally.

Let $780 = a$ Then $x + y + z = a$

$$\therefore 82 = b \quad \therefore x = a - y - z$$

$$\therefore 43 = c$$

Again $x + y - z = x + b$. And $y + z - x = z - c$

$$\therefore y - z = b \quad \therefore y = x - c$$

$$\therefore y = b + z \quad \therefore x = y + c$$

$$\text{Now } a - y - z = y + c \quad \text{Then } \frac{a - c - z}{2} = b + z$$

$$\therefore 2y = a - c - z \quad \therefore a - c - z = 2b + 2z$$

$$\therefore y = \frac{a - c - z}{2} \quad \therefore a - c = 2b + 3z$$

$$\therefore 3z = a - c - 2b$$

$$\therefore z = \frac{a - c - 2b}{3} = 191$$

$$\begin{array}{rcl}
 780 & = & a \\
 43 & = & c \\
 \hline
 737 & & \\
 164 & = & 2b \\
 3 \overline{) 573} & & \\
 191 & &
 \end{array}
 \qquad
 \begin{array}{rcl}
 z & = & 191 \\
 b & = & 82 \\
 \hline
 y = b + z & = & 273 \\
 c & = & 43 \\
 \hline
 x = y + c & = & 316
 \end{array}$$

39. Three *Persons*, *A*, *B*, and *C*, owe a certain Sum of Money, so that *A* and *B* together owe 210 *Crowns*, *B* and *C* 290, and *C* and *A* 300. What did each of them owe?

Numerically.

x, *Crowns*, the Debt of *A*.

y, *Crowns*, the Debt of *B*.

z, *Crowns*, the Debt of *C*.

Then $x + y = 210$. And $y + z = 290$ & $x + z = 300$

$\therefore x = 210 - y$ $\therefore y = 290 - z$ $\therefore x = 300 - z$

Now $210 - y = 300 - z$ Then $z - 90 = 290 - z$

$\therefore 210 = y + 300 - z$ $\therefore 2z - 90 = 290$

$\therefore z - 90 = y$ $\therefore 2z = 380$

$y = 290 - z = 100$ $\therefore z = \frac{380}{2} = 190$

$x = 210 - y = 110$

100

100

110

100

190

190

210

290

300

Literally.

Let $210 = a$ Then $x + y = a$ & $y + z = b$

$290 = b$ $\therefore x = a - y$ $\therefore y = b - z$

$300 = c$ Again $x + z = c$

$\therefore x = c - z$

Now $a - y = c - z$ Then $a - c + z = b - z$

$\therefore a = y + c - z$ $\therefore a - c + 2z = b$

$\therefore y = a - c + z$ $\therefore 2z = b + c - a$

$\therefore z = \frac{b + c - a}{2} = 190$

O o

290

$$\begin{array}{r}
 290 = b \\
 300 = c \\
 \hline
 590 \\
 210 = a \\
 \hline
 2 \overline{) 380} \\
 190
 \end{array}$$

$$y = b - z = 100$$

$$x = c - z = 110$$

$$300 = c$$

$$190 = z$$

$$110$$

$$\begin{array}{r}
 290 = b \\
 190 = z \\
 \hline
 100
 \end{array}$$

40. To find *three* Numbers, so that the *first* with $\frac{1}{2}$ the rest, the *second* with $\frac{1}{3}$ the rest, and the *third* with $\frac{1}{4}$ the rest; may each make 34.

Numerically.

x , the first Number.

y , the second Number.

z , the third Number.

$$\text{Then } x + \frac{y+z}{2} = 34, \text{ and } y + \frac{x+z}{3} = 34$$

$$\therefore 2x + y + z = 68$$

$$\therefore 2x = 68 - y - z$$

$$\therefore x = \frac{68 - y - z}{2}$$

$$\therefore 3y + x + z = 102$$

$$\therefore x = 102 - 3y - z$$

$$\text{Again } z + \frac{x+y}{4} = 34$$

$$\therefore 4z + x + y = 136$$

$$\therefore x = 136 - 4z - y$$

$$\text{Now } \frac{68 - y - z}{2} = 102 - 3y - z$$

$$\therefore 68 - y - z = 204 - 6y - 2z$$

$$\therefore 68 + 5y - z = 204 - 2z$$

$$\therefore 5y - z = 136 - 2z$$

$$\therefore 5y = 136 - z$$

$$\therefore y = \frac{136 - z}{5}$$

5

Again

$$\text{Again } \frac{68 - y - z}{2} = 136 - 4z - y$$

$$\therefore 68 - y - z = 272 - 8z - 2y$$

$$\therefore 68 + y - z = 272 - 8z$$

$$\therefore y - z = 204 - 8z$$

$$\therefore y = 204 - 7z$$

$$\text{Then } \frac{136 - z}{5} = 204 - 7z$$

$$\therefore 136 - z = 1020 - 35z$$

$$\therefore 136 + 34z = 1020$$

$$\therefore 34z = 884$$

$$\therefore z = \frac{884}{34} = 26$$

$$y = 204 - 7z = 22$$

$$x = 102 - 3y = 10$$

$$\begin{array}{r} 34 \overline{)884} (26 \\ \underline{204} \\ 0 \end{array}$$

$$\begin{array}{r} 22 \quad 26 \\ 3 \quad 7 \\ \hline 66 \quad 182 \\ 26 \quad 204 \\ \hline 92 \quad 22 \\ 102 \quad 22 \\ \hline 10 \quad 10 \\ 4 \overline{)32} \\ 8 \end{array}$$

$$\begin{array}{r} 10 \quad 22 \quad 26 \quad 22 \quad 10 \\ 24 \quad 12 \quad 8 \quad 2 \overline{)48} \quad 3 \overline{)36} \\ \hline 34 \quad 34 \quad 34 \quad 24 \quad 12 \end{array}$$

Literally.

$$\text{Let } 34 = a. \quad \text{Then } x + \frac{y + z}{2} = a$$

$$\therefore 2x + y + z = 2a$$

$$\therefore 2x = 2a - y - z$$

$$\therefore x = \frac{2a - y - z}{2}$$

$$\text{Again } y + \frac{x + z}{3} = a, \text{ and } z + \frac{x + y}{4} = a$$

$$\therefore 3y + x + z = 3a \quad \therefore 4z + x + y = 4a$$

$$\therefore x = 3a - 3y - z \quad \therefore x = 4a - 4z - y$$

O o 2

Now

$$\text{Now } \frac{2a - y - z}{2} = 3a - 3y - z$$

$$\therefore 2a - y - z = 6a - 6y - 2z$$

$$\therefore 2a + 5y - z = 6a - 2z$$

$$\therefore 5y - z = 4a - 2z$$

$$\therefore 5y = 4a - z$$

$$\therefore y = \frac{4a - z}{5}$$

$$\text{Again } \frac{2a - y - z}{2} = 4a - 4z - y$$

$$\therefore 2a - y - z = 8a - 8z - 2y$$

$$\therefore 2a + y - z = 8a - 8z$$

$$\therefore y - z = 6a - 8z$$

$$\therefore y = 6a - 7z$$

$$\text{Then } \frac{4a - z}{5} = 6a - 7z$$

$$\therefore 4a - z = 30a - 35z$$

$$\therefore 4a + 34z = 30a$$

$$\therefore 34z = 26a$$

$$\therefore z = \frac{26a}{34} = \frac{13a}{17}$$

$$y = 6a - 7z = 22$$

$$x = 3a - 3y - z = 10$$

$$34 = a$$

$$26$$

$$204$$

$$68$$

$$34)884(26$$

$$204$$

$$0$$

$$26 = z \quad 34 = a$$

$$182 = 7z \quad 102 = 3a$$

$$22 = y \quad 66 = 3y$$

$$66 = 3y \quad 26 = z$$

$$10$$

The

The EXTRACTION of ROOTS.

Note, **W**HEN a small Figure is set at the Head of a Letter in any Quantity, it denotes the *Power* of that Letter, viz. If 2, the *Square* of it; if 3, the *Cube*, &c.

Thus a^2 , is aa ; a^3 , is aaa ; a^4 , is $aaaa$, &c.

Powers, by some called *Involution* and *Evolution*, are nothing else but *Products*, arising from a continual *Multiplication*, of any assumed *Side* or *Root*: For supposing, that any *Root* is the *first Power* of itself, or that the *first Power* arises when the *Root* is multiplied into *Unity*; if then the same *Root* be multiplied into itself, it produces its *Square*, which is the *second Power*, whose *Index*, or Numerical Exponent, is 2. The *Square*, multiplied by the same *Root*, produces its *Cube*, which is the *third Power*, whose *Index* is 3; and so on, *ad infinitum*, according to the Number of their Dimensions, of which they are compounded. But the more compounded the *Powers* are, the higher they are esteemed to be; and therefore they are not distinguished, by bare Name or Denomination only, but by their Order and Degrees, which constitute a Sort of *Progressional Scale*, in which the *Quantities* do, as it were, ascend and descend.

But we must also observe, that the *Powers*, which are above the *Cubick*, are not only produced from the *Root* itself, multiplied into the Degree next going before, but also from the mutual *Multiplication* of the *inferior Powers*, as their very Names declare: For the *Biquadrate*, or *Square Square*, which is the *fourth Power*, arises not only from the *Root*, multiplied into the *Cube*, but also from the *Square*, multiplied into itself. Hence this *Power*, that is the *Biquadrate*, is also a *Square*; and the like you may judge of the rest.

Every *Root* is either simple, as a , $2a$, $3a$, &c. or compounded, that is, *Binomials* or *Trinomials*, &c. as $a+b$, $a-b$, $a+b+c$, &c.

The *Genesis* of any *Power*, is the Production of the same *Power*, by *Multiplication* from a given *Root*, and therefore is very easy from a *Simple Root*. For Instance, if the *Root* be a , its *Square* will be aa , *Cube* aaa , *Biquadrate* $aaaa$, &c. If the *Root* be $2a$, the *Square* will be $4aa$, *Cube* $8aaa$, *Biquadrate* $16aaaa$, &c. for the Numbers set before the Letters are multiplied by themselves.

Nor is the Production of *Powers* from a *Compound Root* difficult, tho' it require more Labour, as will appear by the *Involution* of the *Root* $a+b$, as followeth.

$$\begin{array}{r}
 a+b \\
 a+b \\
 \hline
 aa+ab \\
 ab+bb \\
 \hline
 aa+2ab+bb. \text{ Square.} \\
 a+b
 \end{array}$$

$$\begin{array}{r}
 aaa+2aab+abb \\
 aab+2abb+bbb \\
 \hline
 aaa+3aab+3abb+bbb \text{ Cube.} \\
 a+b
 \end{array}$$

$$\begin{array}{r}
 a^4+3a^3b+3a^2b^2+ab^3 \\
 a^3b+3a^2b^2+3ab^3+b^4 \\
 \hline
 a^4+4a^3b+6a^2b^2+4ab^3+b^4 \text{ Biquadrat.} \\
 a+b
 \end{array}$$

$$\begin{array}{r}
 a^5+4a^4b+6a^3b^2+4a^2b^3+ab^4 \\
 a^4b+4a^3b^2+6a^2b^3+4ab^4+b^5 \\
 \hline
 a^5+5a^4b+10a^3b^2+10a^2b^3+5ab^4+b^5 \text{ 5th Power.} \\
 a+b
 \end{array}$$

$$\begin{array}{r}
 a^6+5a^5b+10a^4b^2+10a^3b^3+5a^2b^4+ab^5 \\
 a^5b+5a^4b^2+10a^3b^3+10a^2b^4+5ab^5+b^6 \\
 \hline
 a^6+6a^5b+15a^4b^2+20a^3b^3+15a^2b^4+6ab^5+b^6 \text{ 6th Power}
 \end{array}$$

Thus $a+b$ involved, gives a *Theorem* by which the *Root* of any *Power* may be extracted, and will be evident by the following Examples.

Note : In the Extraction of the *Root* of any *Quantity*, you must place a *Point* over the last *Figure* ; so pointing off *two* *Figures* for the *Square*, *three* for the *Cube*, *four* for the *Biquadrat*, &c. the *Number* of *Points* shews the *Number* of *Figures* of which the *Root* will consist.

$$aa + 2ab + bb$$

$\begin{array}{r} 2 \quad 1 \quad 9 \quad 6 \quad 9 \\ \sqrt{48267.9482} \\ 4=aa \\ \hline 1 : 2a=4)8 \\ \quad 4=2ab \\ \hline 2=a \\ 1=b \quad 4^2 \\ \quad 1=bb \\ \hline 2 : 2a=42)416 \\ \quad 378=2ab \\ \hline 21=a \\ 9=b \quad 387 \\ \quad 81=bb \\ \hline 3 : 2a=438)3069 \\ \quad 2628=2ab \\ \hline 219=a \\ 6=b \quad 4414 \\ \quad 36=bb \\ \hline 4 : 2a=4392)43788 \\ \quad 39528=2ab \\ \hline 2196=a \\ 9=b \quad 42602 \\ \quad 81=bb \\ \hline 42521 \end{array}$			$\begin{array}{r} 2=a \\ 2=a \\ \hline 4=aa \\ \hline 21=a \\ 2 \\ \hline 42=2a \\ 9 \\ \hline 378=2ab \\ \hline 2196=a \\ 2 \\ \hline 4392=2a \\ 9 \\ \hline 39528=2ab \end{array}$	$\begin{array}{r} 2=a \\ 2 \\ \hline 4=2a \\ 1=b \\ \hline 4=2ab \\ \hline 219=a \\ 2 \\ \hline 438=2a \\ 6=b \\ \hline 2628=2ab \end{array}$
---	--	--	--	---

$$aaa + 3aab + 3abb + bbb$$

$$\begin{array}{r} 907.86 \\ \sqrt{748269714.832654} \\ 729 = aaa \end{array}$$

In the first Step.

$$\begin{array}{r} 1. \\ 3aa = 24300 \quad 192697 \\ 170100 = 3aab \end{array}$$

$$\begin{array}{r} 90 = a \\ 7 = b \\ 225971 \\ 13230 = 3abb \\ 2127414 \end{array}$$

$$\begin{array}{r} 90 = a \\ 90 = a \\ 8100 = aa \\ 3 \\ 24300 = 3aa \\ 7 \end{array}$$

$$\begin{array}{r} 2. \\ 3aa = 2467947 \quad 21270718 \\ 19743576 = 3aab \\ 107 = a \\ 8 = b \\ 15271423 \\ 174144 = 3abb \end{array}$$

$$\begin{array}{r} 170100 = 3aab \\ 90 = a \\ 3 \\ 270 = 3a \\ 49 = bb \end{array}$$

$$\begin{array}{r} 3. \\ 3aa = 247230252 \quad 1509722806 \\ 1483381512 = 3aab \end{array}$$

$$\begin{array}{r} 9078 = a \\ 6 = b \\ 263412945 \\ 980424 = 3abb \\ 2624325214 \\ 216 = bbb \\ 2624324998 \end{array}$$

$$\begin{array}{r} 13230 = 3abb \\ 7 = b \\ 7 \\ 49 = bb \\ 7 \\ 343 = bbb \end{array}$$

The second and third Steps are wrought after the same Manner as the first Step, which I leave to the Learner.

$$aaaa \mid 4aaab \mid 6aabb \mid 4abbb \mid bbbb$$

$$\sqrt[4]{\begin{array}{r} 5 \quad 3 \quad . \quad 0 \quad 9 \\ 7948267.00000000 \end{array}}$$

$$625 = aaaa$$

$$5 = a$$

$$5 = a$$

1.

$$4aaa = 500) 1698$$

$$25 = aa$$

$$25 = aa$$

$$a = 5$$

$$1500 = 4aaab$$

$$6$$

$$5$$

$$b = 3$$

$$1982$$

$$150 = 6aa$$

$$125 = aaa$$

$$125 = aaa$$

$$1350 = 6aabb$$

$$9 = bb$$

$$5$$

$$4$$

$$500 = 4aaa$$

$$6326$$

$$1350 = 6aabb$$

$$625 = aaaa$$

$$540 = 4abbb$$

$$3$$

$$1500 = 4aaab$$

$$57867$$

$$20 = 4a$$

2.

$$81 = bbbb$$

$$27 = bbb$$

$$4aaa = 595508000) 5778600000$$

$$540 = 4abbb$$

$$5359572000 = 4aaab$$

$$530 = a$$

$$bb = 9$$

$$9 = b$$

$$4190280000$$

$$bb = 9$$

$$136517400 = 6aab$$

$$81 = bbbb$$

$$40537626000$$

$$1545480 = 4abbb$$

$$405360805200$$

$$6561 = bbbb$$

$$405360798639$$

The first Step, in its Operation exhibited, I think sufficient to shew the whole Proceeding.

Extraction of Roots.

$$a^5 + 5a^4b + 10a^3b^2 + 10a^2b^3 + 5ab^4 + b^5$$

$$\sqrt[5]{\begin{array}{cccccc} & 2 & & 3 & & 7 & & 4 \\ 754287694278.42597 \end{array}}$$

$$1 : 5a^4 = 80)434$$

$$2 = a$$

$$240 = 5a^4b$$

$$2 = a$$

$$2 = a$$

$$1942$$

$$4 = a^2$$

$$3 = b$$

$$720 = 10a^3bb$$

$$2 = a$$

$$16 = a^4$$

$$12228$$

$$8 = a^3$$

$$5$$

$$1080 = 10a^2b^3$$

$$2 = a$$

$$80 = 5a^4$$

$$111487$$

$$16 = a^4$$

$$3$$

$$810 = 5ab^4$$

$$2 = a$$

$$240 = 3a^4b$$

$$1106776$$

$$32 = a^5$$

$$243 = b^5$$

$$2 : 5a^4 = 1399205)11065339$$

$$3 = b$$

$$9794435 = 5a^4b$$

$$3 = b$$

$$23 = a$$

$$12709044$$

$$9 = b^2$$

$$7 = b$$

$$5961830 = 10a^3b^2$$

$$3 = b$$

$$67472142$$

$$27 = b^3$$

$$1814470 = 10a^2b^3$$

$$3 = b$$

$$656576727$$

$$81 = b^4$$

$$276115 = 5ab^4$$

$$3$$

$$6563006128$$

$$243 = b^5$$

$$16807 = b^5$$

$$3 : 5a^4 = 15774782805)65629893214$$

$$63099131220 = 5a^4b$$

$$237 = a$$

$$25307619942$$

$$4 = b$$

$$2129928480 = 10a^3b^2$$

$$231776914625$$

$$35948160 = 10a^2b^3$$

$$2317409664659$$

$$303360 = 5ab^4$$

$$23174093612997$$

$$1024 = b^5$$

$$23174093611973$$

$$\begin{array}{r} 8 = a^3 \\ 10 \\ \hline 80 = 10a^3 \\ 9 = bb \\ \hline 720 = 10a^3bb \end{array}$$

$$\begin{array}{r} 4 = a^2 \\ 10 \\ \hline 40 = 10a^2 \\ 27 = b^3 \\ \hline 1080 = 10a^2b^3 \end{array}$$

$$\begin{array}{r} 81 = b^4 \\ 10 = 5a \\ \hline 810 = 5ab^4 \end{array}$$

In the like Manner, the *Root* of any *Quantity* in any *Power*, may be obtained; the which I shall leave to the *Learner*, holding the preceeding *Examples* sufficient for his *Instruction*.

The *Proof* of the preceeding *Examples* I have omitted, as needless; but I would advise the *Learner* to prove every *Step* as he proceeds, because it will facilitate the *Work* of the ensuing *Steps*, as will appear by the *Proof* of the *first Step* in the preceeding *Example*, as followeth.

$$\begin{array}{r} 23 = a \\ 23 = a \\ \hline 69 \\ 46 \\ \hline 529 = aa \\ 23 = a \\ \hline 1587 \\ 1058 \\ \hline 12167 = aaa \\ 23 = a \\ \hline 36501 \\ 24334 \\ \hline 279841 = aaaa \\ 23 = a \\ \hline 839523 \\ 559682 \\ \hline 6436343 = aaaaa \\ 1106533 \\ \hline 7542876 \end{array}$$

The Remainder added - - -

Thus having before you $279841 = aaaa$, which multiplied by 5, you very easily obtain the *Divisor* for the next Step: Then as easily follows $5aaaaab$, $10aaabbb$, $10aabbbb$, $5abbbbb$, and $bbbbb$, as is plain in the preceeding *Example*.



ARITHMETICAL PROGRESSION,

IS when any *Rank*, or *Series* of Numbers, do either *increase*, or *decrease*, by an *equal Interval*, or *common Difference*.

As $1 : 2 : 3 : 4 : 5$, &c. Or $2 : 4 : 6 : 8 : 10 : 12$, &c.

Or as $1 : 3 : 5 : 7 : 9$, &c. Or $1 : 4 : 7 : 10 : 13$, &c.

If any *three* Numbers be in *Arithmetical Progression*, the Sum of the *two Extremes* will be equal to the *Double* of the *Mean*.

$$\begin{array}{rcl}
 \text{As } 2 : 3 : 4. & \text{Or } 5 : 7 : 9. & \text{Or } 7 : 10 : 13 \\
 \frac{4}{6} = \frac{2}{6} & \frac{9}{14} = \frac{2}{14} & \frac{2}{20} = \frac{7}{20}
 \end{array}$$

If any *four* Numbers be in *Arithmetical Progression*, the Sum of the *two Extremes* will be equal to the Sum of the *two Means*.

$$\begin{array}{rcl}
 \text{As } 2 : 3 : 4 : 5. & \text{Or } 1 : 4 : 7 : 10 \\
 \frac{5}{7} = \frac{3}{7} & \frac{10}{11} = \frac{4}{11}
 \end{array}$$

From hence it is evident, that if never so many Numbers be in *Arithmetical Progression*, the Sum of the *two Extremes* will always be equal to the Sum of any *two Means*, that are *equally* distant from

from those *Extremes*; and if the Number of Terms be odd, the Sum of the *two Extremes* will be *double* to the *middle Term*.

As $2 : 4 : 6 : 8 : 10 : 12 : 14 : 16 : 18 : 20$

$$\begin{array}{r} 20 \\ \hline 22 \end{array} \quad \begin{array}{r} 16 \\ \hline 22 \end{array} \quad \begin{array}{r} 12 \\ \hline 22 \end{array} \quad \begin{array}{r} 8 \\ \hline 22 \end{array} \quad \begin{array}{r} 4 \\ \hline 22 \end{array}$$

Again, $4 : 7 : 10 : 13 : 16 : 19 : 22$

$$\begin{array}{r} 22 \\ \hline 26 \end{array} \quad \begin{array}{r} 2 \\ \hline 26 \end{array} \quad \begin{array}{r} 7 \\ \hline 26 \end{array}$$

Now it will be very easy to conceive, that if the Sum of the *two Extremes* be multiplied by the Number of Terms, the Product will be *double* the Sum of all the Terms; and also, that if the Difference betwixt the *two Extremes* be divided by the Number of Terms, less 1, the *Quotient* will be the *Common Difference* of the *Series*.

As $3 : 6 : 9 : 12 : 15 : 18$

$$\begin{array}{r} 18 \\ \hline 21 \\ \hline 6 \text{ the Number of Terms.} \\ \hline 126 \end{array} \quad \begin{array}{r} 3 \\ 6 \\ 9 \\ 12 \\ 15 \\ 18 \\ \hline 63 \\ 2 \\ \hline 126 \end{array}$$

From 18
Take 3
Numb. of Terms, } 5)15(3 the Common
less 1, is - - - } Difference.

Now for the easier resolving such *Questions* as may depend upon *Progressional Quantities*,

Let the first Term of any *Progressional Quantity* be equal to - - - - - a

The *Last Term* - - - - - u

The *Number* of Terms - - - - - n

The *Common Difference* - - - - - d

The *Sum* of all the Terms - - - - - s

Then

Then $\frac{u-a}{n-1} = d$. That is, the Difference betwixt the *Extremes*, divided by the *Number* of *Terms*, less 1, is equal to the *Common Difference*.

Again, $na + nu = 2s$. That is, the Sum of the *Extremes*, multiplied by the *Number of Terms*, is equal to twice the Sum of all the *Terms*.

Now by the Help of these *two Equations*, if any *three* of the said *five Parts* be given, the other *two* may be very easily found, as will appear by the following *Questions*.

1. Suppose a *Basket* were fixed in a certain Place, and 20 *Stones* laid in a right Line, the *first Stone* 2 *Yard* from the *Basket*, and the rest each a *Yard* from one another: It is required, what Distance a Man must go, to gather up those *Stones* singly, one after another, returning with every *Stone*, to put it into the *Basket*.

In this Question we have the *first Term* 2 *Yards*, the *last Term* 40 *Yards*, and the *Number* of *Terms* 20, to find the Sum of all the *Terms*.

$$\text{Then } na + nu = 2s$$

$$\therefore s = \frac{na + nu}{2} = 420 \text{ Yards.}$$

$$20 = n$$

$$2 = a$$

$$40 = na$$

$$40 = u$$

$$20 = n$$

$$800 = nu$$

$$40$$

$$2)840(420$$

$$a = 2$$

$$u = 40$$

$$n = 20$$

2. A *Debtor* agrees with his *Creditor*, to pay his Debt at 12 several *Payments*, in *Arithmetical Progression*. The *first Payment* to be 10 *l.* and the *last*

last 98. What was the whole Debt, and what must be each Payment?

In this *Question* we have the *first Term* 10*l.* and the *last* 98, and the *Number of Terms* 12, to find the *Sum* of all the *Terms*, and *common Difference*.

$$\begin{array}{l} a = 10 \quad \text{Then } na + nu = 2s \\ u = 98 \quad \therefore s = \frac{na + nu}{2} = 648 \text{ £.} \\ n = 12 \end{array} \quad \begin{array}{l} 98 = u \\ 12 = n \\ 1176 = nu \end{array}$$

$$\begin{array}{l} \text{Again } \frac{u-a}{n-1} = d = 8 \\ n-1 = 11 \end{array} \quad \begin{array}{l} 98 = u \quad 12 = n \\ 10 = a \quad 10 = a \\ 120 = na \\ 1176 = nu \\ 120 = na \\ 2 \overline{) 1296} \\ 648 \text{ £.} \end{array}$$

1 Payment - - - 10	7 Payment - - - 58
2 - - - - - 18	8 - - - - - 66
3 - - - - - 26	9 - - - - - 74
4 - - - - - 34	10 - - - - - 82
5 - - - - - 42	11 - - - - - 90
6 - - - - - 50	12 - - - - - 98
	<hr/>
180	468
468	
<hr/>	
648	

3. Of 8 *Brethren*, the *youngest* was 27 *Years* old, the *eldest* 50; each differed alike in *Age* from *o-*ther. What was the *Age* of each?

$$\begin{array}{l} a = 27 \quad \text{Then } \frac{u-a}{n-1} = d = 3\frac{2}{7} \\ u = 50 \\ n = 8 \end{array} \quad \begin{array}{l} 50 = n \\ 27 = a \\ n-1 = 7 \end{array}$$

1 - - - - 27	5 - - - - 40 $\frac{2}{7}$
2 - - - - 30 $\frac{2}{7}$	6 - - - - 43 $\frac{4}{7}$
3 - - - - 33 $\frac{4}{7}$	7 - - - - 46 $\frac{6}{7}$
4 - - - - 36 $\frac{6}{7}$	8 - - - - 50

4. One had divers Sons, the *youngest* 6 Years old, the *eldest* 40; each exceeding the other 2 Years. How many Sons had he?

$$\begin{aligned} a &= 6 \\ u &= 40 \\ d &= 2 \end{aligned}$$

$$\text{Then } \frac{u-a}{n-1} = d$$

$$\therefore u-a = nd-d$$

$$\therefore u-a+d = nd$$

$$\therefore n = \frac{u-a+d}{d}$$

$$\frac{40-6+2}{2} = 18$$

$$\begin{aligned} 34 &= u-a \\ 2 &= d \end{aligned}$$

$$d = 2 \overline{) 36} \\ \underline{18}$$

5. One travelled 50 Miles, encreasing his Journey every Day 2 Miles, till at 5 Days end he finished it. How many Miles did he travel every Day?

$$s = 50$$

$$d = 2$$

$$n = 5$$

$$\text{Then } na + nu = 2s$$

$$\therefore na = 2s - nu$$

$$\therefore a = \frac{2s - nu}{n}$$

$$\text{Again } \frac{u-a}{n-1} = d$$

$$\therefore u-a = nd-d$$

$$\therefore u = nd-d+a$$

$$\therefore a = u-nd+d$$

$$\text{Then } \frac{2s-nu}{n} = u+d-nd$$

$$100 = 2s$$

$$50 = nnd$$

$$150$$

$$10 = nd$$

$$10 \overline{) 140} \quad 14$$

$$\underline{0}$$

$$\therefore 2s-nu = nu-+nd-nnd$$

$$\therefore 2s = nd-nnd+2nu$$

$$\therefore 2nu = 2s-nnd+nd$$

$$\therefore u = \frac{2s-nnd+nd}{2} = 14$$

6. One had 40 Cloths, worth 5*l.* each. He sold the *first* for 1 Crown, and increased his Piece 2 Crowns more for every Cloth, than was paid for the former. What did he receive for his Cloths, and how much did he gain by them?

$$\begin{array}{lll} n=40 & \text{Then } nd-d=u-a & a=1 \\ a=1 & \therefore a+nd-d=u=79 & nd=80 \\ d=2 & \text{Again } na+nu=2s & \frac{81}{81} \\ & \therefore s=\frac{na+nu}{2}=1600 & d=2 \\ & & \underline{79} \end{array}$$

$$\begin{array}{rcl} 40=na & 400 & 79=u \\ 3160=nu & 200 & 40=n \\ 2 \overline{)3200} & \text{£. 200 gained} & 3160=nu \\ 4 \overline{)1600} \text{ Crowns} & & \\ \underline{400} \text{ £.} & & \end{array}$$

7. A Sum of Money is to be paid in 12 Days, paying the *first* Day 10 *l.* and increasing every Payment after 5 *l.* till the Days are expired. What is the Debt?

$$\begin{array}{lll} n=12 & \text{Then } a+nd-d=u=65 & a=10 \\ d=5 & \text{And } s=\frac{na+nu}{2}=450 & nd=60 \\ a=10 & & \underline{70} \\ 120=na & 65=u & d=5 \\ 780=nu & 12=n & \underline{65} \\ 2 \overline{)1900} & 780=nu & \\ \underline{450} & & \end{array}$$

8. 685*l.* is to be disbursed by *Aritbmetical Progression*, to how many I know not; but the *first* had 19*l.* and the *last* 118*l.* How many did receive the Money, and how much had each?

Qq

$s=685$

$$s = 685$$

$$a = 19$$

$$u = 118$$

$$\text{Then } na + nu = 2s$$

$$\therefore n = \frac{2s}{a+u} = 10$$

$$118 = u$$

$$685 = s$$

$$19 = a$$

$$2$$

$$n-1=9 \begin{array}{r} 99 \\ 11 \\ 0 \end{array} \quad 137 \begin{array}{r} 1370 \\ 10 \\ 0 \end{array}$$

$$1 - - 19$$

$$2 - - 30$$

$$3 - - 41$$

$$4 - - 52$$

$$5 - - 63$$

$$6 - - 74$$

$$7 - - 85$$

$$8 - - 96$$

$$9 - 107$$

$$10 - 118$$

$$\underline{\quad\quad\quad} 685$$

$$\text{Again } d = 11 \frac{u-a}{n-1}$$

$$118 = u$$

$$19 = a$$

$$137 = a+u$$



GEOMETRICAL PROGRESSION,

IS when any *Rank*, or *Series* of Numbers, do increase by one *common Multiplier*, or decrease by one *common Divisor*, called the *Ratio*.

As 2, 4, 8, 16, &c. here 2 is the *common Multiplier*. Or 128, 64, 32, 16, &c. here 2 is the *common Divisor*.

Again, 2, 6, 18, 54, &c. here 3 is the *common Multiplier*. Or 128, 32, 8, 2. here 4 is the *common Divisor*.

1. If *three* Numbers are *proportional*, the *Product* of the *Extremes* will be *equal* to the *Square* of the *Mean*.

$$\begin{array}{ccccccc} 2 : 4 : 8 & 16 : 8 : & 128 & : & 32 : 8 \\ \frac{4}{2} & \frac{2}{4} & \frac{4}{8} & & \frac{8}{32} \\ \frac{16}{16} = 16 & \frac{64}{64} = 64 & \frac{1024}{1024} & & \frac{96}{1024} \end{array}$$

2. If

2. If four Numbers are *proportional*, the *Product* of the *Extremes* will be equal to the *Product* of the *Means*.

$$\begin{array}{ccccccc} 2 : 4 : 8 : 16 & 32 : 16 : 8 : 4 & 128 : 32 : 8 : 2 \\ \underline{8} & \underline{2} & \underline{4} & \underline{8} & \underline{2} & \underline{8} \\ 32 = 32 & 128 = 128 & 256 = 256 \end{array}$$

From hence it follows, that if never so many Numbers be in *continual Proportion*, the *Product* of the *two Extremes* will be equal to the *Product* of any *two Means*, equally distant from the *Extremes*.

3. If never so many Numbers are *proportional*, it will be, as any one of the *Antecedents* is to its *Consequent*; so will the *Sum* of all the *Antecedents* be, to the *Sum* of all the *Consequents*.

Note, all the Terms except the *last*, are *Antecedents*; and all the Terms except the *first*, are *Consequents*.

$$\begin{array}{ccccccc} 2 : 4 : 8 : 16 : 32 : 64 : 128 : 256 \\ \underline{8} & & & & & & \underline{64} \\ 16 & & & & & & 32 \\ 32 & & & & & & 16 \\ 64 & & & & & & 8 \\ 128 & & & & & & 4 \\ 256 & & & & & & 2 \\ \hline \text{Sum } 508 \text{ Consequents} & & & & \text{Sum } 254 \text{ Antecedents} \end{array}$$

$$\begin{array}{ccccccc} \text{As } 2 \dots 4 :: 254 \dots 508 & \text{As } 32 \dots 64 :: 254 \dots 508 \\ \underline{4} & \underline{2} & & \underline{64} & \underline{32} \\ 1016 = 1016 & & & 1016 & 1016 \\ & & & 1524 & 1524 \\ & & & 16256 = 16256 \end{array}$$

Q q 2

These

These Things being premised, such *Equations* may be deduced, as will solve all such *Questions*, as are usually proposed about *Quantities* in *Geometrical Proportion*, continued.

Let the *first* Term be \equiv to ----- a
 The *common* Ratio ----- r
 The *last* Term ----- u
 The *Sum* of all the Terms ----- s

$$\text{As } a \dots ar :: s - u \dots s - a$$

$$\text{Then } ars - aru = as - aa$$

$$\therefore rs - ru = s - a$$

To find the *last* Term of any *Geometrical Series* of Numbers.

If the *first* Term of any *Geometrical Series* be an *Unit*; then the *first* Term of the *Arithmetical Series* must be a *Cypher*.

If the *first* Term of any *Geometrical Series* be *more* than an *Unit*, then the *first* Term of the *Arithmetical* must be an *Unit*.

1. Arithmetical Terms 0 : 1 : 2 : 3 : 4 : 5 : 6
 Geometrical Terms 1 : 2 : 4 : 8 : 16 : 32 : 64
2. Arithmetical Terms 1 : 2 : 3 : 4 : 5 : 6 : 7
 Geometrical Terms 3 : 9 : 27 : 81 : 243 : 729 : 2187

The *Product* of any two *Geometrical Terms*, gives that *Geometrical Term*, which is the *Sum* of the two *Arithmetical Terms* over them.

Thus, in the *first* Case, $16 \times 4 =$ to 64, the *Sum* of 4 and 2, viz. the 6th Term.

Again, in the *second* Case, $81 \times 9 = 729$, the 6th Term.

Now, 6th Term multiplied by the 4th, will produce the 10th, and the 10th multiplied by the 10th will produce the 20th, &c. And therefore, from hence

hence it is plain, that any Term in a Geometrical Series, by the Help of some of the beginning Terms, may be produced, without shewing all the Terms.

1. One bought an *Horse* after this Manner. The *Horse* had 4 *Shoes*, every *Shoe* contained 6 *Nails*, in all 24. He thinking to have a good Bargain, agreed to pay a *Farthing* for the *first* Nail, and double it to the *last*. What did he pay for the *Horse*?

$$a = 1$$

$$r = 2$$

Arithmetic Terms 0: 1: 2: 3: 4: 5: 6

Geometrical Terms 1: 2: 4: 8: 16: 32: 64

6th Term - - - - 64

6th Term - - - - 64

256

384

12th Term - 4096

6th Term - - 64

16384

24576

18th - - 262144

5th - - - - 32

524288

786432

23d - - 8388608 = u

The Price of the Horse - - - £. 17476: 5: 3½

s. d.

Then $rs - ru = s - a$

∴ $rs - s - ru = -a$

∴ $rs - s = ru - a$

∴ $s = ru - a$

$r - u$

8388608 = u

2 = r

16777216 = ru

1 = a

4 | 16777215 = s

12 | 4194303 : ½

20 | 34952½ : 3

2. A cunning *Servant* agreed with his *Master*, unskilled in Numbers, to serve him 11 *Years*, without any other Reward for his Service, but the Produ

Produce of a *Wheat Corn*, and that Product to be sowed the *next Year*, and so on, from Year to Year, until the End of the *Time*, taking the Increase, but in a *ten-fold Proportion*. It is required, to find the Sum of the whole Produce?

Note, 7680 *Corns*, contain a Statute Pint, and let the *Wheat* be rated at 16 s. per *Quarter*.

Arith.	1	2	3	4	5	6
Geo.	10	: 100	: 1000	: 10000	: 100000	: 1000000

6th Term 1000000 Then $s = ru - a$ $a = 10$

5th Term 100000 $r = 10$

11th Term 100000000000 = u

$10 = r$

1000000000000 = ru

$10 = a$

$r - 1 = 9 | 999999999990$

Pints

7680 | 111111111110 = s | 14467592

qua.

28257 at 16 s. per *Quarter*

is 22605 £. 12 s.

4 | 7233796 qts.

2 | 1808449 gall.

4 | 904224 : 1 pecks

8 | 226056 bush.

28257 qrs.

3. When *Sessa Ebn Dahir*, an *Indian*, presented the Play of the *Chefs-board* to King *Scheramus*, he did so admire the Excellence of the Invention, that he bid *Dahir* ask whatever he pleased, and it should be granted him. He asked no other Gift, but a *Grain of Wheat* for the first *Square*, and that it might be doubled, as often as there were *Squares* on the Board, which was 64.

The

The King being greatly displeased at his asking so small a Reward, commanded his Servants to give him just that Number and not *one Grain* more; which, when his *Secretary* had computed, they brought him Word that there was not to be found on the whole Earth, much less in his Kingdom, a Quantity of Wheat equal to that Sum.

When the King heard this, he was as much pleased with the Ingenuity of the *Petition*, as he was before with that of the *Invention*.

How many *Tons* of Wheat were there, and how much would it amount to at *5 l. per Ton*; and how many such Ships as our *Royal Sovereign* of 1000 *Tons*, would be loaden therewith?

Arithmetic $0 : 1 : 2 : 3 : 4 : 5 : 6$

Geometric $1 : 2 : 4 : 8 : 16 : 32 : 64$

$$a = 1$$

$$4 - - 16$$

$$r = 2$$

$$4 - - 16$$

$$\text{Then } s = \frac{ru - a}{r - 1}$$

$$8 - 256$$

$$8 - 256$$

$$16 - - - 65536$$

$$16 - - - 65536$$

$$32 - - - 4294967296$$

$$16 - - - - - 65536$$

$$48 - 281474976710656$$

$$12 - - - - - 4096$$

$$60 - 1152921504606846976$$

$$3 - - - - - 8$$

$$63 - 9223372036854775808 = u$$

$$2 = r$$

$$18446744073709551616 = ru$$

$$1 = a$$

$$r - 1 = 1 | 18446744073709551615$$

INTEREST.

7680)18446744073709551615 Grains.

2)2401919801264264 Pints.

4)1200959900632132 Quarts.

2)300239975158033 Gallons.

4)150119987579016 : 1 Pecks

8)37529996894754 Bushels.

4)4691249611844 : 2 Quarters.

1172812402961 Tons.

Ton. Q. B. P. G.

1172812402961 : 0 : 2 : 0 : 1 at 5 l. per
Ton, is 5864062014805 l. 6 s. 4 d.

1000)1172812402961(

1172812402 Ships.



INTEREST.

LET the *Principal*, or Sum put out to Interest,
be = p .

The *Ratio* of the Rate *per Cent. per Ann.* be = r .

The *Time* of Continuance at Interest be = t .

The *Amount* of the Principal and Interest be = a .

Note : The *Ratio* of the Rate is only the *Simple Interest* of 1 l. for one Year, and is thus found.

As 100 : .6 :: 1 : .06. As 100 : 5 :: .05

As 100 : 7.5 :: 1 : .075. As 100 : 7.75 :: 1 : .0775

Theorem.

$$trp + p = a$$

i. At

INTEREST. 307

1. At 5 per Cent. per Ann. what is the Amount of 279 l. 15 s. 8 d. forborn 13 Years?

Then $trp \div p = a$

$$\begin{aligned} r &= .05 \\ t &= 13 \\ p &= 279.783333 \end{aligned}$$

$$\begin{aligned} p &= 279.783333 \\ r &= .05 \\ pr &= 13.98916665 \\ t &= 13 \end{aligned}$$

$$\begin{array}{r} 15 : 8 \\ 12 \\ \hline 188 \\ \hline 240 \overline{) 188.000000} \\ \underline{.783333} \end{array}$$

$$\begin{aligned} prt &= 181.85916645 \\ p &= 279.78333333 \\ a &= \text{£. } 461.64249978 \end{aligned}$$

$$\begin{array}{r} 20 \\ \hline \text{Sh. } 12.84999560 \\ \hline 12 \end{array}$$

$$d. 10.19994720$$

2. At 6 per Cent. per Ann. what will 354 l. 14 s. 7 d. amount to in 7 Years, 25 Days?

Then $trp \div p = a$

$$\begin{aligned} r &= .06 \\ p &= 354.729166 \\ t &= 7.068493 \end{aligned}$$

$$\begin{aligned} 25 \\ \hline 365 \overline{) 25.000000} \\ \underline{.068493} \end{aligned}$$

$$\begin{array}{r} s. \quad d. \\ 14 : 7 \\ 12 \\ \hline 175 \\ \hline 240 \overline{) 175.000000} \\ \underline{.729166} \end{array}$$

$$\begin{aligned} p &= 354.729166 \\ r &= .06 \\ pr &= 21.28374996 \\ t &= 7.068493 \\ prt &= 150.44403760601028 \\ p &= 354.7291666 \\ a &= \text{£. } 505.1732042 \end{aligned}$$

$$\begin{array}{r} 20 \\ \hline \text{Sh. } 3.4640840 \\ \hline 12 \end{array}$$

$$d. 5.5690080$$

$$\begin{array}{r} 4 \\ \hline f. 2.276 \\ R \quad r \end{array}$$

3. What is 728 l. 16 s. 8 d. payable at the End of $5\frac{1}{2}$ Years, worth in ready Money, at 5 per Cent. Discount?

$$\text{Then } trp + p = a$$

$$t = 5.5$$

$$\therefore p = \frac{a}{tr + 1}$$

$$r = .05$$

$$r = .05$$

$$tr + 1$$

$$tr .275$$

$$t = 5.5$$

$$1.$$

$$a = 728.833333$$

$$1.275$$

$$tr + 1 = 1.275) 728.8333333 = a$$

$$16 : 8$$

$$p = 571.63398$$

$$12$$

$$20$$

$$200$$

$$12.67960$$

$$240) 200.000000$$

$$12$$

$$.833333$$

$$8.15520$$

4. At what Rate per Cent. will 256 l. 10 s. amount to 309 l. 16 s. 10 d. in 3 Years, 1 Quarter, 2 Months, 18 Days?

$$\text{Then } trp + p = a$$

$$p = 256.5$$

$$\therefore trp = a - p$$

$$a = 309.841666$$

$$\therefore r = \frac{a - p}{tp}$$

$$t = 3.465981$$

$$tp$$

INTEREST.

309

$$16 : 10$$

$$12$$

$$202$$

$$240)202.000000$$

$$.841666$$

$$a=309.841666$$

$$p=256.5$$

$$a-p=53.341666$$

$$889.0241265)53.341666666$$

$$r=.06$$

$$1 \text{ Quarter } .25$$

$$2 \text{ Months } .166666$$

$$18 \text{ Days } .049315$$

$$t=3.465981$$

$$p=256.5$$

$$17329905$$

$$20795886$$

$$17329905$$

$$6931962$$

$$tp=889.0241265$$

5. In what Time will 350 l. raise a Stock of 400 l. at $6\frac{1}{2}$ per Cent. per Annum?

$$a=400$$

$$\text{Then } trp + p = a$$

$$\therefore trp = a - p$$

$$p=350$$

$$\therefore trp = a - p$$

$$r=.065$$

$$\therefore t = \frac{a-p}{rp}$$

$$a=400$$

$$p=350$$

$$a-p=50$$

$$p=350$$

$$r=.065$$

$$1750$$

$$2100$$

$$rp=22.750)50.0000000$$

$$\text{Years } 2.1978$$

$$365$$

$$\text{Days } 72.1970$$

R r 2

To

To compute *Annuities* or *Pensions* in Arrear.

Let the *yearly Rent* of an Annuity or Pension be $=y$.

The *Time* of its Continuance, or being unpaid, $=t$.

The *Ratio*, or Interest of 1 *l.* one Year, be $=r$.

The *Amount* of the Annuity, with its Interest, $=a$.

$$\text{Now } \frac{ttr - tr}{2} = \frac{a - ty}{y}$$

1. If 250 *l.* yearly Rent, be forborn or unpaid 7 Years, what will it amount to, at 6 per Cent. for each Payment, as it becomes due?

$$y = 250 \quad t = 7 \quad r = .06 \quad \text{Then } \frac{ttr - tr}{2} = \frac{a - ty}{y}$$

$$\therefore ttry - try = 2a - 2ty$$

$$\therefore ttry - try + 2ty = 2a$$

$$\therefore a = \frac{ttry - try + 2ty}{2} = 2065 \text{ £.}$$

$49 = tt$	$7 = t$	$14 = 2t$	$735 = ttry$
$.06 = r$	$.06 = r$	$250 = y$	$3500 = 2ty$
<hr/>	<hr/>	<hr/>	<hr/>
$2.94 = ttr$	$.42 = tr$	700	4235
$250 = y$	$250 = y$	28	105
<hr/>	<hr/>	<hr/>	<hr/>
14700	2100	$3500 = 2ty$	$2)4130$
588	84		2065
<hr/>	<hr/>		
$735.00 = ttry$	$105.00 = try$		

2. What yearly Rent, forborn 9 Years, will raise a Stock of 3000 *l.* allowing $5\frac{1}{4}$ per Cent?

$$t = 9 \quad a = 3000 \quad r = .0525 \quad \text{Then } \frac{ttr - tr}{2} = \frac{a - ty}{y}$$

$$\therefore ttry - try = 2a - 2ty$$

$$\therefore ttry - try + 2ty = 2a$$

$$\therefore y = \frac{2a}{ttr - tr + 2t} = 275.482$$

INTEREST.

311

$$\begin{array}{r}
 .0525=r \\
 81=tt \\
 \hline
 4.2525=ttr \\
 .4725=tr \\
 \hline
 3.7800 \\
 18.=2t \\
 \hline
 21.7800
 \end{array}
 \begin{array}{r}
 .0525=r \\
 9=t \\
 \hline
 .4725=tr \\
 \hline
 18=2t \\
 \hline
 6000=2a \\
 \hline
 21.7800
 \end{array}
 \begin{array}{r}
 9=t \\
 2 \\
 \hline
 18=2t \\
 \hline
 6000=2a \\
 \hline
 21.7800
 \end{array}$$

$$21.7800)6000.0000000$$

$$\begin{array}{r} \text{£. } 275.482 \\ 20 \end{array}$$

$$\begin{array}{r} \text{Sh. } 9.640 \\ 12 \end{array}$$

$$\begin{array}{r} \text{d. } 7.680 \\ 4 \end{array}$$

$$\text{f. } 2.720$$

If 250 l. yearly Rent, forborn 7 Years, amounts to 2065 l. what must the Rate of Interest be per Cent ?

$$\begin{array}{l}
 y=250 \\
 t=7 \\
 a=2065
 \end{array}$$

$$\text{Then } \frac{ttr-tr}{2} = \frac{a-ty}{y}$$

$$\therefore ttry-try=2a-2ty$$

$$\therefore r = \frac{2a-2ty}{tty-ty} = .06$$

$$\begin{array}{r} 2065=a \\ 2 \end{array}$$

$$\begin{array}{r} 250=y \\ 7=t \end{array}$$

$$\begin{array}{r} 250=y \\ 14=2t \end{array}$$

$$\begin{array}{r} 250=y \\ 49=ty \end{array}$$

$$\begin{array}{r} 4130=2a \\ 3500=2ty \end{array}$$

$$1750=ty$$

$$3500=2ty$$

$$\begin{array}{r} 12250=tty \\ 1750=ty \end{array}$$

$$10500)630.00$$

$$.06$$

$$\frac{100}{6.00}$$

$$6.00$$

u 1

36 m. 4.6

The present Worth of *Annuities, Pensions, &c.* may be computed by the following Equation.

$$\frac{ttry - try + 2ty}{2} = ptr + p$$

1. What is 59 l. yearly Rent, to continue 5 Years, worth in ready Money, at 6 per Cent. per Annum?

$$y = 59$$

$$t = 5$$

$$r = .06$$

$$\text{Then } \frac{ttry - try + 2ty}{2} = ptr + p$$

$$\therefore ttry - try + 2ty = 2ptr + 2p$$

$$\therefore p = \frac{ttry - try + 2ty}{2tr + 2} = 254.1538$$

$$25 = tt$$

$$.06 = r$$

$$59 = y$$

$$10 = 2t$$

$$.06 = r$$

$$5 = t$$

$$10 = 2t$$

$$.06 = r$$

$$1.50 = ttr$$

$$.30 = tr$$

$$590 = 2ty$$

$$.60 = 2tr$$

$$59 = y$$

$$59 = y$$

$$2.$$

$$88.50 = ttry$$

$$17.70 = try$$

$$2.60$$

$$17.70 = try$$

$$70.80$$

$$590 = 2ty$$

$$2.60 \overline{) 660.800000}$$

$$\pounds. 254.1538$$

$$20$$

$$Sh. 3.0760$$

$$12$$

$$d. .9120$$

$$4$$

$$f. 3.6480$$

2. What

INTEREST.

313

2. What *Annuity* may be purchased for 60*l.* to continue 21 *Years*, at 6 *per Cent. per Annum.*

$$\begin{aligned} p &= 60 \\ t &= 21 \\ r &= .06 \end{aligned}$$

$$\text{Then } \frac{ttry - try + 2ty}{2} = ptr + p$$

$$\therefore ttry - try + 2ty = 2ptr + 2p$$

$$\therefore y = \frac{2ptr + 2p}{ttr - tr + 2t} = 4.0357$$

$$\begin{array}{r} 60 = p \\ 2 \end{array} \quad \begin{array}{r} 60 = p \\ 2 \end{array} \quad \begin{array}{r} 21 = t \\ 21 = t \end{array} \quad \begin{array}{r} 21 = t \\ .06 = r \end{array}$$

$$\begin{array}{r} 120 = 2p \\ 21 = t \end{array} \quad \begin{array}{r} 120 = 2p \\ 21 = t \end{array} \quad \begin{array}{r} 441 = tt \\ .06 = r \end{array} \quad \begin{array}{r} 1.26 = tr \end{array}$$

$$\begin{array}{r} 2520 = 2pt \\ .06 = r \end{array}$$

$$\begin{array}{r} 26.46 = ttr \\ 1.26 \end{array}$$

$$\begin{array}{r} 151.20 = 2ptr \\ 120 = 2p \end{array}$$

$$\begin{array}{r} 25.20 \\ 42 = 2t \end{array}$$

$$67.20 \overline{) 271.200000}$$

$$67.20$$

$$\text{£. } 4.0357$$

$$20$$

$$.7140$$

$$12$$

$$d. 8.5680$$

$$4$$

$$\text{far. } 2.2720$$

QUADRATIC

QUADRATIC EQUATIONS.

THERE are *three* Cases, or Forms of *Quadratic Equations*.

1 Case $xx = ax + bb$

$\therefore xx - ax = bb$

Note, In the Equation, $xx - ax = bb$, a , is called the *Coefficient*; and the *Square* of $\frac{1}{2}$ the *Coefficient* added to $xx - ax$, makes it a *compleat Square*; and therefore must be added to both Sides of the Equation,

Then $xx - ax = bb$

$\therefore xx - ax + \frac{aa}{4} = \frac{aa}{4} + bb$

$\therefore x - \frac{a}{2} + \sqrt{\frac{aa}{4}} = \frac{aa}{4} + bb$

$\therefore x = \frac{a}{2} + \sqrt{\frac{aa}{4} + bb}$

Proof.

$$\begin{array}{r}
 x - \frac{a}{2} \text{ is } \frac{4xx - 4ax + aa}{2 \times 2} \quad \frac{2x - a}{2x - a} \\
 \hline
 \frac{4xx - 4ax + aa}{4} \quad \frac{4xx - 2ax - 2ax + aa}{4xx - 4ax + aa}
 \end{array}$$

Then $\frac{4xx - 4ax + aa}{4}$ is $xx - ax + \frac{aa}{4}$

Now $xx - ax + \frac{aa}{4} = \frac{aa}{4} + bb$

$\therefore xx - ax = bb$

$\therefore xx = ax + bb$. Which was to be proved.

2 : Case $xx = -ax + bb$

$$\therefore xx + ax = bb$$

By adding the Square of $\frac{1}{2}$ the Coefficient to each Side of the Equation, it will be

$$xx + ax + \frac{aa}{4} = \frac{aa}{4} + bb$$

$$\therefore x + \frac{a}{2} \sqrt{\frac{aa}{4} + bb}$$

$$\therefore x = \sqrt{\frac{aa}{4} + bb} - \frac{a}{2}$$

$$\text{Or thus, } x = -\frac{a}{2} + \sqrt{\frac{aa}{4} + bb}$$

3 : Case $xx = ax - bb$

$$\therefore xx - ax = -bb$$

By adding the Square of $\frac{1}{2}$ the Coefficient as afore, it will be

$$xx - ax + \frac{aa}{4} = \frac{aa}{4} - bb$$

$$\therefore x - \frac{a}{2} \sqrt{\frac{aa}{4} - bb}$$

$$\therefore x = \frac{a}{2} + \sqrt{\frac{aa}{4} - bb}$$

$$\text{Or } x = \frac{a}{2} - \sqrt{\frac{aa}{4} - bb}$$

Note, This Case having *two Roots*, Use and Practice will teach us, which of them is to be applied in any *Question* proposed. But often, both of them will be useful in the Construction.

S s

1. To

1. To find a *Number*, which being multiplied by 6; and the *Product* subtracted from the *Square* of the said *Number*, shall leave 280.

Numerically.

x , the *Number* sought.

Then $xx - 6x = 280$

$$\therefore xx - 6x + 9 = 289$$

$$\therefore x - 3 = \sqrt{289}$$

$$\therefore x = 3 + \sqrt{289} = 20$$

$$\begin{array}{r} 20 \\ 20 \\ \hline 400 \end{array}$$

$$\begin{array}{r} 20 \\ 6 \\ \hline 120 \end{array}$$

$$\begin{array}{r} 400 \\ 120 \\ \hline 280 \end{array}$$

$$\begin{array}{r} \sqrt{289} | 17 \\ 1 \quad 3 \\ 27 \quad 189 \quad 20 \\ \hline 189 \\ 0 \end{array}$$

Literally.

Let $6 = a$
 $280 = b$

Then $xx - ax = b$

$$\therefore xx - ax + \frac{aa}{4} = \frac{aa}{4} + b$$

$$\begin{array}{r} 4 \overline{) 36} = aa \\ 9 \end{array}$$

$$\therefore x - \frac{a}{2} = \sqrt{\frac{aa}{4} + b}$$

$$\therefore \frac{280}{4} = b$$

$$\therefore x = \frac{a}{2} + \sqrt{\frac{aa}{4} + b} = 20$$

$$\begin{array}{r} \sqrt{289} | 17 \\ 0 \quad 3 \\ 20 \end{array}$$

2. To find a *Number*, which being multiplied by 8, and the *Product* added to the *Square* of the *Number* sought shall produce 660.

Numeri-

Numerically.

x , The Number sought.

Then $xx + 8x = 660$

$\therefore xx + 8x + 16 = 676$

$\therefore x + 4 = \sqrt{676}$

$\therefore x = -4 + \sqrt{676} = 22$

$$\sqrt[2]{676} \overline{)26}$$

$$\begin{array}{r} 4 4 \\ 46 \overline{)276} 22 \\ \underline{276} \\ 0 \end{array}$$

$$\begin{array}{r} 22 \\ 22 \\ \hline 44 \\ 44 \\ \hline 484 \end{array}$$

$$\begin{array}{r} 22 \\ 8 \\ \hline 176 \end{array}$$

$$\begin{array}{r} 484 \\ 176 \\ \hline 660 \end{array}$$

Literally.

Let $8 = a$ Then $xx + ax = b$
 $660 = b$

$\therefore xx + ax + \frac{aa}{4} = \frac{aa}{4} + b$

$\frac{4 \overline{)64}}{16} = aa$

$\therefore x + \frac{a}{2} + \sqrt{\frac{aa}{4} + b}$

$660 = b$

$\sqrt[2]{676} \overline{)26}$

$\therefore x = -\frac{a}{2} + \sqrt{\frac{aa}{4} + b} = 22$

$\frac{4}{225}$

3. Let 969 Soldiers, be drawn up into an oblong Square; so that the Difference between the greater and lesser Sides be 40. I demand the Number of Soldiers in each Rank, Length and Breadth?

Numerically.

 x , the Number of Soldiers in the Length. $x+40$, the Number in Breadth.

$$\begin{array}{r}
 x \\
 \hline
 xx+40x \\
 \hline
 \sqrt{1369} \quad 37 \\
 \begin{array}{r}
 9 \quad 20 \\
 67 \quad 496 \quad 17 \\
 469 \\
 \hline
 0
 \end{array}
 \end{array}
 \quad
 \begin{array}{l}
 \text{Then } xx+40x=969 \\
 \therefore xx+40x+400=1369 \\
 \therefore x+20=\sqrt{1369} \\
 \therefore x=-20+\sqrt{1369}=17
 \end{array}$$

Length 17 Breadth 57

$$\begin{array}{r}
 17 \\
 \hline
 399 \\
 57 \\
 \hline
 969
 \end{array}$$

Literally.

Let $969=a$ Then $xx+bx=a$ $40=b$

$$\therefore xx+bx+\frac{bb}{4}=\frac{bb}{4}+a$$

 $40=b$ $40=b$

$$4 \overline{) 1600} = bb$$

$$\therefore x+\frac{b}{2}\sqrt{\frac{bb}{4}}+a$$

$$\sqrt{1369}=17$$

$$\therefore x=-\frac{b}{2}+\sqrt{\frac{bb}{4}}+a=17$$

4. Let 480 Soldiers, be drawn up, into an oblong Square; so that the Sum of the greater and lesser Sides be 52. I demand the Number of each Rank, in Length and Breadth?

Numeri-

had bought every *Ell* 2 *Crowns* cheaper. How many *Ells* did he buy?

Numerically.

x , the Number of *Ells* bought.

As $x : 70 :: 1 : \frac{70}{x}$ the Price of an *Ell*.

$$\text{As } x+4 : 70 :: 1 : \frac{70}{x+4}$$

$$\frac{70}{x+4} - 2, \text{ is } \frac{70+2x+8}{x+4}$$

$$\text{Then } \frac{70+2x+8}{x+4} = \frac{70}{x}$$

$$\therefore 70x+2xx+8x = 70x+280$$

$$\therefore 78x+2xx = 70x+280$$

$$\therefore 8x+2xx = 280$$

$$\therefore xx+4x = 140$$

$$\therefore xx+4x+4 = 144$$

$$\therefore x+2 = \sqrt{144}$$

$$\therefore x = -2 + \sqrt{144} = 10$$

$$\text{As } 10 : 70 :: 1 : 7$$

$$\text{As } 14 : 70 :: 1 : 5$$

Literally.

$$\text{Let } 70 = a$$

$$4 = b$$

$$2 = c$$

$$\text{as } x : a :: 1 : \frac{a}{x}$$

$$\text{as } x+b : a :: 1 : \frac{a}{x+b}$$

$$\frac{a}{x+b} + c, \text{ is } \frac{a+cx+bc}{x+b}$$

Then

Quadratic Equations.

321

$$\text{Then } \frac{a+cx+bc}{x+b} = \frac{a}{x}$$

$$\therefore ax+cx+bcx= ax+ab$$

$$\therefore cx+bcx=ab$$

$$\therefore xx+bx=\frac{ab}{c}$$

$$\therefore xx+bx+\frac{bb}{4}=\frac{bb}{4}+\frac{ab}{c}$$

$$\therefore x+\frac{b}{2}=\sqrt{\frac{bb}{4}}+\frac{ab}{c}$$

$$\therefore x=-\frac{b}{2}+\sqrt{\frac{bb}{4}}+\frac{ab}{c}=10$$

$$\begin{array}{r} 4 \overline{) 16 = bb} \\ 4 \end{array}$$

$$\begin{array}{r} 70 = a \\ 4 = b \end{array}$$

$$140$$

$$c=2 \overline{) 280 = ab} \\ 140$$

$$\begin{array}{r} \sqrt{144} \overline{) 12} \\ 2 \\ 10 \end{array}$$

6. A Sett of Boon Companions, dining at an Inn. The Reckoning came to 175 Pence; but before the Bill was paid, two of them slipt away, and then the club of them that remained, came to 10 Pence per Man more. How many were there in Company?

Numerically.

x , the Number of Companions.

$\frac{175}{x}$ Each Man's Reckoning.

$\frac{175}{x-2}$ Each Man's Reckoning left behind.

Then

Then $\frac{175}{x} + 10$, is $\frac{175+10x}{x} = \frac{175}{x-2}$

$$\therefore 175x + 10xx - 350 - 20x = 175x$$

$$\therefore 10xx - 350 - 20x = 0$$

$$\therefore 10xx - 20x = 350 \quad 7 \overline{)175}$$

$$\therefore xx - 2x = 35 \quad 25$$

$$\therefore xx - 2x + 1 = 36$$

$$\therefore x - 1 = \sqrt[2]{36} \quad 5 \overline{)175}$$

$$35$$

$$\therefore x = 1 + \sqrt[2]{36} = 7 \quad 25 + 10 = 35$$

Literally.

Let $175 = a$
 $2 = b$
 $10 = c$
 Then $\frac{a}{x} + c$, is $\frac{a+cx}{x} = \frac{a}{x-b}$

$$175 = a \quad \therefore ax + cxx - ab - bcx = ax$$

$$2 = b \quad \therefore cxx - ab - bcx = 0$$

$$c = 10 \overline{)350} \quad \therefore cxx - bcx = ab$$

$$35 \quad \therefore xx - bx = \frac{ab}{c}$$

$$\sqrt[2]{36} \quad \therefore xx - bx + \frac{bb}{4} = \frac{bb}{4} + \frac{ab}{c} \quad \begin{matrix} 2=b \\ 2=b \end{matrix}$$

$$7 \therefore x - \frac{b}{2} = \sqrt[2]{\frac{bb}{4} + \frac{ab}{c}} \quad 4 \overline{)4} = bb$$

$$\therefore x = \frac{b}{2} + \sqrt[2]{\frac{bb}{4} + \frac{ab}{c}} = 7$$

7. To divide the Number 21, into *two Parts* ; so that if the *greater* be divided by the *lesser* ; and again, the *lesser* by the *greater* ; and then the *first Quotient* if multiplied by 4, and the *second* by 25, the *Products* may be equal.

Numeri-

Numerically.

$$[x] \quad 21-x$$

$$\begin{array}{r} 4x \\ \hline x \\ 21-x \times \frac{4}{1} \\ \hline 21-x \end{array}$$

$$\begin{array}{r} 525-25x \\ \hline x \\ 21-x \times \frac{25}{1} \\ \hline x \end{array}$$

Then $\frac{4x}{21-x} = \frac{525-25x}{x}$

$$\begin{array}{l} \therefore 4xx = 11025 - 525x - 525x + 25xx \\ \therefore 4xx = 11025 - 1050x + 25xx \quad 21)1050(50 \\ \therefore 25xx - 1050x = 4xx - 11025 \quad 0 \\ \therefore 21xx - 1050x = -11025 \quad 21)11025(525 \\ \therefore xx - 50x = -525 \quad 52 \\ \therefore xx - 50x + 625 = 100 \quad 105 \\ \therefore x - 25 = \sqrt{100} \quad 0 \\ \therefore x = 25 + \sqrt{100} = 35, \text{ too great.} \\ \therefore x = 25 - \sqrt{100} = 15 \end{array}$$

$$[15] \quad 6$$

$$\begin{array}{r} 60 \\ \hline 15 \times \frac{4}{6} \\ \hline 6)60(10 \\ 0 \end{array}$$

$$\begin{array}{r} 150 \\ \hline 6 \times \frac{25}{15} \\ \hline 15)150(10 \\ 0 \end{array}$$

Literally.

Let $21=a$
 $4=b$
 $25=c$

$$[x] \quad a-x$$

$$\begin{array}{r} ca-cx \\ \hline a-x \times \frac{c}{x} \\ \hline x \end{array}$$

$$\begin{array}{r} 6x \\ \hline x \times \frac{b}{a-x} \\ \hline a-x \end{array}$$

T t

Then

$$\text{Then } \frac{ca-cx}{x} = \frac{bx}{a-x}$$

$$\therefore caa-cax-cax+cx x=bx x$$

$$\therefore caa-2cax+cx x=bx x$$

$$\therefore cx x-2cax=bx x-caa$$

$$\therefore cx x-bx x-2cax=-caa. \text{ Let } c-b=d=21$$

$$\therefore dx x-2cax=-caa$$

$$\therefore xx-\frac{2cax}{d}=-\frac{caa}{d}$$

$$\therefore xx-\frac{2cax}{d}+\frac{4ccaa}{4dd}=\frac{4ccaa}{4dd}-\frac{caa}{d}$$

$$\therefore x-\frac{2ca}{2d}=\sqrt{\frac{4ccaa}{4dd}-\frac{caa}{d}}$$

$$\therefore x=\frac{2ca}{2d}-\sqrt{\frac{4ccaa}{4dd}-\frac{caa}{d}}=15$$

$$25=c$$

$$25=c$$

$$125$$

$$50$$

$$625=cc$$

$$4$$

$$2500=4cc$$

$$220500$$

$$882$$

$$4dd=1764)1102500(625$$

$$4410 \quad 525$$

$$8820 \quad \sqrt{100(10}$$

$$25$$

$$15$$

$$0$$

$$441=dd$$

$$4$$

$$1764=4dd$$

$$d=21)11025(525$$

$$52$$

$$105$$

$$0$$

$$25=c$$

$$21=a$$

$$25$$

$$50$$

$$525=ca$$

$$2$$

$$2d=42)1050(25$$

$$210$$

$$0$$

8. Let 600 Soldiers be disposed into an oblong Square, which the Colonel, willing to make broader, finds that if he takes away 10 Ranks from the Length, he shall increase the Breadth with 2 Ranks. What was the Number of his Soldiers in Length and Breadth?

Numerically.

x , the Number of Soldiers in Length.

$\frac{600}{x}$ the Number in Breadth.

$$\text{Then } \frac{600}{x} + 2, \text{ is } \frac{600x + 2xx - 6000 - 20x}{x} \times \frac{x - 10}{1}$$

$$\therefore \frac{600x + 2xx - 6000 - 20x}{x} = 600$$

$$\therefore 600x + 2xx - 6000 - 20x = 600x$$

$$\therefore 2xx - 6000 - 20x = 0$$

$$\therefore 2xx - 20x = 6000$$

$$\therefore xx - 10x = 3000$$

$$\therefore xx - 10x + 25 = 3025$$

$$\therefore x - 5 = \sqrt{3025}$$

$$\therefore x = 5 + \sqrt{3025} = 60$$

$$\begin{array}{r} \sqrt{3025} \\ 25 \quad 5 \\ 105 \overline{) 525} \quad 60 \\ \underline{525} \end{array}$$

Length 60

Breadth 10

$$\begin{array}{r} 10 \\ \underline{\quad} \\ 50 \\ \underline{\quad} \\ 12 \\ \underline{\quad} \\ 600 \end{array}$$

$$\begin{array}{r} 2 \\ \underline{\quad} \\ 12 \end{array}$$

T t 2

Literally.

Literally.

$$\text{Let } \begin{array}{l} 600=a \\ 10=b \\ 2=c \end{array} \quad \frac{ax-ab+cx-x-bcx}{x-b} \times \frac{a+cx}{x} \quad \frac{a}{x} + c \text{ is } \frac{a+cx}{x}$$

$$\text{Then } \frac{ax-ab+cx-x-bcx}{x} = a$$

$$\begin{array}{lll} \therefore ax-ab+cx-x-bcx=ax & 600=a & 10=b \\ \therefore cx-x-bcx-ab=0 & 10=b & 10=b \\ \therefore cx-bcx=ab & & \\ \therefore xx-bx=\frac{ab}{c} & c=2 \quad 6000=ab & 4)100=bb \end{array}$$

$$\begin{array}{ll} \therefore xx-bx+\frac{bb}{4}=\frac{bb}{4}+\frac{ab}{c} & \begin{array}{r} 3000 \\ \sqrt{3025} \end{array} \\ \therefore x-\frac{b}{2}=\sqrt{\frac{bb}{4}+\frac{ab}{c}} & \begin{array}{r} 55 \\ 5 \\ 60 \end{array} \end{array}$$

$$\therefore x=\frac{b}{2}+\sqrt{\frac{bb}{4}+\frac{ab}{c}}=60$$

9. A Man buys a Horse, which he sells again for 56*l.* and gains as many Pounds in the Hundred as the Horse cost him. What did he give for the Horse?

Numerically.

x , the Price of the Horse.

$56-x$, gained.

As $100 : x :: x : 56-x$

$$\begin{array}{l} \therefore xx=5600-100x \\ \therefore xx+100x=5600 \\ \therefore xx+100x+2500=8100 \end{array}$$

$$\therefore x+50=\sqrt{8100}$$

$$\therefore x=-50+\sqrt{8100}=40$$

As 100 : 40 : 40 : 16

$$\frac{16}{1600} \quad \frac{40}{1600}$$

Literally.

Let $56 = a$ As $b : x :: x : a - x$
 $100 = b$ $\therefore xx = ba - bx$

$$\begin{array}{l} 56 = a \quad 100 = b \quad \therefore xx + bx = ba \\ \frac{100}{5600} = b \quad \frac{100}{410000} = bb \quad \therefore xx + bx + \frac{bb}{4} = \frac{bb}{4} + ba \\ \frac{5600}{\sqrt{8100}} = 90 \quad \therefore x + \frac{b}{2} = \frac{\sqrt{bb}}{4} + ba \\ \frac{50}{40} \quad \therefore x = -\frac{b}{2} + \frac{\sqrt{bb}}{4} + ba \end{array}$$

10. A Linnen Draper, buys 2 Sorts of Linnen for 30*l*. one Sort *Fine*, the other *Coarse*. An *Ell* of the *Fine* cost as many Pounds as he had *Ells*. He bought 28 *Ells* of the *Coarse* at such a Price, that 8 *Ells* of the *Coarse* cost as many Pounds as one *Ell* of the *Fine*. How many *Ells* of the *Fine* did he buy, and what Price did he give for them both.

Numerically.

x , the Number of fine *Ells*.

xx , the Price of the *Fine*.

As $8 : x :: 28 : \frac{28x}{8}$ the Price of the *Coarse*.

$$\text{Then } xx + \frac{28x}{8} = 30$$

$$\therefore xx + \frac{7x}{2} = 30$$

$$\therefore xx + \frac{7x}{2} = \frac{49}{16} + 30 = \frac{529}{16}$$

$$\therefore x + \frac{7}{4} = \sqrt{\frac{529}{16}}$$

$$\therefore = -\frac{7}{4} + \sqrt{\frac{529}{16}} = 4$$

$$\sqrt[3]{529|23}$$

$$\begin{array}{r} 4 \\ 43 \overline{)129} \\ 0 \end{array}$$

$$\frac{23}{4} - \frac{7}{4} \text{ is } \frac{16}{4} = 4$$

$$\text{As } 8 : 4 :: 28 : 14$$

$$\begin{array}{r} 4 \\ 4 \end{array}$$

$\frac{16}{16}$ Price of Fine.

$\frac{14}{14}$ Price of Coarse.

$$\frac{30}{30}$$

Literally.

$$\text{Let } \begin{array}{l} 30 = a \\ 28 = b \\ 8 = c \end{array} \quad \text{As } c : x :: b : \frac{bx}{c}$$

$$\text{Then } xx + \frac{bx}{c} = a$$

$$\begin{array}{l} 64 = cc \\ \frac{4}{256} = 4cc \end{array} \quad \begin{array}{l} 28 = b \\ 28 = b \\ 224 \end{array}$$

$$\therefore xx + \frac{bx}{c} + \frac{bb}{4cc} = \frac{bb}{4cc} + a$$

$$\begin{array}{l} \frac{56}{784} = bb \\ \frac{256}{256} = 4cc \end{array} \quad \therefore x + \frac{b}{2c} = \sqrt{\frac{bb}{4cc}} + a$$

$$\begin{array}{r} 8464 \\ 784 \overline{)7680} \\ 784 \overline{)30} \\ 256 \overline{)1} \\ 256 \end{array}$$

$$\therefore x = -\frac{b}{2c} + \sqrt{\frac{bb}{4cc}} + a = 4$$

$$\begin{array}{r} \sqrt[3]{256|16} \\ 1 \\ 26 \overline{)156} \\ 156 \\ 0 \end{array}$$

$$\begin{array}{r} \sqrt[3]{8464|92} \\ 81 \\ 182 \overline{)364} \\ 364 \\ 0 \end{array}$$

$$\frac{92}{16} - \frac{28}{16} \text{ is } \frac{64}{16} = 4$$

11. Of

Quadratic Equations.

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11. Of 3 *proportional* Numbers; the *middle* Term is 12; and the Difference between the *Ex-* *streams* is 10. I demand the Numbers?

Numerically.

$$\begin{matrix} (1) & (2) & (3) \\ x & : 12 & : x + 10 \end{matrix}$$

$$\text{Then } \frac{x}{xx+10x} = 144$$

$$\therefore xx+10x+25=169$$

$$\therefore x+5=\sqrt{169}$$

$$\therefore x=-5+\sqrt{169}=8$$

$$\begin{array}{r} \sqrt{169} \overline{) 13} \\ 1 \\ \underline{23} \\ 69 \\ \underline{0} \end{array}$$

$$\begin{array}{r} 8 : 12 : 18 \\ 12 \\ \underline{144} = 144 \end{array}$$

Literally.

$$\begin{array}{l} \text{Let } 12 = a \\ 10 = b \end{array}$$

$$\begin{matrix} 1 & 2 & 3 \\ x & : a & : x + b \end{matrix}$$

$$10 = b$$

$$\text{Then } \frac{x}{xx+bx} = aa$$

$$\begin{array}{r} 10 = b \\ 4 \overline{) 100} = bb \end{array}$$

$$\therefore xx+bx+\frac{bb}{4}=\frac{bb}{4}+aa$$

$$\begin{array}{r} 25 \\ 2 \overline{) 144} = aa \\ \sqrt{169} \overline{) 13} \\ 5 \\ \underline{8} \end{array}$$

$$\therefore x+\frac{b}{2}=\frac{\sqrt{bb}}{4}+aa$$

$$\therefore x=-\frac{b}{2}+\frac{\sqrt{bb}}{4}+aa=8$$

12. Of *three* *proportional* Numbers; the Sum of the *first* and *second* is 10; and the Difference between the *second* and *third* is 24. What are the Numbers?

Numeri-

Numerically.

$$10 - x : x : x + 24$$

$$\begin{array}{r} 10 - x \\ 10x + 240 \\ -xx - 24x \\ \hline \end{array}$$

$$\text{Then } xx = -14x + 240 - xx$$

$$\therefore 2xx + 14x = 240$$

$$\therefore xx + 7x = 120$$

$$\therefore xx + 7x + \frac{49}{4} = \frac{49}{4} + 120 = \frac{529}{4}$$

$$\therefore x + \frac{7}{2} = \sqrt{\frac{529}{4}}$$

$$\therefore x = -\frac{7}{2} + \sqrt{\frac{529}{4}} = 8$$

$$2 : 8 : 32$$

$$\frac{8}{64} = \frac{2}{64}$$

$$\frac{64}{64} = \frac{64}{64}$$

Literally.

$$\text{Let } 10 = a$$

$$24 = b$$

$$a - x : x : x + b$$

$$x + b$$

$$\text{Then } ax - xx + ab - bx = xx$$

$$14 = c \therefore ax + ab - bx = 2xx$$

$$14 = c \therefore ab = 2xx + bx - ax. \text{ Let } b - a = c = 14$$

$$\frac{56}{56} \therefore 2xx - cx = ab$$

$$\frac{14}{cc} = \frac{196}{16} = \frac{49}{4}$$

$$\therefore xx - \frac{cx}{2} = \frac{ab}{2}$$

$$24 = b$$

$$10 = a$$

$$2 \overline{) 240 = ab}$$

$$120$$

$$\frac{529}{49 \ 480}$$

$$\frac{49}{4} + \frac{120}{1}$$

$$\therefore xx - \frac{cx}{2} + \frac{cc}{16} = \frac{cc}{16} + \frac{ab}{2}$$

$$\therefore x - \frac{c}{4} = \sqrt{\frac{cc}{16} + \frac{ab}{2}} = 8$$

$$\sqrt{\frac{529}{4}} - \frac{7}{2} = \frac{16}{2} = 8$$

13. Of four proportional Numbers, the *third* is 12, and the Sum of the *first* and *second* is 8. Besides, if the *second* Number be subtracted from its *Square*, it will leave the *fourth*. What are the Numbers?

Numerically.

$$\begin{array}{cccc} 1 & 2 & 3 & 4 \\ 8-x & : & x & : & 12 & : & xx-x \\ & & 12 & & 8-x \end{array}$$

$$\text{Then } 12x = 8xx - 8x - xxx + xx$$

$$\therefore 12 = 8x - 8 - xx + x$$

$$\therefore 12 = 9x - 8 - xx$$

$$\therefore xx - 9x + 12 = -8$$

$$\therefore xx - 9x = -20$$

$$\therefore xx - 9x + \frac{81}{4} = \frac{81}{4} - 20$$

$$\frac{81}{4} - 20 = \frac{1}{4}$$

$$\sqrt{\frac{1}{4}} = \frac{1}{2}$$

$$\therefore x = \frac{9}{2} \pm \sqrt{\frac{81}{4} - 20} \quad \frac{9}{2} + \frac{1}{2} = \frac{10}{2} = 5$$

$$\therefore x = \frac{9}{2} \pm \sqrt{\frac{81}{4} - 20} = 5$$

$$3 : 5 : 12 : 20$$

$$\frac{5}{60} = \frac{3}{60}$$

Literally.

$$\begin{array}{l} \text{Let } 12 = a \\ 8 = b \end{array}$$

$$\begin{array}{cccc} 1 & 2 & 3 & 4 \\ b-x & : & x & : & a & : & xx-x \\ & & x & & \end{array}$$

$$\frac{xx-x}{ax}$$

$$bxx - xxx - bx + xx = ax$$

$$\therefore bx - xx - b + x = a$$

$$\therefore xx - bx - b - x = -a$$

U u

Let

Let $-b-1 = -c = -9 \therefore xx - bx - x = -a - b$
 $\therefore xx - cx = -a - b$

$$\begin{array}{r} 81 \quad 80 \quad 9=c \\ 81 \quad 20 \quad 9=c \\ \hline 4 \quad 1 \quad 81=cc \\ 4 \quad 4 \end{array} \quad \therefore xx - cx - \frac{cc}{4} = \frac{cc}{4} - a - b$$

$$\therefore x - \frac{c}{2} = \sqrt{\frac{cc}{4} - a - b}$$

$$\sqrt{\frac{1}{4}} \frac{1}{2} + \frac{9}{2} = 5 \quad \therefore x = \frac{c}{2} + \sqrt{\frac{cc}{4} - a - b} = 5$$

14. Of four proportional Numbers, there is given the Sum of the *Means* 24, and likewise the Sum of the *Extremes* 56. What are the Numbers?

Numerically.

$$\begin{array}{cccc} 1 & 2 & 3 & 4 \\ \frac{xx}{24-x} : x : 24-x : 56-x \end{array} \quad \text{that is} \quad \frac{1344-56x-xx}{24-x}$$

$$\begin{array}{r} 1344xx - 56xxx - xxxx \\ 1344 - 56x - xx \\ \hline 24-x \end{array} \times \frac{xx}{24-x} \quad \begin{array}{r} 56 \\ 24 \\ \hline 224 \\ 112 \\ \hline 1344 \end{array} \quad \begin{array}{r} 24-x \\ x \\ \hline 24x-xx \end{array}$$

Then $\frac{1344xx - 56xxx - xxxx}{576 - 48x - xx} = 24x - xx$

$$\begin{array}{r} 576 - 48x - xx \\ 24x - xx \\ \hline 13824x - 1152xx - 24xxx \\ - 576xx - 48xxx - xxxx \\ \hline 13824x - 1728xx - 72xxx - xxxx \end{array} \quad \begin{array}{r} 576 \quad 48 \\ 24 \quad 24 \\ \hline 2304 \quad 192 \\ 1152 \quad 96 \\ \hline 13824 \quad 1152 \end{array}$$

Now

Now

$$1344xx - 56xxx - xxxx = 13824x - 1728xx + 72xxx - xxxx$$

$$\therefore 1344xx - 56xxx = 13824x - 1728xx + 72xxx$$

$$\therefore 1344x - 56xx = 13824 - 1728x + 72xx$$

$$\therefore 1344x = 13824 - 1728x + 128xx$$

$$\therefore 128xx - 3072x = -13824$$

$$\therefore xx - 24x = -108$$

$$\therefore xx - 24x + 144 = 36$$

$$\therefore x - 12 = \sqrt[2]{36}$$

$$\therefore x = 12 + \sqrt[2]{36} = 18$$

$$\begin{array}{cccc} 1 & 2 & 3 & 4 \\ 54 & : & 18 & : & 6 & : & 2 \\ 2 & & 6 & & & & \end{array}$$

$$108 = 108$$

$$18$$

$$18$$

$$144$$

$$18$$

$$6)324$$

$$54$$

Literally.

$$\text{Let } 24 = a \frac{xx}{a-x} : x : a-x : b - \frac{xx}{a-x} \text{ is } \frac{ba-bx-xx}{a-x}$$

$$\frac{baxx-bxxx-xxxx}{ba-bx-xx} \times \frac{xx}{a-x}$$

$$\frac{a-a-2ax+xx}{ax-xx}$$

$$\text{Then } \frac{baxx-bxxx-xxxx}{aa-2ax+xx} = ax-xx$$

$$\frac{aa-2ax+xx}{ax-xx}$$

$$\frac{aaax-2aaxx+axxx}{-aaxx-2axxx-xxxx}$$

$$\frac{aaax-3aaxx+3axxx-xxxx}{aaax-3aaxx+3axxx-xxxx}$$

$$\text{Now } baxx-bxxx-xxxx = aaax-3aaxx+3axxx-xxxx$$

$$\therefore baxx-bxxx = aaax-3aaxx+3axxx$$

$$\therefore bax-bxx = aaa-3aax+3axx$$

$$\therefore 3axx+bxx-3aax-bax = -aaa$$

$$\text{Let } 3a+b=c; \text{ and } -3aa-ba=-d$$

U u 2

Then

Then $cx - dx = -aaa$

$$\therefore xx - \frac{dx}{c} = -\frac{aaa}{c}$$

$$\therefore xx - \frac{dx}{c} + \frac{dd}{4cc} = \frac{dd}{4cc} - \frac{aaa}{c}$$

$$\therefore x - \frac{d}{2c} = \sqrt{\frac{dd}{4cc} - \frac{aaa}{c}} \quad \begin{array}{l} 24=a \\ 24=a \\ 56=b \end{array}$$

$$\therefore x = \frac{d}{2c} + \sqrt{\frac{dd}{4cc} - \frac{aaa}{c}} = 18 \quad \begin{array}{l} 96 \\ 48 \\ 120 \\ 576=aa \\ 1344=ab \end{array}$$

$$\begin{array}{r} 3072=d \\ 3072=d \\ 6144 \\ 21504 \\ 92260 \\ \hline 65536 \end{array} \quad \begin{array}{r} 24=a \\ 3 \\ -1728=3aa \\ 72=3a-1344=ab \\ 56=b \\ \hline 3072=d \\ 128=c \end{array}$$

$$\begin{array}{r} 65536)9447184(144 \\ 289358 \\ \hline 272144 \\ \hline 0 \end{array} \quad \begin{array}{r} 108 \\ \sqrt{3616256} \\ \hline 128 \end{array} \quad \begin{array}{r} 128 \ 256)3072(12 \\ 512 \\ \hline 0 \end{array} \quad \begin{array}{l} 24=a \\ 24=a \\ 576=aa \\ 24=a \end{array}$$

$$\begin{array}{r} 16384=cc \\ 4 \\ \hline 65536=4cc \end{array} \quad \begin{array}{r} 2304 \\ 1152 \\ \hline 13824(108 \\ 1024 \\ \hline 0 \end{array}$$

$$6 \div 12 = 18$$

$$65536 = 4cc \quad 128)13824(108$$

15. Two Country-women, *A* and *B*, carry together 100 Eggs to Market; and in the Sale of them, one took as much Money as the other. But *A*, who had the largest Eggs, says to *B*, had I carried as many Eggs as you, I should have had 18 Pence for them. But, replies *B*, if I had brought as

as many as you, I should have had but 8 Pence for them. How many Eggs had each?

Numerically.

x , the Eggs A had.

$100 - x$, the Eggs B had.

$$\text{As } 100 - x : 18 :: x : \frac{18x}{100 - x}$$

$$\text{As } x : 8 :: 100 - x : \frac{800 - 8x}{x}$$

$$\text{Then } \frac{18x}{100 - x} = \frac{800 - 8x}{x} \quad \frac{800 - 8x}{100 - x}$$

$$\therefore 18xx = 8000 - 1600x + 8xx \quad \frac{80000 - 800x}{-800x + 8xx}$$

$$\therefore 10xx = 80000 - 1600x \quad -800x + 8xx$$

$$\therefore 10xx + 1600x = 80000$$

$$\therefore xx + 160x = 8000$$

$$\therefore xx + 160x + 6400 = 14400$$

$$\therefore x + 80 = \sqrt{14400}$$

$$\therefore x = -80 + \sqrt{14400} = 40$$

A had 40 Eggs, and B 60.

$$\text{As } 60 : 18 :: 40 : 12$$

$$\begin{array}{r} 40 \\ \hline 6 \overline{) 720} \\ \underline{60} \\ 12 \end{array}$$

$$\text{As } 40 : 8 : 60$$

$$\begin{array}{r} 60 \\ \hline 4 \overline{) 480} \\ \underline{40} \\ 12 \end{array}$$

Literally.

Literally.

$$\text{Let } 100 = a$$

$$18 = b$$

$$8 = c$$

$$\text{As } a - x : b :: x : \frac{bx}{a - x}$$

$$\text{Then } \frac{bx}{a - x} = \frac{ca - cx}{x} \quad \text{As } x : c :: a - x : \frac{ca - cx}{x}$$

$$\therefore bxx = caa - 2cax + cxx$$

$$\therefore bxx - cxx = caa - 2cax$$

$$\text{Let } b - c = d = 10$$

$$\text{Then } dxx = caa - 2cax$$

$$\therefore dxx + 2cax = caa$$

$$\therefore xx + \frac{2cax}{d} = \frac{caa}{d}$$

$$\therefore xx + \frac{2cax}{d} + \frac{caa}{dd} = \frac{caa}{dd} + \frac{caa}{d}$$

$$\therefore x + \frac{ca}{d} = \sqrt{\frac{caa}{dd} + \frac{caa}{d}}$$

$$\therefore x = -\frac{ca}{d} + \sqrt{\frac{caa}{dd} + \frac{caa}{d}} = 40$$

$$100 = a$$

$$100 = a$$

$$10000 = aa$$

$$64 = cc$$

$$dd = 100) 640000 (6400$$

$$\quad \quad \quad 8000$$

$$\sqrt{14400} (120$$

$$\quad \quad \quad 80$$

$$\quad \quad \quad 40$$

$$10000 = aa$$

$$8 = c$$

$$100 = a$$

$$8 = c$$

$$d = 10) 80000 (8000$$

$$0$$

16. Two Countrymen, *A* and *B*, sell their Corn at different Prices. *A* sold 20 *Busbels*, and *B* re-

B received for 1 Bushel, as many Shillings as he sold Bushels. *A* perceives, that if he had sold as many Bushels as *B* received Shillings, he should then have received 252 Shillings: But both together received 176 Shillings. How many Bushels did *B* sell, and what Price had *A*?

Numerically.

x , the Price of *B*'s Bushels.

$176 - x$, the Price of *A*'s.

As $176 - x : 252 :: 20 : x$

$x \quad 20$

88

88

Then $176x - xx = 5040$

704

$\therefore xx - 176x = -5040$

704

$\therefore xx - 176x + 7744 = 2704$

7744

$\therefore xx - 88 = \sqrt{2704}$

5040

$\therefore x = 88 + \sqrt{2704} = 140$

$\sqrt{2704}(52$

25 88

176

102)204 140

140

204

0

36, the Price of *B*'s Bushels. \therefore *B* had 6 Bush.

As $36 : 252 : 20 : 140$

20

36

5040

840

420

5040

Literally.

Let $20 = a$

x , the Price of *A*'s Bushels.

$252 = b$

$c - x$, the Price of *B*'s.

$176 = c$

Then, as $c - x : b :: a : x$

$$176 = c \quad \therefore cx - xx = ab$$

$$176 = c \quad \therefore xx - cx = -ab$$

$$1056 \quad \therefore xx - cx + \frac{cc}{4} = \frac{cc}{4} - ab$$

$$1232$$

$$176$$

$$4 \overline{) 30976} = cc \quad \therefore x - \frac{c}{2} = \frac{\sqrt{cc - ab}}{4} \quad \begin{array}{r} 252 = b \\ 20 = a \\ \hline 5040 = ab \end{array}$$

$$7744$$

$$25040$$

$$\sqrt{2704} 52 + 88 = 140$$

$$\therefore x = \frac{c}{2} + \frac{\sqrt{cc - ab}}{4} = 140 \quad 2 \overline{) 176 = c} \\ 88$$

17. Two Merchants sell 21 Ells of Cloth : The first sells one Ell for as many Crowns as is $\frac{1}{5}$ of the Number Ells that the second had ; and the second sells one Ell for as many Crowns as is $\frac{1}{3}$ of the Number of Ells that the first had. The Sale being over, they had taken 48 Crowns. How many Ells did each sell, and at what Price ?

Numerically.

x , the Number of Ells the first sold.

$21 - x$, the Number the second sold.

$$\text{As } 1 : \frac{21 - x}{5} :: x : \frac{21x - xx}{5} \quad \begin{array}{r} 48 \\ 15 \\ \hline \end{array}$$

$$\text{As } 1 : \frac{x}{3} :: 21 - x : \frac{21x - xx}{3} \quad \begin{array}{r} 240 \\ 48 \\ \hline \end{array}$$

$$\text{Then } \frac{21x - xx}{5} + \frac{21x - xx}{3} = 48 \quad 720$$

$$\therefore 63x - 3xx + 105x - 5xx = 720$$

$$\therefore 168x - 8xx = 720$$

$$\therefore 8xx - 168x = -720$$

$$\therefore xx - 21x = -90$$

$\therefore xx$

$$\therefore xx - 21x + \frac{441}{4} = \frac{441}{4} - 90 = \frac{81}{4}$$

$$\therefore x - \frac{21}{4} = \frac{\sqrt{81}}{4}$$

$$\therefore x = \frac{21}{2} + \frac{\sqrt{81}}{4} = 15, \text{ too great.}$$

$$\therefore x = \frac{21}{2} - \frac{\sqrt{81}}{4} = 6, \text{ the first's Ells.}$$

15, the second's Ells.

$$\frac{\sqrt{81}}{4} - \frac{9}{2} + \frac{21}{2} \text{ is } \frac{12}{2} = b$$

The Price of the first 18

The Price of the second 30

48

Literally.

Let $21 = a$

$48 = b$

x , the Number of Ells the first had.

$a - x$, the Number the second had.

$$\text{As } 1 : \frac{a-x}{5} :: x : \frac{ax-xx}{5}$$

$$\text{As } 1 : \frac{x}{3} :: a-x : \frac{ax-xx}{3}$$

$$\text{Then } \frac{ax-xx}{5} + \frac{ax-xx}{3} = b$$

$$\therefore 3ax - 3xx + 5ax - 5xx = 15b$$

$$\therefore 8ax - 8xx = 15b \quad 48 = b$$

$$\therefore ax - xx = \frac{15b}{8} \quad \frac{15}{240}$$

$$\therefore xx - ax = \frac{15b}{8} \quad \frac{48}{8|720}$$

$$\therefore xx - ax + \frac{aa}{4} = \frac{aa}{4} - \frac{15b}{8}$$

X x

$$21 = a$$

$$21 = a$$

$$21$$

$$42$$

$$441 = aa$$

$$4$$

$$\therefore x -$$

$$\therefore x - \frac{a}{2} = \frac{\sqrt{aa - 15b}}{4 - \frac{15b}{8}}$$

$$\therefore x = \frac{a}{2} + \frac{\sqrt{aa - 15b}}{4 - \frac{15b}{8}} = 15 \text{ too great.}$$

$$\therefore x = \frac{a}{2} - \frac{\sqrt{aa - 15b}}{4 - \frac{15b}{8}} = 6$$

18. Two Men have a Parcel of Silk, the first 40 Ells, the second 30. The first sells for a Guinea $\frac{1}{3}$ of an Ell more than the second. When the Sale was over, they had taken between them 42 Guineas. How many Ells did each of them sell for a Guinea?

Numerically.

x , the Number of Ells the first sold for a Guinea.

$\frac{3x-1}{3}$ the Number of Ells the second sold for a Guinea.

As $x :: 40 : \frac{40}{x}$ the Price of the first.

As $\frac{3x-1}{3} : 1 :: 90 : \frac{270}{3x-1}$ the Price of the second.

Then $\frac{40}{x} + \frac{270}{3x-1}$ is $\frac{120x-40+270x}{3xx-x}$

$$\therefore \frac{120x-40+270x}{3xx-x} = 42$$

$$\therefore 120x-40+270x = 126xx-42x$$

$$\therefore 390x-40 = 126xx-42x$$

$$\therefore 126xx-432x = -40$$

$$18)432(24$$

$$\therefore xx - \frac{432x}{126} = -\frac{40}{126}$$

$$\frac{72}{0}$$

$$\therefore xx - \frac{24x}{7} = -\frac{20}{63}$$

$$18)126(7$$

$$\therefore xx - \frac{24x}{7} + \frac{144}{49} = \frac{144}{49} - \frac{20}{63} = \frac{8092}{3087} = \frac{1156}{441}$$

$$\therefore x - \frac{12}{7} = \sqrt{\frac{1156}{441}}$$

$$\therefore x = \frac{12}{7} + \sqrt{\frac{1156}{441}} = 34$$

$$\begin{array}{r} 490 \\ 238 \overline{) 252} \end{array}$$

$$\begin{array}{r} 34 \\ 21 \overline{) 7} \end{array}$$

$$\begin{array}{r} 49 \\ 147 \overline{) 490} \end{array} \begin{array}{r} 3 \\ 147 \end{array} = 3$$

$$\begin{array}{r} 3 \\ 10 \overline{) 30} \end{array} \begin{array}{r} 3 \\ 10 \overline{) 120} \end{array} \begin{array}{r} 3 \\ 10 \overline{) 120} \end{array} \begin{array}{r} 3 \\ 10 \overline{) 120} \end{array}$$

$$\sqrt{1156} = 34$$

$$\begin{array}{r} 9 \\ 64 \overline{) 256} \end{array}$$

$$\sqrt{441} = 21$$

$$\begin{array}{r} 4 \\ 41 \overline{) 41} \end{array}$$

Literally.

$$\begin{array}{l} \text{Let } 40 = a \\ 90 = b \\ 42 = c \end{array}$$

$$\text{As } x : 1 :: a : \frac{a}{x}$$

$$\text{As } \frac{x^3 - 1}{3} : 1 :: b : \frac{3b}{3x - 1}$$

$$\frac{a}{x} + \frac{3b}{3x - 1} \text{ is } \frac{3ax - a + 3bx}{3xx - x}$$

$$\text{Then } \frac{3ax - a + 3bx}{3xx - x} =$$

$$\therefore 3ax - a + 3bx = 3cx - cx$$

$$\therefore 3cx - cx - 3bx - 3ax = -a$$

$$\text{Let } -c - 3b - 3a = -d = 432$$

$$\text{Then } 3cx - dx = -a$$

$$\therefore xx - \frac{dx}{3c} = -\frac{a}{3c}$$

$$\therefore xx - \frac{dx}{3c} + \frac{dd}{36cc} = \frac{dd}{36cc} - \frac{a}{3c}$$

$$42 = c$$

$$270 = 3b$$

$$120 = 3a$$

$$432 = d$$

$$432 = d$$

$$864$$

$$1296$$

$$1728$$

$$186624 = dd$$

$$\therefore x - \frac{d}{6c} = \sqrt{\frac{dd}{36cc} - \frac{a}{3c}} \quad \begin{array}{r} 42=c \\ 42=c \\ \hline 84 \end{array}$$

$$\therefore x = \frac{d}{6c} + \sqrt{\frac{dd}{36cc} - \frac{a}{3c}} = 3\frac{1}{3} \quad \begin{array}{r} 168 \\ 1764=cc \\ \hline 36 \end{array}$$

$$\frac{186624}{63504} = \frac{144}{49} - \frac{20}{63} = \frac{1156}{441} \quad \begin{array}{r} 10584 \\ 5292 \\ \hline 63504=36cc \end{array}$$

$$\sqrt{\frac{1156}{441}} \left| \frac{34}{21} + \frac{36}{21} = \frac{70}{21} = 3\frac{1}{3} \quad \begin{array}{r} 432 \\ 256 \\ \hline 21 \end{array} \right.$$

19. To find a *Number*, to the *Quadruple* of which, if you add 91, the Whole shall be, to the *Square* of the *Number* sought, as 3 to 4.

Numerically.

x , the *Number* sought.

Then, as $4x+91 : xx :: 3 : 4$

$$\therefore \frac{4}{16x+364} = \frac{xx}{3xx}$$

$$3xx - 16x = 364$$

$$\therefore xx - \frac{16x}{3} = \frac{364}{3}$$

$$\therefore xx - \frac{16x}{3} + \frac{64}{9} = \frac{64}{9} + \frac{364}{3} \quad \begin{array}{r} 14 \\ 14 \\ \hline 56 \end{array}$$

$$\sqrt{\frac{1156}{441}} \left| 34 \quad \therefore x - \frac{8}{3} = \sqrt{\frac{64}{9} + \frac{364}{3}} \quad \begin{array}{r} 14 \\ 196 \end{array} \right.$$

$$\begin{array}{r} 9 \\ 64 \overline{)256} \\ \underline{156} \end{array} \quad \therefore x = \frac{8}{3} + \sqrt{\frac{64}{9} + \frac{364}{3}} + 14$$

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$$\frac{64}{9} + \frac{364}{3} \text{ is } \frac{3468}{27} = \frac{1156}{9}$$

14

4

56

91

147

$$\sqrt[3]{\frac{1156}{9}} \left| \frac{34}{3} + \frac{8}{3} \text{ is } \frac{42}{3} = 14 \right.$$

$$\text{As } 147 : 196 :: 3 : 4$$

$$\frac{4}{588} = \frac{3}{588}$$

Literally.

$$\text{Let } 91=a \quad \text{As } cx+a : xx : b : c$$

$$3=b$$

$$4=c$$

$$\text{Then } \frac{c}{cx+ca} = \frac{b}{bxx}$$

$$\therefore bxx - ccx = ca$$

$$91$$

$$4$$

$$364$$

$$3$$

$$9=bb$$

$$4=c$$

$$\therefore xx - \frac{ccx}{b} = \frac{ca}{b}$$

$$\frac{4}{36}$$

$$=4bb$$

$$\frac{4}{16}$$

$$=cc$$

$$\therefore xx - \frac{ccx}{b} + \frac{cccc}{4bb} = \frac{cccc}{4bb} + \frac{ca}{b}$$

$$\frac{4}{64}$$

$$=ccc$$

$$\therefore x - \frac{cc}{2b} = \sqrt{\frac{cccc}{4bb} + \frac{ca}{b}}$$

$$cccc = \frac{256}{36} = \frac{64}{9}$$

$$4bb = \frac{36}{9}$$

$$\therefore x = \frac{cc}{2b} + \sqrt{\frac{cccc}{4bb} + \frac{ca}{b}} = 14$$

$$\frac{64}{9} + \frac{364}{3} \text{ is, as above, } \&c.$$

20. To find a *Number*, from the *Double* of which, if you subtract 12, the *Square* of the Remainder shall be 9 Times the *Number* sought, *less* 1.

Numerically.

x , the *Number* sought.

Then $4xx - 48x + 144 - 1 = 9x$

$$\begin{array}{r} 2x - 12 \\ 2x - 12 \\ \hline 4xx - 24x \\ -24x + 144 \\ \hline 4xx - 48x + 144 \end{array}$$

$$\begin{array}{r} \therefore 4xx - 48x + 143 = 9x \quad 57 \\ \therefore 4xx - 57x + 143 = 0 \quad 57 \\ \therefore 4xx - 57x = -143 \quad 399 \\ \therefore xx - \frac{57x}{4} = -\frac{143}{4} \quad 285 \\ \phantom{\frac{57x}{4}} = -\frac{143}{4} \quad 3249 \end{array}$$

$$\begin{array}{r} 3844 \\ 12996 \quad 9152 \quad 143 \\ \hline 3249 \quad 143 \\ 64 \quad 4 \quad 572 \\ \hline 256 \quad 9152 \end{array}$$

$$\therefore xx - \frac{57x}{4} + \frac{3249}{64} = \frac{3249}{64} - \frac{143}{4}$$

$$\therefore x - \frac{57}{8} = \frac{\sqrt{3844}}{256}$$

$$\sqrt{3844} \begin{array}{r} 62 \\ 36 \end{array} \quad \sqrt{256} \begin{array}{r} 16 \\ 1 \end{array}$$

$$\therefore x = \frac{57}{8} + \frac{\sqrt{3844}}{256} = 11$$

$$\begin{array}{r} 122 \overline{)244} \\ 244 \\ \hline 0 \end{array} \quad \begin{array}{r} 26 \overline{)156} \\ 156 \\ \hline 0 \end{array} \quad \frac{62}{16} \text{ is } \frac{31}{8} + \frac{57}{8} \text{ is } \frac{88}{8} = 11$$

$$\begin{array}{r} 11 \\ 9 \\ \hline 99 \end{array} \quad \begin{array}{r} 10 \\ 10 \\ \hline 100 - 1, \text{ is } 99 \end{array} \quad \begin{array}{r} 22 \\ 12 \\ \hline 10 \end{array}$$

Literally.

Then $4xx - 4ax + aa - b = cx$

$$\begin{array}{r} 12 = a \\ 1 = b \\ 9 = c \end{array} \quad \begin{array}{r} x \\ 2 \\ \hline 2x - a \\ 2x - a \\ \hline 4xx - 2ax \\ -2ax + aa \\ \hline 4xx - 4ax + aa \end{array}$$

$$\begin{array}{r} \therefore 4xx - 4ax - cx + aa - b = 0 \\ \therefore 4xx - 4ax - cx = b - aa \end{array}$$

Let $-4a - c = -d = -57$

Then $4xx - dx = b - aa$

$$\therefore xx - \frac{dx}{4} = \frac{b - aa}{4}$$

$$48 = 4a$$

$$\begin{aligned}
 48 &= 4a & 1 &= b \\
 9 &= c & -144 &= aa \\
 57 &= d & -143 & \\
 57 &= d & & \\
 dd &= \frac{3249}{64} = \frac{143}{4} \text{ \&c.} & \therefore x - \frac{d}{8} &= \sqrt{\frac{dd}{64} + \frac{b-aa}{4}} \\
 \text{as foregoing.} & & \therefore x &= \frac{d}{8} + \sqrt{\frac{dd}{64} + \frac{b-aa}{4}} = 11
 \end{aligned}$$

21. To divide the Number 19 into 2 Parts, so that the Sum of the Squares of both Parts may be 193.

Numerically.

$$\begin{array}{r}
 \boxed{x} \quad \boxed{19-x} \\
 \underline{19-x} \\
 361-19x \\
 -19x+xx \\
 \hline
 361-38x+xx
 \end{array}
 \begin{array}{r}
 19 \\
 19 \\
 171 \\
 19 \\
 \hline
 361
 \end{array}$$

$$\text{Then } xx+361-38x+xx=193 \quad \begin{array}{r} 25 \\ \hline 361 \quad 336 \end{array}$$

$$\therefore 2xx-38x+361=193$$

$$\therefore 2xx-38x=-168$$

$$\therefore xx-19x=-84$$

$$\begin{array}{r}
 361 \\
 \hline
 361-84 \\
 \hline
 4 \\
 \hline
 4
 \end{array}
 \begin{array}{r}
 19 \\
 19 \\
 171
 \end{array}$$

$$\therefore xx-19x+\frac{361}{4}=\frac{361}{4}-84=\frac{25}{4} \quad \begin{array}{r} 19 \\ \hline 361 \end{array}$$

$$\therefore x-\frac{19}{2}=\sqrt{\frac{25}{4}} \quad \sqrt{\frac{25}{4}}=\frac{5}{2} \quad \therefore \frac{5}{2}+\frac{19}{2} \text{ is } \frac{24}{2}=12$$

$$\therefore x=\frac{19}{2}+\sqrt{\frac{25}{4}}=12$$

$$\boxed{12} \quad \boxed{7}$$

$$\frac{12}{144} + \frac{7}{49} = 193$$

Literally.

$$\begin{aligned} \text{Let } 19 &= a \\ 193 &= b \end{aligned}$$

$$\begin{array}{r} \boxed{x} \quad \boxed{a-x} \\ a-x \\ \hline aa-2ax+xx \end{array}$$

$$\text{Then } xx + aa - 2ax + xx = b$$

$$\therefore 2xx + aa - 2ax = b$$

$$\therefore 2xx - 2ax = b - aa$$

$$\therefore xx - ax = \frac{b - aa}{2}$$

$$\therefore xx - ax + \frac{aa}{4} = \frac{aa}{4} + \frac{b - aa}{2}$$

$$\therefore x - \frac{a}{2} = \frac{\sqrt{aa}}{4} + \frac{b - aa}{2}$$

$$\therefore x = \frac{a}{2} + \frac{\sqrt{aa}}{4} + \frac{b - aa}{2} = 12$$

$$\frac{361}{4} = 84, \text{ \&c. as above.}$$

22. To divide 7 into two Parts, so that the Difference of the Squares made by the treble of the lesser, and double of the greater may be 17.

Numerically.

$$\begin{array}{r} \boxed{x} \quad \boxed{7-x} \\ 2 \quad \quad 3 \\ \hline 2x \quad 21-3x \\ 2x \quad 21-3x \\ \hline 4xx \quad 441-63x \\ \quad -63x+9xx \\ \hline 441-126x+9xx \end{array}$$

$$\begin{array}{r} 21 \\ 21 \\ \hline 21 \\ 42 \\ \hline 441 \end{array}$$

$$\begin{array}{r} 21 \\ 21 \\ \hline 21 \\ 42 \\ \hline 441 \end{array}$$

$$\begin{array}{r} 21 \\ 21 \\ \hline 21 \\ 42 \\ \hline 441 \end{array}$$

$$\begin{array}{r} 441-126x+9xx \\ -63x+9xx \\ \hline 441-126x+9xx \end{array}$$

$$\begin{array}{r} 441-126x+9xx \\ -63x+9xx \\ \hline 441-126x+9xx \end{array}$$

$$\begin{array}{r} 441-126x+9xx \\ -63x+9xx \\ \hline 441-126x+9xx \end{array}$$

Then

Quadratic Equations.

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Then $441 - 126x + 9xx - 4xx = 17$

$\therefore 5xx + 441 - 126x = 17$

$\therefore 5xx - 126x = -424$

$\therefore xx - \frac{126x}{5} = -\frac{424}{5}$

$\therefore xx - \frac{126x}{5} + \frac{3969}{25} = \frac{3969}{25} - \frac{424}{5}$

$\frac{9245}{19845} \quad \frac{10600}{3969} \quad \frac{424}{25} \quad \frac{5}{5}$

$\therefore x = \frac{63}{5} - \sqrt{\frac{1849}{25}} = 4$

$\frac{9245}{125} = \frac{1849}{25}$

$\sqrt{1849} = 43$

$\frac{16}{83} \quad \frac{249}{249} \quad 0$

$\frac{63}{5} - \frac{43}{5} \text{ is } \frac{20}{5} = 4$

$\begin{array}{r} 4 \\ 2 \\ 8 \\ 8 \\ 64 \end{array} \quad \begin{array}{r} 3 \\ 3 \\ 9 \\ 9 \\ 91 \end{array}$

$81 - 64 = 17$

Literally.

Let $7 = a$
 $17 = b$

$\begin{array}{r} x \\ 2 \\ 2x \\ 2x \\ 4xx \end{array} \quad \begin{array}{r} a-x \\ 3 \\ 3a-3x \\ 3a-3x \\ 9aa-18ax+9xx \end{array}$

Y y

Then

$$\text{Then } 9aa - 18ax + 9xx - 4xx = b$$

$$\therefore 9aa - 18ax + 5xx = b \quad 49 = aa \quad 7 = a$$

$$\therefore 5xx - 18ax = b - 9aa \quad \underline{9} \quad \underline{7 = a}$$

$$\therefore xx - \frac{18a}{5} = \frac{b - 9aa}{5} \quad \begin{array}{r} 441 = 9aa \\ 17 = b \\ \hline 424 \end{array} \quad \begin{array}{r} 49 \\ 81 \\ \hline 49 \end{array}$$

$$\begin{array}{r} 392 \\ \hline 3969 = 81aa \end{array}$$

$$\therefore xx - \frac{18a}{5} + \frac{81aa}{25} = \frac{81aa}{25} + \frac{b - 9aa}{5}$$

$$\therefore x - \frac{9a}{5} = \sqrt{\frac{81aa}{25} + \frac{b - 9aa}{5}}$$

$$\therefore x = \frac{9a}{5} + \sqrt{\frac{81aa}{25} + \frac{b - 9aa}{5}} = 4$$

$$\frac{3969}{25} - \frac{424}{5} \text{ is as afore going, \&c.}$$

23. A Man buys a Piece of *Linnen*, and by felling it again, he gains 12 *l.* — $\frac{1}{10}$ of what he bought it for; and finds by this Means, that he gained as much in the 100 as the *Linnen* cost him. What Price was the *Linnen* bought and sold at?

Numerically.

x , what the Piece cost.

Gained 12 — $\frac{x}{10}$ that is $\frac{120 - x}{10}$

Then, as $x : 100 :: \frac{120 - x}{10} : x$

$$\therefore xx = \frac{12000 - 100x}{10}$$

$$\therefore 10xx = 12000 - 100x$$

$$\therefore 10xx + 100x = 12000$$

$$\therefore xx + 10x = 1200$$

$$\therefore xx + 10x + 25 = 1225$$

$$\sqrt{1225} (35$$

$$\begin{array}{r} 9 \quad 5 \\ \hline \end{array}$$

$$9)325 \quad 30$$

$$\underline{325}$$

$$\underline{0}$$

$$\therefore x+5 = \sqrt[2]{1225}$$

$$\therefore x = -5 + \sqrt[2]{1225} = 30$$

$$12-3 = 9 \text{ l. gained.}$$

$$\text{As } 30 : 100 :: 9 : 30$$

$$\frac{30}{900} = \frac{9}{900}$$

Literally.

$$\text{Let } 12 = a$$

x , the Price of the Piece.

$$10 = b$$

$$100 = c$$

$a - \frac{x}{6}$ that is $\frac{ab-x}{b}$ gained.

$$\text{Then, as } x : c :: \frac{ab-x}{b} : x$$

$$\therefore xx = \frac{cab-cx}{b}$$

$$100 = bb$$

$$100 = c$$

$$\therefore bxx = cab - cx$$

$$4bb = 400$$

$$4|00|100|00 = cc$$

$$\therefore bxx + cx = cab$$

$$26 = 20|100 = c$$

$$\begin{array}{r} 25 \\ \sqrt{1225|35} \\ 5 \\ 30 \end{array}$$

$$\therefore xx + \frac{cx}{b} = ca$$

$$\therefore xx + \frac{cx}{b} + \frac{cc}{4bb} = \frac{cc}{4bb} + ca$$

$$\therefore x + \frac{c}{2b} = \sqrt{\frac{cc}{4bb} + ca}$$

$$\therefore x = -\frac{c}{2b} + \sqrt{\frac{cc}{4bb} + ca} = 30$$

24. A Man buys 18 Ells of Cloth, some Red, the rest Black. What he bought of each cost 40 Shillings. He paid for every Ell of Red, one

Y y 2

Shilling

Selling more, than for the Black. How many Ells of each Sort did he buy?

Numerically.

x , the Number of Ells Black.

$18-x$, the Number of Ells Red.

$$\text{As } x : 40 :: 1 : \frac{40}{x}$$

$$\text{As } 18-x : 40 :: 1 : \frac{40}{18-x}$$

$$\text{Then } \frac{40}{x} + 1 = \frac{40}{18-x}$$

$$\therefore \frac{40+x}{x} = \frac{40}{18-x}$$

$$\therefore 720-22x-xx=40x$$

$$\therefore 720-62x-xx=0$$

$$\therefore 720=xx+62x$$

$$\therefore xx+62x+961=1681$$

$$\therefore x+31=\sqrt{1681}$$

$$\therefore x=-31+\sqrt{1681}=10$$

$$\text{As } 10 : 40 :: 1 : 4$$

$$\text{As } 8 : 40 :: 1 : 5$$

$$\begin{array}{r} 40+x \\ 18-x \\ \hline 720+18x \\ -40x-xx \\ \hline 720-22x-xx \end{array}$$

$$31$$

$$31$$

$$31$$

$$93$$

$$961$$

$$2720$$

$$\sqrt{1681}(41$$

$$16$$

$$81)81$$

$$81$$

$$0$$

$$31$$

$$10$$

Literally.

$$\text{Let } 18=a \\ 40=b$$

$$\text{As } x : b :: 1 : \frac{b}{x}$$

$$\text{As } a-x : b :: 1 : \frac{5}{a-x}$$

Then

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$$\text{Then } \frac{b}{x} + 1 = \frac{b}{a-x}$$

$$80 = 2b$$

$$62 = c$$

$$18 = 2$$

$$62 = c$$

$$62$$

$$124$$

$$\therefore \frac{b+x}{x} = \frac{b}{a-x}$$

$$372$$

$$4 \overline{) 4844} = cc$$

$$\therefore ab + ax - bx - xx = bx$$

$$961$$

$$\therefore ab = xx + 2bx - ax$$

$$\text{Let } 2b - a = c = 62$$

$$2 \quad 720 = bb$$

$$\text{Then } xx + cx = ab$$

$$\sqrt{1681} (41$$

$$\therefore xx + cx + \frac{cc}{4} = \frac{cc}{4} + ab$$

$$31$$

$$10$$

$$\therefore x + \frac{c}{2} = \sqrt{\frac{cc}{4} + ab}$$

$$\therefore x = -\frac{c}{2} + \sqrt{\frac{cc}{4} + ab} = 10$$

25. A Man buys 120 lb. of Pepper, and as much Ginger, and received for a Crown, one Pound of Ginger, more than of Pepper; so that the whole Price of the Pepper, came to 6 Crowns more than the Price of the Ginger. How many Pounds of each did he buy for a Crown?

Numerically.

x , the Pounds of Pepper for a Crown.

$x+1$, the Pounds of Ginger for a Crown.

$$\text{As } x : 1 :: 120 : \frac{120}{x}$$

$$\text{As } x+1 : 1 :: 120 : \frac{120}{x+1}$$

$$\text{Then } \frac{120}{x+1} + 6 = \frac{120}{x}$$

$$\therefore 120x + 6xx - 6x = 120x + 120$$

$$\therefore 126x + 6xx = 120x + 120$$

$$\therefore bx$$

$$\therefore 6x + 6xx = 120$$

$$\therefore xx + x = 20$$

$$\therefore xx + x + \frac{1}{4} = \frac{1}{4} + 20$$

$$\therefore x + \frac{1}{2} = \sqrt{\frac{1}{4} + 20}$$

$$\therefore x = -\frac{1}{2} + \sqrt{\frac{1}{4} + 20} = 4$$

$$\text{As } 4 : 1 :: 120 : 30$$

$$\text{As } 5 : 1 :: 120 : 24$$

Literally.

$$\text{Let } 120 = a \\ 6 = b$$

$$\text{As } x : 1 :: a : \frac{a}{x}$$

$$\text{As } x+1 : 1 :: a : \frac{a}{x+1}$$

$$\text{Then } \frac{a}{x+1} + b = \frac{a}{x}$$

$$\therefore \frac{a + bx + b}{x+1} = \frac{a}{x}$$

$$\therefore ax + bxx + bx = ax + a$$

$$\therefore bxx + bx = a$$

$$\therefore xx + x = \frac{a}{b}$$

$$\therefore xx + x + \frac{1}{4} = \frac{1}{4} + \frac{a}{b} \quad b=6 \mid 120 = a$$

$$\therefore x + \frac{1}{2} = \sqrt{\frac{1}{4} + \frac{a}{b}} \quad \frac{1}{4} + 20, \text{ is as afore.}$$

$$\therefore x = -\frac{1}{2} + \sqrt{\frac{1}{4} + \frac{a}{b}} = 4$$

F I N I S.

ERRATA.

PAGE 3. line 31. read *it may be.*

Page 16. line last, read 65505.

Page 25. Example 4. read $3825 : 18 : 10\frac{1}{2}$.

Page 42. Example 6. read $2 : 12 : 11$

In the Table of equal Parts of a Stone, blot out $3 \dots \frac{1}{2}$

Page 58. line 7. Example 2. read $19 : 4$

Example 4. read
$$\begin{array}{r} 2) 5 : 7\frac{1}{2} \\ \hline 2 : 9\frac{1}{4} \end{array}$$

Page 66. line 2. read $3829 : 3 : 20$

Page 66. line 3. read $257 : 5 : 1$

Page 77. read 250 *Reas.* $1 : 4 : \frac{2}{3}$

line 4. Example 4. read $2 : 15 : 11\frac{2}{3}$

Page 86. Example 3. blot out $0\frac{1}{4}$

Page 88. line 9. Example 1. read $1g \dots \frac{1}{5}$

Page 93. line 7. read $\frac{1}{2}d$ is $\frac{3}{5} \text{ } \frac{2}{5}$

Page 101. line 9. Question 4. read $.02\frac{1}{2}$

line 10. Question 5. read $.04\frac{1}{2}$

Page 108. line 7. read $13 : 4$ *per Hund. Doz.*

Page 119. read Thus $\frac{2}{3}$ of $\frac{3}{4}$ is $\frac{6}{12}$

Page 120. line 9. read $7\frac{1}{2}$

Page 135. line last, read .836136666

Page 136. line 4. read .875520833

Page 145. line 6. at the End, read $= \frac{1}{400}$

line 9. read 131131

line 15. read 30620

Page 150. line last, read 1.93168

Page 163. line 10. Decimally, read 4

Page 166. line 5. Dec. read 36.8866

Page 170. line 9. Dec. read 3492

Page 175. line 14. Dec. read 20

Page 176. line 9. Dec. read 241.07139

$$\begin{array}{r} 4 \\ \hline .28556 \\ 28 \end{array}$$

Page 197. line 26. read *April 30*

Page 198. line 14. after *July 31.* read *August 31*

Page 200. line 13. read 346.9375

Page 216. line 5. read 2000

Page 223. line 7. read 822557

Page 228. line 9. read 1500

Page 250. line 3. Quest. 6. Numerically, read 322

line 4. *disto*, read $16x$

Page 253. line 13. read $c + a - bb$

Page 256. line 4. read $108 = ac$

line 7. Numerically, read $x = \sqrt[2]{1024} = 32$

Page 261. line 2, at the End, read 36

line 3. Quest. 18. Num. read $+\frac{32x - 2x}{3}$

16. k104

1911.007

Page 262. line 5. Literally, read $\frac{3x+4a-4x}{6}$

Page 265. line 6. Lit. read $bca = 1050$
line 8. ditto, read $x + bca - ba = bcx$

Page 269. line 5. Nnm. read $100 - y = y$

Page 272. line 5. Lit. read $4 = bb$

line 8. ditto, read $ba = bb + bbb - b$

Page 273. line 7. read $y = \sqrt{144}$

line 6. Lit. read $12 = y$

Page 274. line 10. Lit. read $2400 = 2a - y$

Page 275. line last, read $bb - cc = 5$, and blot out $= bb - cc$ after 23

Page 285. line 16. read $y = 102 - 3x - x = 10$

Page 290. in 2d Step, read $907 = 2$

Page 297. line 1. Quest. 3. read $50 = u$

Page 299. line 3. at the End, read Price.

Page 300. line 10. read $d = \frac{u-a}{n-1} = 11$

line 13. Geo. Pro. read $16 : 8 : 4$

Page 315. line 7. read $x + a = \sqrt{\frac{aa}{4} + bb}$

line 6. Case 3. read $\frac{x-a}{2} = \sqrt{\frac{aa}{4} + bb}$

Page 317. line 3. Quest. 2. Lit. read $x + a = \sqrt{\frac{aa}{4} + b}$

line last. read 22.

Page 318. line 3. Quest. 3. Lit. read $\frac{x+b}{2} = \sqrt{\frac{aa}{4} + b}$

Page 321. line 3. Quest. 6. Lit. read $\frac{175}{x-2}$

Page 323. line 3. Lit. read bx

Page 327. line last, read $xx + \frac{7x}{2} + \frac{49}{16} =$

Page 339. line 3. read $x - \frac{21}{2} = \sqrt{\frac{81}{4}}$

line 10. Lit. read $xx - ax = -\frac{15b}{8}$

Page 341. line 3. read $x = \frac{12}{7} + \sqrt{\frac{1156}{41}} = 3\frac{2}{3}$

line 2. Lit. read As $\frac{3x-1}{3} : 1 :: b - \frac{3b}{3x-1}$

line 4. Lit. read Then $\frac{3ax-a+3bx}{3xx-x} = 0$

Page 345. line 6. read $dd = \frac{3249}{64} - \frac{143}{4}$



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